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MODELLING AND ANALYSING OF THE TRANSVERSE VIBRATIONS OF THE TRACTOR TRAILER

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Summary: *Among important dynamic problems of the tractor-trailer unit are traverse vibrations of the trailer. The problems is so important because it concerns not only trailers of tractors, but also other units which can move quicker, and therefore the vibrations can be very dangerous. The important task of this analysis was the estimation of the effects of parameter variations.*

This investigation showed that stability of the system is determined mostly by the damping of the coupling. The roots lines of the equations for many various damping and velocities were therefore plotted. The optimum damping was found. At the end of this paper, as a result of investigation, the active control system was introduced. In the solution of this problem, the damping was changing according to the travelling speed. The block diagram of this follow-up control was presented.

1. Introduction

Many experiences, and also model testing, show that shaking vibrations of the tractor's trailer mostly depend on stiffness of the type, as well as the coupling tractor trailer. As it is proved, the vibrations do not arise as the result of kinematics input from road, but as the result of self-excited

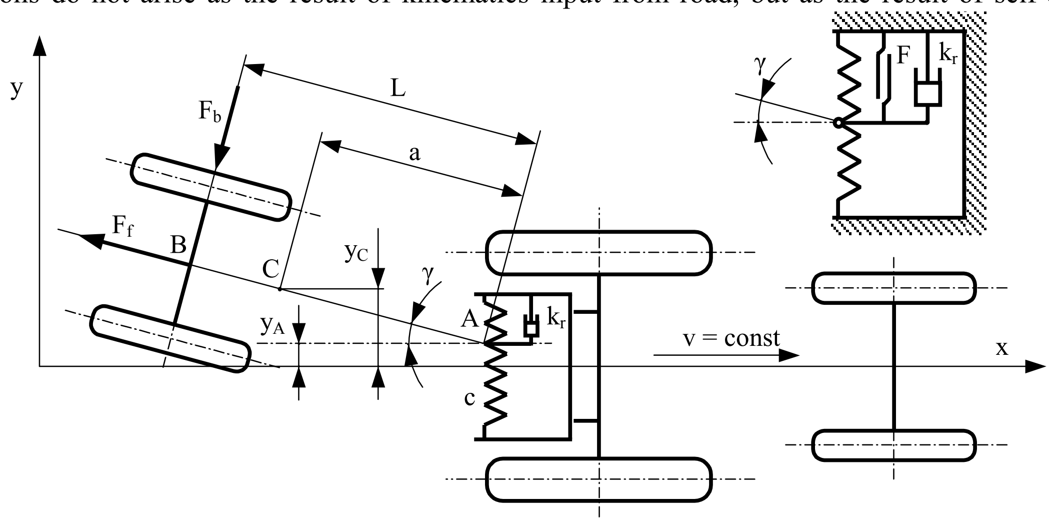

Fig.1. Two degree of freedom model of a trailer

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processes. At the beginning a simple two degrees of freedom model has been used, Fig.1.

In order to examine all the effects, which occur during the motion, it was assumed that the unit is moving along a straight smooth and horizontal road with the constant speed v.

This simple model can estimate the influence of all the important parameters on vibrations of the trailer.

Assuming $\sin\gamma = \gamma$, and $\cos\gamma = 1$ the equation of motion is as follows:

$$\begin{aligned} & m(\ddot{y}_A + a\ddot{\gamma}) = -k_r y_A - c y_A - f_r F_f + F_f \gamma - F_b \\ & am\ddot{y}_A + J_A \ddot{\gamma} = -LF_b \end{aligned} \tag{1}$$

The unknown transverse force can be obtained from the form, [1];

$$F_b = k_y \Psi \text{, and } \Psi = \gamma + \frac{\dot{y}_A + L\dot{\gamma}}{v} \tag{2}$$

A stability condition can be investigated from the characteristic equation, Eq. (3):

$$\begin{vmatrix} m\lambda^2 + \left(k_r + \frac{k_y}{v}\right)s + c & ams^2 + \frac{k_y L}{v}s + \left(k_y - F_f\right) \\ ams^2 + \frac{k_y L}{v}s & J_A s^2 + \frac{k_y L^2}{v}s + k_y L \end{vmatrix} = 0 \tag{3}$$

However, for the simplicity, a model with one degree of freedom was introduced. The equation of this model shows the effect of coupling on dynamics of this system, equally well.

