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ON LATERAL PRESSURE AT REST OF GRANULAR MATERIALS

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Summary: *The paper concentrates on one of the three main problems of the lateral pressure theory of granular materials, i.e. the pressure at rest. Both the proposed view at the pressure at rest and the analysis procedure are based on the General Lateral Pressure (GLP) theory. The paper contains also more detailed theoretical explanation of the problem and some experimental results and their analysis.*

1 INTRODUCTION

The present conventional theory of earth pressure contains several discrepancies, which are more or less known or obvious, but have not received due attention either in theory or in practice. The fundamental objections to this theory include the following points:

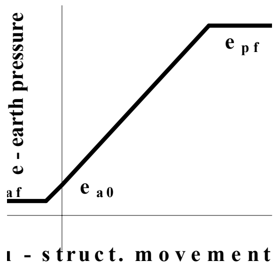

Fig. 1 Conventional dependence earth press/ structure movement.

a) Only a single value of the pressure at rest is considered in the area of zero or very small movements of the retaining structure. The magnitude of the value almost always corresponds with the value of active pressure at rest (Jaky [3] or furthers – see Fig.1), although the theoretical existence and the approximate value of the passive pressure at rest have been known for over the past 25 years Pruška [16], Koudelka [4[, [5].
b) The idea of a single (mostly plane) shear or slip surface in the mass and the full mobilization of the shear strength on it in an otherwise not deforming (granular) soil mass as the condition of the *general* effect of extreme values of active (minimum) or passive (maximum) pressure affecting the *whole* retaining structure is unrealistic particularly for geometric, but also for other reasons [6], [7], [8], [13]. This concept introduces into analyses the most advantageous assumption, which generally is not correct and can be real in the very special cases only.

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c) In the area of current movements of the retaining structure only the values of active or passive pressure (extreme values during the shear strength peak mobilization) are considered. However, it is generally known that during shear tests, after the respective peak displacement has been exceeded, the shear stress drops to the residual value. The residual strength is significantly lower than the peak strength, which is illustrated by the well-known diagrams of shear stress/movement. Thus, this assumption is very optimistic and, therefore, risky [7], [12].

The paper concentrates on the first of these three main problems of the lateral pressure theory of granular materials, i.e. the pressure at rest.

2 ADVANCED CONCEPT OF PRESSURE AT REST

The lateral pressure at rest originates under the condition of zero or very small movements of the retaining structure whether they head away from the granular mass (active pressure at rest) or into the mass (passive pressure at rest). The value of the pressure at rest for the same granular material may vary within an interval appropriate for the given material. For the magnitude of the limit values of this interval in *non-cohesive* materials and for the horizontal surface of the granular mass two known formulas were derived by means of the coefficients of the pressure at rest expressing the ratio between horizontal and vertical stresses. The first is the Jáky [3] equation (1a) and its simplified and worldwide extended form (1b) (e.g. by EC7-1, Art. 9.5.2):

$$K_{01} = \frac{1-\sin\phi'}{1+\sin\phi'}\left(1+\frac{2}{3}\sin\phi'\right) = K_{0a} \qquad (1a),$$

$$K_{0a} = (1-\sin\phi') \qquad (1b),$$

The second is Pruška´s [16] formula (2a) applicable to the upper limit of the interval of the pressure at rest (passive pressure at rest) and its simplified form (2b) :

$$K_{02} = \frac{1+\sin\phi'}{1-\sin\phi'}\left(1-\frac{2}{3}\sin\phi'\right) = K_{0p} \qquad (2a),$$

$$K_{0p} = (1+\sin\phi') \qquad (2b).$$

This concept leads mathematically to a singularity for the zero value of retaining structure movement. In fact the singularity is not possible but the press value acting against a rear face point of the perfect rigid structure can be different due to a number of conditions.

The GLP theory supposes (see Fig. 2) that the initial lateral press value $K_0\ \sigma_1$ should be in the interval limited by the coefficients $K_{0a} \leq K_0 \leq K_{0p}$

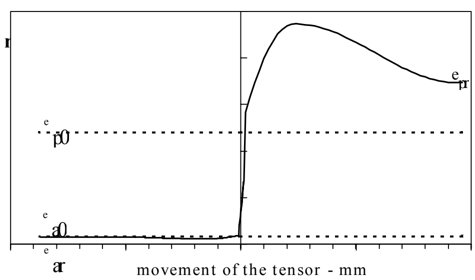


Fig. 2. Theoretical relation between the normal component of lateral pressure and the structure rare face point in the depth of 0.265 m, or in the location of No.2 tensor.

