Dear colleagues.

In my presentation I will deal with problem

of STRUCTURAL MODAL MODIFICATION OF NON-PROPORTIONALLY DAMPED SYSTEM.

In the design of a structure which must vibrate within acceptable levels during operation, a suitable concept must be chosen. For example, based on numeric analysis and optimization of individual structural components, it is possible to create a real structure which satisfies the operational conditions set upon it. But in general, the real structure partially exhibits differing properties than those predicted computationally. It is therefore necessary to modify critical structural elements in order to fulfil the desired properties.

This operation can be performed through modal synthesis, where the original structure (usualy experimentally identified) is modified by an additional component (usually analytically defined) . This approach combines the modal properties of the real structure obtained through measurements and the modal properties of additional components obtained computationally. Through optimization of the additional components it is possible to obtain the desired properties of a modified structure while reducing the computational requirements and increasing accuracy of the results.

The procedure of structural modal modification can be explained throw next steps.

The original structure is represented by the proportionally damped system whose eigenvalue solution has the following form:

$$\left(\boldsymbol{K}_0 - \boldsymbol{M}_0\omega_{0j}^2\right)\boldsymbol{v}_{0j} = \boldsymbol{0} \tag{1}$$

where stiffness and mass parameters are represented by coefficient matrices $\boldsymbol{K}_0$ and $\boldsymbol{M}_0$ and jth eigen mode and natural frequency for this mode by terms $\boldsymbol{v}_{0j}$ and ω_{0j}.
Following equations describe conditions for orthonormality.

$$\boldsymbol{V}_0^T\boldsymbol{K}_0\boldsymbol{V}_0 = \boldsymbol{\Omega}_0^2,\ \boldsymbol{V}_0^T\boldsymbol{M}_0\boldsymbol{V}_0 = \boldsymbol{I},\ \boldsymbol{V}_0^T\boldsymbol{B}_0\boldsymbol{V}_0 = 2\boldsymbol{\Delta} = 2(\alpha\boldsymbol{I} + \beta\boldsymbol{\Omega}_0^2) \tag{2}$$

$$\delta_j = \xi\omega_{0j} \tag{3}$$

The symbol ξ introduces damping ratio of the structure.

Because modifying substructures are not located evenly along the original structure, the resulting structure can be interpreted as a disproportionally damped system of 2n dimensional space represented by the coefficient matrices N and P. The vibration of this system can be described by the following second order differential equation:

$$\boldsymbol{N}\dot{\boldsymbol{x}} - \boldsymbol{P}\boldsymbol{x} = \boldsymbol{r} \tag{4}$$

$$\boldsymbol{P} = \begin{bmatrix} -\boldsymbol{K} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{M} \end{bmatrix},\ \boldsymbol{N} = \begin{bmatrix} \boldsymbol{B} & \boldsymbol{M} \\ \boldsymbol{M} & \boldsymbol{0} \end{bmatrix},\ \boldsymbol{x} = \begin{bmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \end{bmatrix}, \boldsymbol{r} = \begin{bmatrix} \boldsymbol{f} \\ \boldsymbol{0} \end{bmatrix} \tag{5}$$

where coefficient matrices are represented by $\boldsymbol{K}, \boldsymbol{M}$ and $\boldsymbol{B}$ and the variables $\boldsymbol{q}$, $\boldsymbol{f}$ express generalised displacement vector and the excitation force.

Eigenvalue problem solution for this system is introduced here:

$$\left(\boldsymbol{P} - s_j\boldsymbol{N}\right)\boldsymbol{w}_j = \boldsymbol{0}, \left(\boldsymbol{K} + s_j\boldsymbol{B} + s_j^2\boldsymbol{M}\right)\boldsymbol{v}_j = \boldsymbol{0} \tag{7}$$

where $\boldsymbol{v}_j$ interpret *j*-th eigen mode and imaginary part of its eigenvalue s_j express the angular frequency of damped system ω_{Dj}.