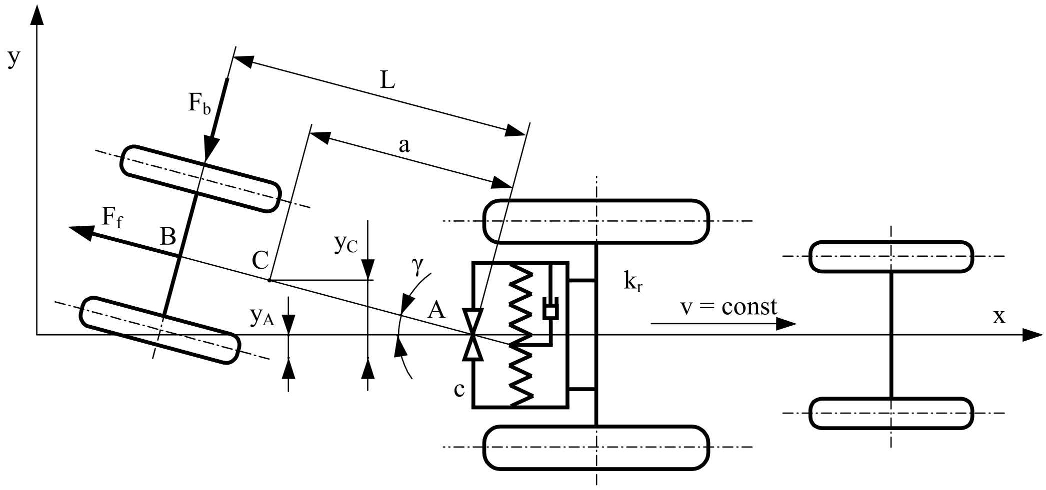

Fig.2. One degree of freedom model of a trailer

2. Different equation and solution

Let us consider the model of the connection, which is illustrated in Fig. 1. We can see that co-ordinate y_a has lower influence on the displacement of a back axle. Consequently, a model can be considerate with one degree of freed, Fig. 2.

The model presented above is simpler. It is only one equation of motion and due to that the influence of the parameters of a coupling on vibrations is clearer. Equation of motion has got the given form:

$$J_A\ddot{\gamma} + \left(k_r b^2 + \frac{k_y L^2}{v}\right)\dot{\gamma} + \left(cb^2 + k_y L\right)\gamma = 0 \tag{4}$$

where as we know $F_b = k_y \psi$ and $\psi = \gamma + \frac{\dot{y}_A + L\dot{\gamma}}{v}$

Equation (4) can be written as:

$$a_2\lambda^2 + a_1\lambda + a_0 = 0 \tag{5}$$

where $a_2 = J_A$

$a_1 = k_t b^2 + k_y L^2 / v$

$a_0 = cb^2 + k_y L$

The equation (5) has a stable solution (Hurwitzs Law) if:

$$a_1 > 0 \text{ and } a_1 a_2 > 0 \tag{6}$$

As we can see, this equation has always stable solution, unless: $v < 0$ or $k_r < 0$

Denotes

$$\frac{\left[k_r b^2 + \frac{k_y L^2}{v}\right]}{J_A} = 2h$$

$$\frac{\left(cb^2 + k_y L\right)}{J_A} = \omega^2 \tag{7}$$

equation (4) has another form:

$$\ddot{\gamma} + 2h\dot{\gamma} + \omega^2 = 0 \tag{8}$$

and its characteristic equation:

$$\lambda^2 + 2h\lambda + \omega^2 = 0 \tag{9}$$

The roots of characteristic equation:

$$\lambda_{1,2} = -h \pm \sqrt{h^2 - \omega^2} \tag{10}$$

if $h^2 \geq \omega^2$ - real roots,

$h^2 < \omega^2$ - complex roots.

It is important to remark that for complex roots a trailer perform a harmonic damped vibration, while for the real roots, that is to say for over critical damping, a exponential damped motion has been done. The boundary between these two kinds of vibration is when $h = \omega$.

$$k_r b^2 + \frac{k_y L^2}{v} = 2\sqrt{\left(cb^2 + k_y L^2\right)J_A} \tag{11}$$

The formula (11) contains a construction and operation parameter of a coupling.

As a user, the trailer ought to go quick to the state of equilibrium. The main interest of this investigation is therefore to find out such parameters, which will assure this movement.

From (11) we can express the critical velocity v_c

$$v_c = \frac{k_y L^2}{2\sqrt{(cb + k_y L^2)J_A} - k_r b^2} \quad (12)$$

It is the highest velocity, which is giving an exponential damped motion. It can been also seen that damping has bigger influence on the v_c than stiffness, due to that, this last is under square root.