For cohesive materials it is advisable to modify the limits of this interval with the influence of cohesion (Myslivec [14]). The actual value of the pressure at rest affecting the given point of the retaining structure in the given time depends on the position of that point, actual conditions (geological composition, geotechnical soil characteristics, movement of the structure, compaction, etc.) and on the history of the stress/movement relation of the structure or the stress/strain relation in the granular mass.

3 SUBSTANTIATION

The existence of various values of the pressure at rest in granular masses (eg. soil and rock) of the same material and of the same shear strength values is proved by the following facts:

3.1 Elevated lateral pressures during over-consolidation and compaction

The existence of various values of the pressure at rest in granular masses of the same material of the same shear strength values is proved in particular by the existence of increased lateral pressures during pre-consolidation, during compaction, etc. During these phenomena, when the retaining structure remains immobile, the increase of lateral pressure at rest attains the values mostly exceeding considerably the pressures at rest according to Eq. (1) or (1a). During compaction or pre-consolidation a minor or major *increase* of the shear strength of the granular mass takes place in most cases. According to the present concept of the pressure at rest the lateral pressure of a granular mass should *drop* in such cases and in the case of unchanged shear strength it should *remain the same*. However, the opposite is true and this paradox is the proof of the existence of several values of the pressure at rest.

This paradox proves that *the same granular mass is capable of transferring different lateral stresses with practically zero deformations even if they are of higher values than those considered according to the formula (1) or (1a), e.g. at present the formula 9.1 in EC 7* [2]. It should be mentioned, that theoretically also the initial lateral pressure at rest can be lower under special conditions.

3.2 Existence of the lower and the upper limits of the pressure at rest

If the existence of an interval has been proved it proves also the existence of its limits. It has been said above that the interval of the pressure at rest is actually not singular. It means that some very small movements take place and both the lower and upper limits are not sharp, but curved with a high curviature. This character is assumed for both limits, although it has been proved for lower limit only (experiments E1 and E2), however, the similar history of the upper limit is probable, e.g. before the displacement of the toes of retaining structures,

.This phenomenon, including the above mentioned paradox, obviously results partly from the structural strength like in the case of vertical load and both the lower and the upper limits of the pressure at rest could be characterized as lateral structural strengths. The upper limit of the pressure at rest can be considered as a phenomenon of the lateral structural compressive strength and the lower one as a phenomenon of the structural expansive strength.

3.3 Results of experiment E1/0

Two experiments (E1 and E2), the first of which is described with the results in the another papers (Koudelka [10], [11]), have proved explicitly the existence of an interval of the pressure at rest and its upper limit in a really non-cohesive mass (flowing, entirely dry glass-making sand) during very small movements of the retaining structure. The basic data of experiments are as follows.

The horizontal dimensions of the tested mass were 1,0 m wide, 1,5 m long and 1,2 m high. The contact surface of the retaining wall was 1,0*1,0 m. The lateral sides of the stand were transparent to enable visual observation of the changes in the mass. The retaining wall was rigid; it could be arbitrarily moved and its movements were measured by standard mechanical indicators in each corner of the retaining wall. Five bi-component tensors were located perpendicularly to the vertical axis.

Pressure sensing was based on the previously tested and proved bi-component tensors which had been designed and produced especially for this research. These tensors enabled simultaneous continuous measurements of the normal and the tangential (shear) components on the rigid contact surfaces of the tensors. The diameter of contact surfaces was 50 mm. The pressure sensor outputs were processed by a 16-channel BMC amplifier and appropriate hardware and software. The visual observation of deformations and movements within the mass was recorded by a photo camera from a stable position and other suitable points.

Both experiments proceeded in three phases. Each of them involved one active of types of basic retaining wall movements (max. value of 8.75 mm). Parts of experiments on the pressure at rest were carried out before the first phases (rotation about the toe) and were marked as phases 0.

