

# MULTI-PARAMETER MTS AND SED CRITERION APPLIED ON A CRACK IN AN ALKALI-ACTIVATED CONCRETE SPECIMEN UNDER MIXED-MODE LOADING

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The work is devoted to an analysis of a **crack propagation** in a **semi-circular concrete specimen** under **three-point bending**. **I-II mixed-mode** loading is ensured via an inclined crack. **Crack deflection angle** is investigated by means of the **generalized/multi-parameter form of the Maximum Tangential Stress (MTS)** criterion and **Strain Energy Density (SED)** criterion as well as **experimentally**. The multi-parameter fracture mechanics concept applied in this work is based on the approximation of the crack-tip stress field by means of the **Williams series expansion (WE)**. It has been proved that this approach can help to describe crack behaviour in some kinds of materials better than the classical (one-parameter: stress intensity factor) fracture mechanics. This conclusion is discussed also in this paper.

## Methodology

The stress tensor components were firstly expressed as a **series expansion by Williams**:

$$\sigma_{ij} = \sum_{n=1}^{\infty} \frac{n}{2} r^{\frac{n}{2}-1} f_{ij}(n, \theta) A_n + \sum_{m=1}^{\infty} \frac{m}{2} r^{\frac{m}{2}-1} g_{ij}(m, \theta) B_m$$

where:

$i, j$  – stress tensor components indexes:  $i, j \in \{x, y\}$ ,  
 $r, \theta$  – polar coordinates considering the centre of the system at the crack tip,  
 $f_{ij}, g_{ij}$  – known stress functions corresponding to the loading mode I and II, respectively,  
 $A_n, B_m$  – unknown coefficients of the higher-order terms of the Williams expansion (WE)

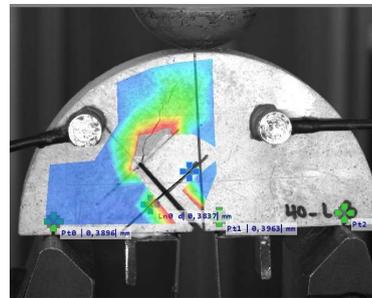
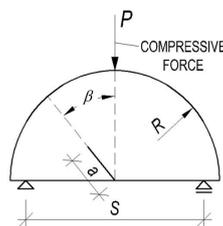
For determination of the WE coefficients, the **over-deterministic method (ODM)** has been applied.

When the WE coefficients are known, an arbitrary stress component can be expressed via multi-parameter power series. In this work, the **maximum tangential stress (MTS) criterion as well as strain energy density (SED) criterion** were applied for determination of further crack propagation angles. The searching for maximum of tangential stress and/or minimum for strain energy density were programmed numerically.

## SCB specimen

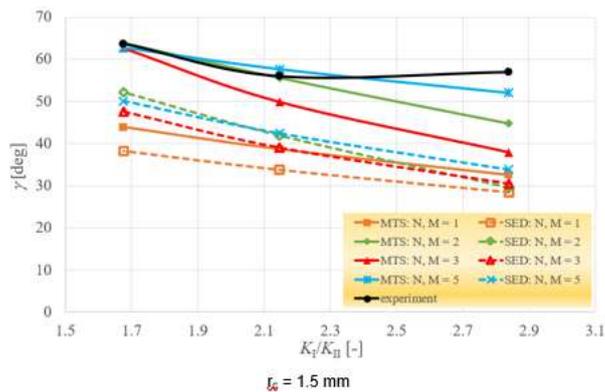
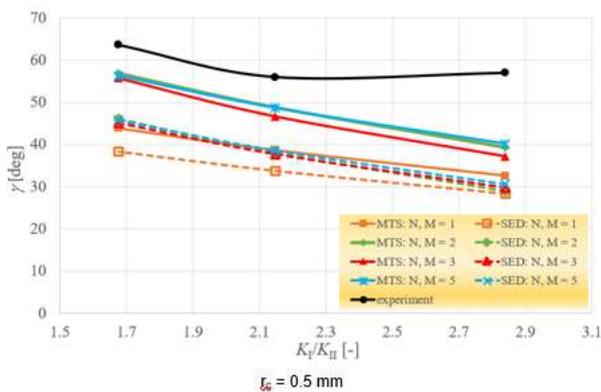
The following configurations were selected for comparison of the experimental and numerical results:

- radius of the disc,  $R = 50$  mm;
- crack length,  $a = 25$  mm (i.e. relative crack length  $\alpha=0.5$ );
- crack inclination angle,  $\beta = 30, 40$  and  $50^\circ$ ;
- span between the supports,  $S = 80$  mm.
- $E = 35$  GPa,  $\nu = 0.23$



## Selected results

Values of the estimated crack deflection angle for the relative crack length,  $a/R = 0.5$ : comparison between the angles  $\gamma$  observed experimentally and values calculated via multi-parameter MTS/SED criterion at critical distances  $r_c = 0.5$  and  $1.5$  mm for various initial crack inclination angles  $\beta$  ( $30, 40$  and  $50^\circ$ ) and the span distance  $S = 80$  mm considering 1, 2, 3 and 5 initial terms of the Williams expansion used for stress tensor components approximation.



## Summary

**Crack deflection angles** have been investigated on an **alkali-activated composite** specimen. Particularly, a **semi-circular disc** with an **inclined crack** was subjected to **three-point bending** experimentally as well as a numerical analysis was performed. Angle of the further crack propagation was estimated by means of the **multi-parameter MTS and SED criterion**. The parametric study was performed via considering **various critical distances** for application of the fracture criteria and **various numbers of the initial terms of the WE** considered for stress tensor components approximation. The results show that **MTS criterion is more suitable** for assessment of the crack propagation than SED criterion. **Additional recommendations** for more precise estimation of the crack path **are larger critical distances** where the fracture criterion is applied and/or assuming **more initial terms of the WE**, because the first singular parameter is not sufficient for this analysis when quasi-brittle materials are treated.