Because the influence of damping on vibration is considerable, it is reasonable to draw out k_c from Eq. (11) – it will be called the critical damping:

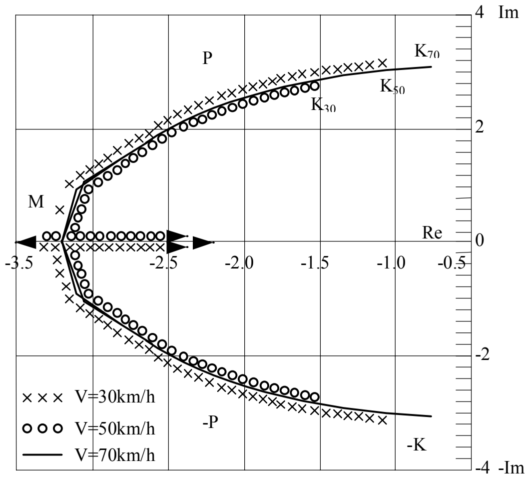

Fig. 3. The roots lines for different values of damping

$$k_c = \frac{2\sqrt{(cb^2 + k_y L)J_A} - \frac{k_y L^2}{v}}{b^2} \quad (13)$$

The upper formula is very important. It describes the relation between the input v and output k_c. That means that it is possible to build the follows-up control system, where k_c would be followed by speed. Let us introduce the complex plane and the roots lines. By observing the roots lines we obtain the full information about the vibrations.

However, for this investigation we have to take a data from a case study [1], Tab. 1.

Table 1. Parameter values assumed for the calculation

m	J	k_y	k_r	c	L	a	b
kg	kg·m^2	N·deg^{-1}	N·s·m^{-1}	N·m^{-1}	m	m	m
3522	3453	80 000	0.5·10^4	58 8860	2.50	2.15	0.20

At the beginning let start to investigate Eq. (12). Results are presented in Table 2.

As we can find out from Table 2, the effect of stiffness on the system is slender. Therefore the stiffness of a coupling can posses the only such value which is really needed for proper construction of the coupling. Let consider now the roots lines for different values of damping.

Table.2. Dependence of stiffness on the velocity v_c.

c	v; km·h^{-1}	
N·m-1	kr = 0; N·s·m-1	kr = 500 000; N·s·m-1
0,0	14,3260	17,0381
40 000	14,2691	16,9576
60 000	14,2401	16,9178
70 000	14,2268	16,8976

Substitution the date to Eq. (12) we obtain the results which as shown in Fig. 3.

As we can see, the roots lines are common for all velocities. The curve starts from point **K** where damping $k_r = 0$, through the point until **P** until **M**.

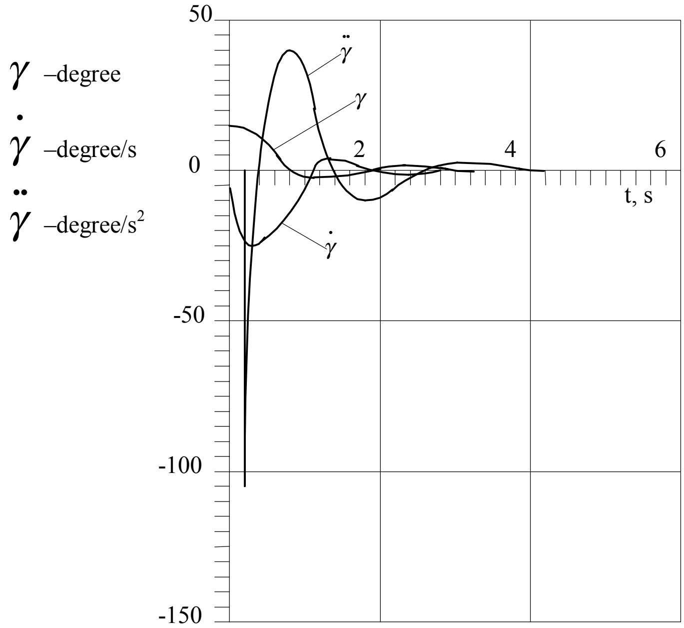

Fig. 4. Vibration of the trailer for the point **K** (v = 30 km/h; k_r = 0,0; c = 0,0)

Let's examine, for example, the curve for $v = 30\ km/h$. For the point **K** the process was finished after 5 s, then 4,5 s for **P**, and 3 s for the point **M**, respectively, Fig. 3, 4 and 5. For the above vibrations the initial angle deflection was 10º.