The first experiment with the pressure at rest E1/0 (see Fig. 3) was concerned with the rotation of the retaining wall about the toe in the region of the passive pressure at rest to residual active pressure. All tensors have shown the existence of an almost singular region of pressures within the narrow interval of displacements around zero during the gradual

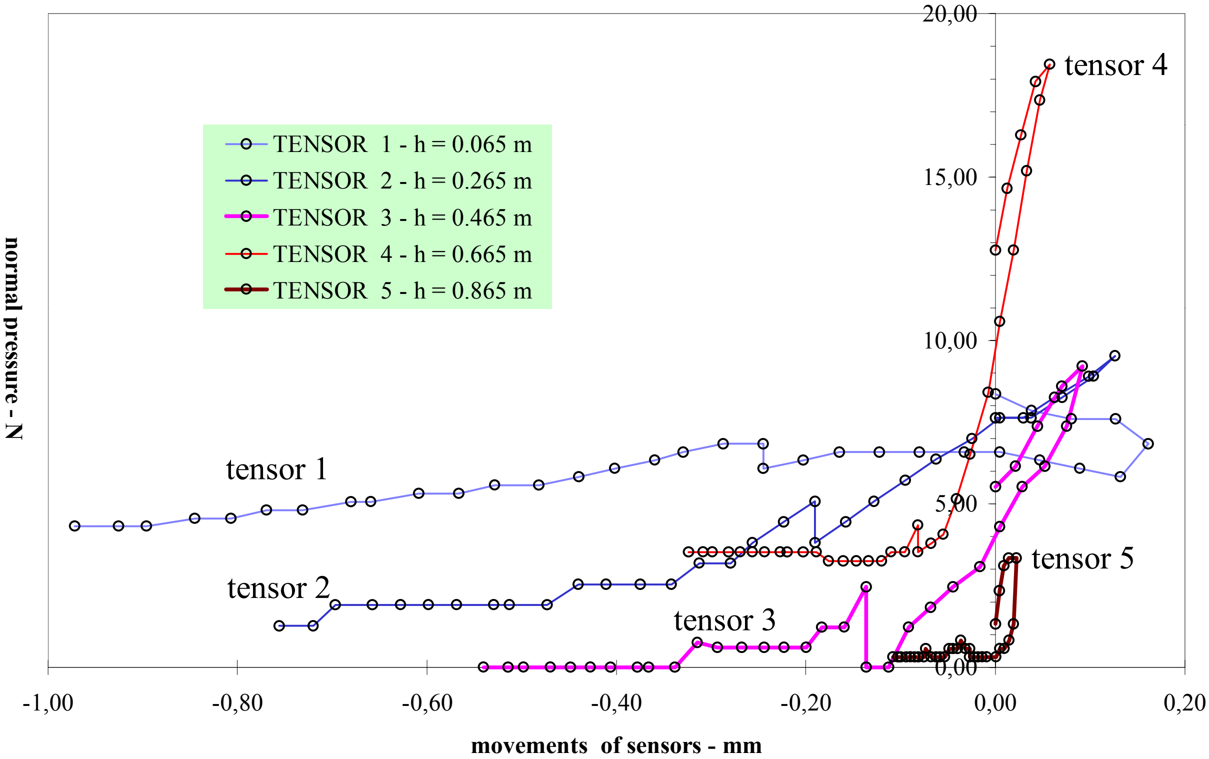


Fig. 3. Experiment E1/0-1 - History of passive (+ movements) pressure at rest during rotation about the toe and history of the initial part of experiment E1/1 - Active (– movements) pressure at rest and the active pressure. Pressures of all tensors in the graph are introduced by their normal components.

displacement of the top of the wall to the passive side (into the mass) by only 0.19 mm and in 4 tensors also during the small rotation of the top about the toe to about 0.25 mm to the active side. The actual movements of the individual tensors were smaller because of the type of the movement and dependent on their distance from the axis of rotation (the toe). The movement of the upper tensor in the passive direction amounted maximally to 0.16 mm, that of the lower tensor to 0.02 mm. In the active interval of this type the movement of the upper tensor amounted to 0.22 mm, that of the lower tensor to 0.02 mm. During passive rotation (into the mass) the pressure in four tensors rose steeply and after the change of the direction of rotation it dropped steeply to a value generally lower than that at the beginning of the experiment, before the movement of the wall.