The following equations represents conditions for orthonormality.

$$\boldsymbol{W^T P\, W = S} \qquad\qquad \boldsymbol{W^T N\, W = I}$$
$$\boldsymbol{S\, V^T M\, V\, S\; - V^T K\, V = \; S} \qquad\qquad \boldsymbol{V^T B\, V + V^T M\, V\, S + S\, V^T\; M\, V = I} \qquad (8)$$

Where $\boldsymbol{W}$ and $\boldsymbol{V}$ are modal matrices and $\boldsymbol{S}$ represents the spectral matrix. These matrices can be expressed by relations:

$$\boldsymbol{W} = \{\boldsymbol{w}_j\} = \begin{bmatrix}\boldsymbol{V}\\ \boldsymbol{V\,S}\end{bmatrix},\; \boldsymbol{V} = \; \{\boldsymbol{v}_j\},\; \boldsymbol{S} = diag(s_j) \qquad (9)$$

Relations (8) and (9) can be used to determine modal-spectral parameters of the resulting structure using original structure modal matrices $\boldsymbol{V}_0$, $\boldsymbol{\Omega}_O$, $2\boldsymbol{\Delta}$ and coefficient matrices of the modifying structure $\boldsymbol{M}_N, \boldsymbol{B}_N, \boldsymbol{K}_N$.

This process represents the modification of original structure defined with $\boldsymbol{\Omega}_{0O}^2\; \boldsymbol{V}_0$, through added substructure $\boldsymbol{M}_A, \boldsymbol{B}_A, \boldsymbol{K}_A$ to resulting structure $\boldsymbol{T_L}, \boldsymbol{S}$ and can be determined by the solution of following eigenvalue problem:

Condition for orthogonality then can be formed as:

$$[\boldsymbol{T}_L^T\; \boldsymbol{S}\;\; \boldsymbol{T}_L^T]\begin{bmatrix}-(\boldsymbol{\Omega}_{0O}^2 + \boldsymbol{V}_0^T\boldsymbol{K}_A\boldsymbol{V}_0) & 0\\ 0 & \boldsymbol{I} + \boldsymbol{V}_0^T\boldsymbol{M}_A\boldsymbol{V}_0\end{bmatrix}\begin{bmatrix}\boldsymbol{T_L}\\ \boldsymbol{T_L\,S}\end{bmatrix} = \boldsymbol{S}$$

$$[\boldsymbol{T}_L^T\; \boldsymbol{S}\;\; \boldsymbol{T}_L^T]\begin{bmatrix}2\Delta_P + \boldsymbol{V}_0^T\boldsymbol{B}_A\boldsymbol{V}_0 & \boldsymbol{I} + \boldsymbol{V}_0^T\boldsymbol{M}_A\boldsymbol{V}_0\\ \boldsymbol{I} + \boldsymbol{V}_0^T\boldsymbol{M}_A\boldsymbol{V}_0 & 0\end{bmatrix}\begin{bmatrix}\boldsymbol{T_L}\\ \boldsymbol{T_L\,S}\end{bmatrix} = \boldsymbol{I} \qquad (10)$$

where properties of original $\boldsymbol{\Omega}_{0O}^2\; \boldsymbol{V}_0$, added substructure matrices $\boldsymbol{M}_A, \boldsymbol{B}_A, \boldsymbol{K}_A$ and resulting structure $\boldsymbol{T_L}, \boldsymbol{S}$ can be determined by the solution of following eigenvalue problem:

$$(\boldsymbol{P}_T - s_j\boldsymbol{N}_T)\boldsymbol{w}_{Tj} = \boldsymbol{0} \qquad (11)$$

The transformation matrix and then the modal matrix can be figured out from equations (10) and (11).

Modal synthesis of the non-proportionately damped layered beam

The above mentioned modal synthesis method can be used also in determining the modal and spectral properties of beam structures with added layers of vibroisolation at specific points. A very simple illustration of such a system can be explained on an existing cantilever beam with a connected (added) beam which creates the modified beam seen in Fig. 1. also shows the schematic illustration of the above mentioned modal synthesis methodology.
Because dimensions of original structure matrices are usually different then coefficient matrices of the modifying structure needs reduction methods.