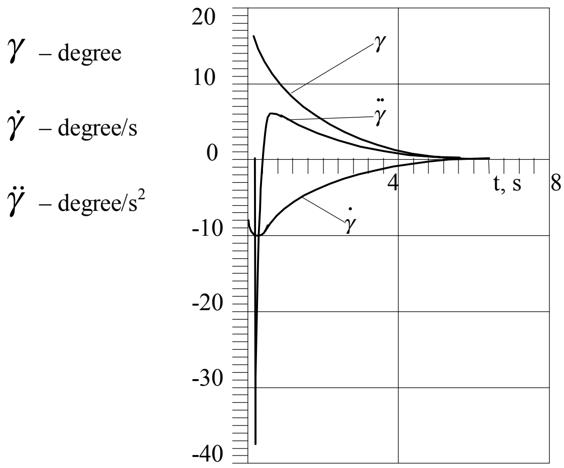

Fig. 5. Vibration of the trailer for the point **P** (v = 30 km/h; k_r = 500,0; c = 0,0)

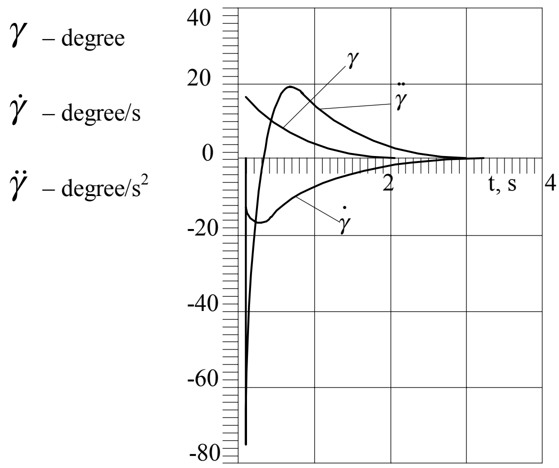

Fig. 6. Vibration of the trailer for the point **M** (v = 30 km/h; k_r = 1600,0; c = 0,0)

On Fig. 7 the dependence of Imλ from velocities is presented. As we can see from this graph is possible to find the critical damping k_r.

According to the results, there is the question about the values of damping. The best solution is the active control system. In that solution the relation between v_c and k_c for Eq.(13) would be still remained. Consequently the coupling tractor-trailer will be working near the roots M, Fig. 3. Acceptable example of this solution is presented in Fig. 8.

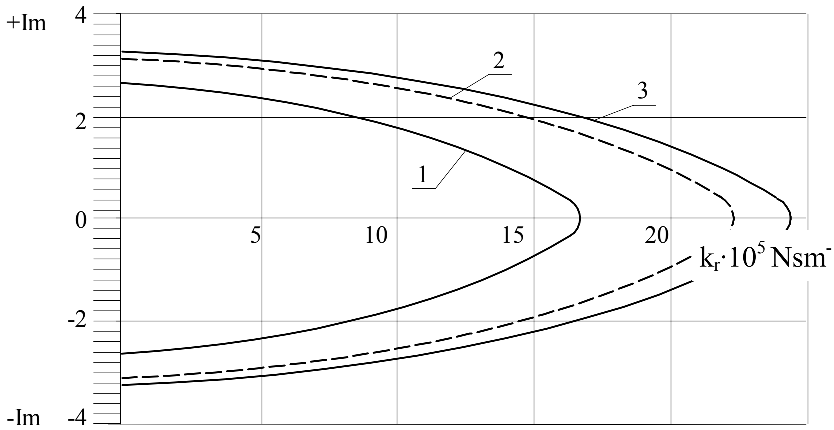

Fig. 7. Dependence Im from different velocities; 1 – v =30 km/h; 2 - 50 km/h; 3 v = 70 km/h

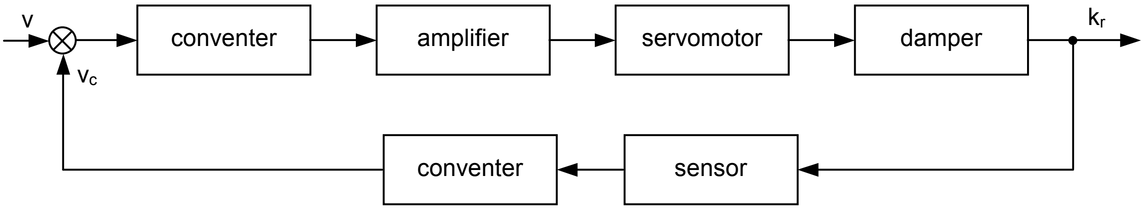

Fig. 8. The block diagram of a feedback follows-up control

Conclusions

- As it was proved, the vibrations of a unit tractor-trailer are strongly determined by the proper selection of the parameters of the coupling.
- The investigation of the roots lines showed that dynamics system of this unit is always stable unless $v < 0$.
- Damping possesses the biggest influence on vibrations. By the proper selection of it we can affect the time of equilibrium.
- The best solution is the active control system where the damping matches the speed.

Notation

m	body mass	kg	J	moment of inertia	$kg \cdot m^2$
c	spring stiffness	$N \cdot m^{-1}$	k_r	damper rate	$N \cdot s \cdot m^{-1}$
k_y	resisting coefficient	$N \cdot deg^{-1}$	v	vehicle speed	$m \cdot s^{-1}$
F_t	transverse force	N	F_f	axial force	N

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