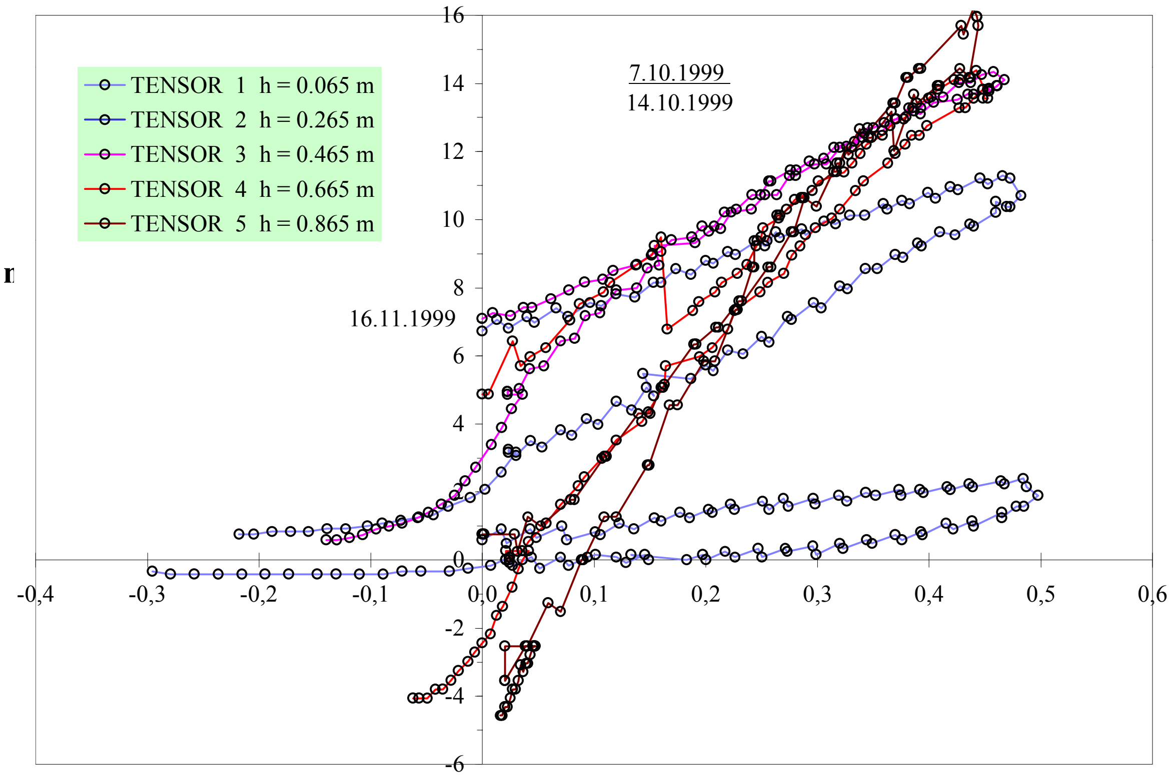
During further rotation about the toe in active direction the pressure dropped more and more mildly until its drop stopped practically within the interval of the movement of the top of the wall between 1.5 mm and 4.5 mm. In the course of further rotation at the top the pressure rose mildly in all tensors (see Fig. 3). This phase of the experiment proceeded with three breaks of 1, 7 and 6 days respectively in the course of which the pressure always increased somewhat. The tensors were numbered from top to bottom and were placed at the depths of 0.165, 0.365 m, 0.565 m, 0.765 m and 0.965 m below the surface of the mass. Further graphs, which illustrate the next phases of the experiment, were published previously [9], [10], [11].

3.4 Results of experiment E 2/0

The equipment, size of mass and procedure of experiment E2 were nearly the identical with experiment E1. The E2 experiment differed by the application of three more sensitive tensors and the type and value of retaining structure movement during the experiment with the passive pressure at rest, in which the translative motion of maximal value of 0.49 mm was applied.

The whole experiment has brought about a great number of results, which were processed and evaluated; their scope is extraordinarily large and extends beyond the theme and limits of this paper. Some visual results of E2 have been presented previously [13] which have shown that the behaviour of the perfectly non-cohesive granular body which has been similar to E1 and was at variance with the present concept of the earth pressure theory again.