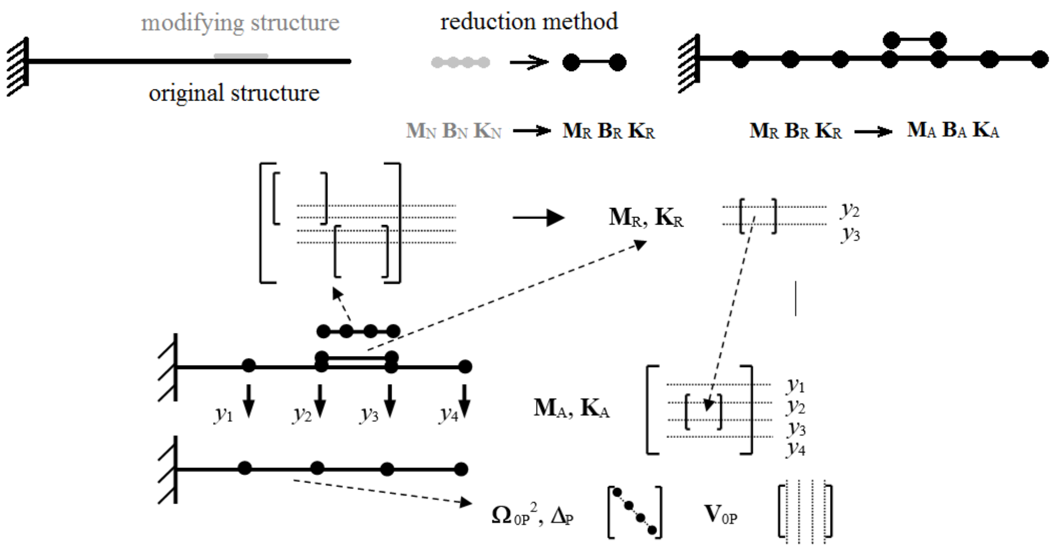

Fig. 1: Principle of modal synthesis of an original and modifying structure.

In conclusion, the method of modal synthesis provides modal and spectral parameters determination of the resulting structure that consist of the original structure interpreted as a proportionally damped system and the modifying substructure described by its modal parameters. The above mentioned modal synthesis method can be used to modify modal and spectral properties of beam structures with added vibroisolating layers to appropriate locations. This situation can be explained on a simple steel beam with an added beam with aluminium foam properties. The presented method can be automated and used for the parameter optimization of vibroisolating layers (orientation, geometry, material properties, etc...). The schematic representation of the optimizing position a and thickness h of the vibroisolating layer with respect to the maximum damping ratio ξ in the second Eigen mode is shown in Fig. 2.

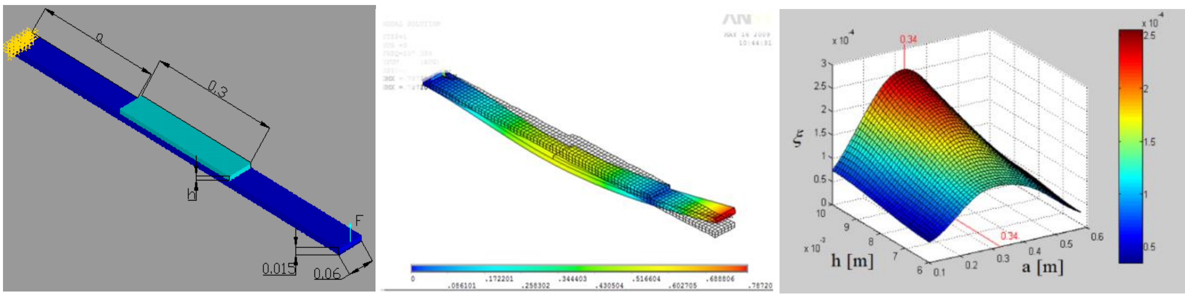

Fig. 2: Optimisation of position "a" and thickness "h" of the vibro-isolating layer with respect to the maximum ratio of damping "ξ"

It is possible to achieve the desired damping of some Eigen modes by choosing the appropriate position and thickness of the vibroisolating layer.

The above mentioned modal synthesis method can be used to modify modal and spectral properties of for example beam structures with added vibroisolating layers to appropriate locations.

Thank you for you attentaion.