Like in experiment E1 the initial part of experiment E2 concerned the proof of the real existence of an interval of the passive pressure at rest. The pressures during translative motion of the retaining wall in the region of the passive pressure at rest and in the rotation about the toe in the region of the active pressure can be seen in Fig. 4. All tensors have shown the existence of an almost singular region of pressures within the narrow interval of displacements around zero during the gradual displacement of the wall to the passive side (into the mass) only 0.49 mm. Similar behaviour was observed in four tensors also during the small rotation of the top about the toe to about 0.25 mm to the active side. The actual movements of the individual tensors towards the active side were smaller because of the type of the movement and the dependence on their distance from the axis of rotation (the toe). Thus, the movements of all tensors in the passive direction amounted to about 0.49 mm and in the active interval of the rotation about the toe the movement of the upper tensor amounted to 0.22 mm, that of the lower tensor to 0.02 mm. During passive motion (into the mass) the pressure in four tensors rose steeply and after the change of the direction of rotation it dropped steeply to a value generally lower than that at the beginning of the experiment, before the movement of the wall. The history was similar as in E1.



rotation about the toe - translative motion - mm

Fig. 4. Experiment E2/0-1 – Measured normal components of the lateral passive and active pressure at rest depended on the actual movements of the tensors in detailed graph.

During further rotation about the toe in active direction (phase E2/1) the pressure dropped more and more mildly until its drop stopped practically within the interval of the movement of the top of the wall between movement of the top 0.5 mm and 5 mm. In the course of further rotation at the top the pressure rose mildly in all tensors except for the first one. This phase of the experiment proceeded without any break.

These results and those of experiment E1/0-1 have shown clearly the existence of the interval of the pressure at rest and its lower limit due to the action of the tested granular mass. This fact lets logically the following supposition: if there is an interval in a granular material of one type, it must exist (with reference to the afore mentioned proofs) more or less also in other granular materials in one form or another.

4 Designed analysis procedure

The pressure at rest takes place under the condition of zero or very small movement of the retaining structure whether in the direction away from the soil mass (active earth pressure at rest) or in the direction into it (passive earth pressure at rest). The value of the pressure at rest may vary within an interval pertaining to the given soil. For loose soils and horizontal soil mass surface the magnitude of the limit values of this interval can be determined by means of the coefficients K_{0a} and K_{0p} , which express the ratio between the horizontal and the vertical (i.e. overburden) effective stresses from the following two formulae:

$$K_{0a} = (1 - \sin \phi') \qquad (1b)$$

for the lower limit (active pressure at rest) and

$$K_{0p} = (1 + \sin \phi') \qquad (2b)$$

for the upper limit (passive pressure at rest). Then equation

$$K_{0a} = (1 - \sin \phi') \sqrt{R_{0c}} \qquad (3),$$

gives the initial value of the pressure at rest, where $K_{0a} \le K_0 \le K_{0p}$ and where R_{0c} is the pre-consolidation number. According to Koudelka´s amendment the formula should be used for the values of $\sqrt{R_{0c}} \le (1+ \sin \phi') / (1- \sin \phi')$.

The interval limits for cohesive soils may be modified by the influence of cohesion with the parameters of shear strength, which can be obtained from the Myslivec´s formulae

$$\phi_0' = \arcsin[\sin \phi' / (2- \sin \phi')] \qquad (4)$$

$$c_0' = c' * tg\, \phi_0' / tg\, \phi' \qquad (5)$$

These parameters may to be used for the earth pressure calculation of Rankin´s stress state.

The introduction of the influence of cohesion is proposed according to Myslivec [14] by the introduction of *the shear strength at rest*, the values of which are derived from Jaky´s equation for active pressure at rest (1a). The value of the internal friction angle could be considered also approximately $\phi_0' \cong 2/3\phi'$. This concept makes it possible to use different procedures and formulae for the pressure at rest calculations, ranging from the simplest to the most advanced. Thus, the influence of slope angle or the general features of ground surface can be calculated directly. Consequently, this solution eliminates the coefficient $K_{0\beta}$ used in the final draft of EC 7-1 (Eq. 9.2) [3].

The amendment recommends conservatively to consider Rankin´s stress state according to Myslivec´s initial solution, although it would be possible to consider δ_0 on the basis of some more recent studies at $\delta_0 = \phi'/2$. The reason for the suggested more conservative $\delta_0 = 0$ is the contemporary uncertainty of the definition of the size of actual movements during the pressure at rest.

The actual value of the pressure at rest applied at the given time to the given spot of the retaining structure depends on the location of this spot, contemporary conditions (geological structure, geotechnical material properties, movement of the structure, compaction etc.) and on the history of the stress/movement ratio of the structure and/or the stress/strain ratio of the granular mass.

5 BEHAVIOUR NON-COHESIVE MASS IN THE AREA OF PRESSURE AT REST

The tested similar granular masses of dry loose sand have behaved obviously differently in the area of very small movements corresponding to the pressure at rest, both to active and passive sides. Moreover, even the behaviour of both the very similar masses (of E1 and E2) has differed with highest probability due to the different types of small passive movements (E1/0 – rotation about the toe, E2/0 – translative motion). Of course, some deviations (probably of the lower order) between both experiments could be caused by the use of the less sensitive tensors for experiment E1. The behaviour of the granular mass due to the different types of passive structure movements can be compared in Tables 1 and 2.

The Tables 1 and 2 show obviously that the rigidity of the masses by the contact responses increases generally together with increasing depth. This course can be noted both on the coefficients of average elasticity k_{e0} and on the coefficients of active elasticity k_{e0a} and the coefficients of passive elasticity k_{e0p} . Tabulated coefficients of the passive elasticity k_{e0p} have different courses, i.e. the mass response to rotation about the toe has more rigidity then the mass response to translative motion. Values of the coefficients of active elasticity k_{e0p}

Ten-	**Depth**	**Struct. movements**			**Normal stresses**			**Coefficients of elasticity**		
sor	h	u_{0a}	u_{0p}	Δu_0	e_0	e_{0a}	e_{0p}	k_{e0}	k_{e0a}	k_{e0p}
no.	m	mm	mm	mm	kPa	kPa	kPa	MNm^{-3}	MNm^{-3}	MNm^{-3}
1	0.065	"-0.40	0.16	0.56	1.064	0.774	0.871	0.172	0.726	1.210
2	0.265	"-0.28	0.13	0.41	0.971	0.405	1.213	1.972	2.023	1.861
3	0.465	"-0.11	0.09	0.20	0.784	0	1.174	5.870	6.400	5.220
4	0.665	"-0.08	0.06	0.14	1.623	0.449	2.348	13.560	14.674	12.075
5	0.865	0	0.02	0.02	0.169	0.039	0.426	19.353	-	12.860

Tab. 1. Experiment E1/0-1. Rigidity of the tested granular mass in the area of the pressure at rest movements by the coefficients of elasticity k_e.

are influenced probably partly by extraordinarily small values of E1 active movements in the lower part of the retaining wall and partly by the different types of previous movements on the passive side.

Ten-	**Depth**	**Struct. movements**			**Normal stresses**			**Coefficients of elasticity**		
sor	h	u_{0a}	u_{0p}	Δu_0	e_0	e_{0a}	e_{0p}	k_{e0}	k_{e0a}	k_{e0p}
no.	m	mm	mm	mm	kPa	kPa	kPa	MNm^{-3}	MNm^{-3}	MNm^{-3}
1	0.065	"-0.03	0.48	0.51	0.071	"-0.042	0.304	0.679	3.862	0.480
2	0.265	"-0.12	0.47	0.59	0.857	0.117	1.437	2.238	6.165	1.235
3	0.465	"-0.07	0.46	0.53	0.903	0.136	1.827	3.190	10.950	2.010
4	0.665	"-0.05	0.42	0.47	0.621	"-0.518	1.745	4.922	22.790	2.795
5	0.865	"-0.03	0.43	0.48	0.097	"-0.806	1.839	5.509	30.091	4.051

Tab. 2. Experiment E2/0-1. Rigidity of the tested granular mass in the area of the pressure at rest movements by the coefficients of elasticity k_e.

6 CONCLUSION

The lateral pressure at rest is one of the principal problems of the General Lateral Pressure (GLP) theory of granular materials (soils, soft rocks and others). The reasons for the formulation of this more advanced theory, in which the dependence of earth pressure magnitude on the movement of the retaining structure according to Fig.2 and the proofs that a retaining structure in a standard general case is affected by lateral pressure of general values, are specified briefly in Chapt. 1 and more detailed previously (Koudelka 1999, 2000 , 2001). The paper outlines the proofs of and the grounds for the proposed solution of pressure at rest in GLP theory and practice which can be considered probably sufficient. This fact, of course, does not eliminate the need of further research of the problem.

7 ACKNOWLEDGEMENT

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