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COMPUTATIONAL METHODS FOR ANALYSIS OF BEHAVIOUR OF ROTORS SUPPORTED BY SQUEEZE-FILM DAMPERS AND FLUID-FILM BEARINGS

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Summary: *Eigenvalues and response on loadings of specified hime histories provide principal information on dynamical properties and behaviour of rotors supported by squeeze-film dampers or fluid-film bearings. In both cases the analysis starts from setting up and solving the equation of motion. The hydraulical forces through which the layers of lubricat act on the the rotor journals are determined from the oil pressure distribution in the gap between the inner and outer rings of the damper or the bearing. A procedure based on repeated linearization of fluid-film forces by means of their expansion into a Taylor series in the neighbourhood of the current position and modification of a Newmark method has been developed and tested. Inertia effects of the lubricant are included in the calculations. The proposed method is marked for good numerical stability.*

1. INTRODUCTION

Rotors working in industrial enterprises are loaded by various time varying forces. To decrease amplitudes of their vibration the shafts are supported by fluid-film bearings or squeeze-film dampers. On the other hand nonlinear properties of these constraint elements can result into operating conditions that are undesirable from the point of view of control or limit state of deformation which is determined by the width of the gap between the discs and the stationary part. The observations show that even a simple excitation can produce a chaotic or self-excited vibration that are marked for large amplitudes. If one or more parameters of the rotor system are close to a stability limit a bifurcation can occur and the motion tends to jump between different attractors.

An important instrument for investigation of behaviour of rotors is a computer modelling method. The model systems are assumed to have the following properties : (i) the shaft is represented by a beam-like body that is discretized into finite elements, (ii) the discs are thin and absolutely rigid, (iii) inertia and gyroscopic effects of the shaft and of the discs are taken into account, (iv) material damping of the shaft is viscous, other kinds of damping are considered to be linear, (v) the rotor is coupled with the stationary part through rolling-element bearings with squeeze-film dampers or through hydrodynamical bearings, (vi) the rotor rotates at constant angular speed and (vii) is loaded by concentrated and distributed forces of general time histories.

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2. VELOCITY PROFILES AT SHORT DAMPERS AND BEARINGS NEGLECTING FLUID-INERTIA FORCES

Neglecting inertia effects of the lubricant the pressure distribution in the gap between the inner and outer rings of the squeeze-film damper or hydrodynamical bearing is described by the Reynolds equation [1]

$$\frac{1}{R}\frac{\partial}{\partial\varphi}\left(\frac{h^3}{R}\frac{\partial p}{\partial\varphi}\right)+\frac{\partial}{\partial x}\left(h^3.\frac{\partial p}{\partial x}\right)=-6\eta\left[e.(2\dot{\gamma}-\Omega_B).\sin\vartheta+2\dot{e}\cos\vartheta\right] \qquad (1)$$

where

$$h=\delta-e.\cos\vartheta \qquad (2)$$

p - pressure of the lubricant,
x - coordinate along the length of the damper or bearing,
R - radius of the damper or the bearing gap,
L - length of the damper or the bearing,
δ - difference between radii of the outer and inner rings of the damper or the bearing,
e - eccentricity,
h - width of the damper or the bearing gap,
η - dynamical viscosity of the lubricant,
Ω_B - angular speed of rotation of the outer ring of the damper or the bearing, (dampers : $\Omega_B = 0$ rad.s^{-1}, bearings : $\Omega_B > 0$ rad.s^{-1}).

Notation of angles φ, γ and ϑ is evident from Fig.1.

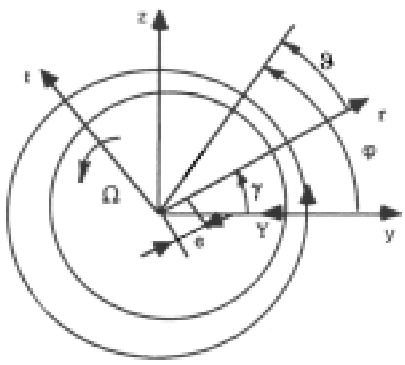

Fig.1 Scheme of the bearing

For relatively short dampers and bearings (in axial direction) the change of pressure in the circumferencial direction is small in relation to the pressure gradient in axial direction. Hence it is possible to neglect the first term in the Reynolds equation

$$\frac{\partial}{\partial x}\left(h^3.\frac{\partial p}{\partial x}\right)=-6\eta\left[e.(2\dot{\gamma}-\Omega_B).\sin\vartheta+2\dot{e}\cos\vartheta\right] \qquad (3)$$

By integrating (3) and taking into account the boundary conditions

$$p=0 \qquad \text{for} \qquad x=\pm\frac{L}{2} \qquad (4)$$

one obtains for the pressure function

$$p=\frac{1}{2}.\left(\frac{L^2}{4}-x^2\right)A_1 \qquad (5)$$

$$A_1=\frac{6.\eta}{h^3}\left[e.(2\dot{\gamma}-\Omega_B).\sin\vartheta+2\dot{e}\cos\vartheta\right] \qquad (6)$$

The velocity profiles in axial (w) and circumferential (u) directions can be calculated from the pressure gradients and equations expressing the force equilibrium of a fluid element

$$u=\frac{Y}{h}\Omega_B.R \qquad (7)$$

$$w = \frac{3.x}{h}\left(\frac{Y}{h} - \frac{Y^2}{h^2}\right)\left[2.\dot{e}.\cos\vartheta + e.(2\dot{\gamma} - \Omega_B).\sin\vartheta\right] \qquad (8)$$

The velocity component v in radial direction (Fig.1) is determined from u, w and from the continuity equation

$$v = 3\left(\frac{Y^3}{3.h^3} - \frac{Y^2}{2.h^2}\right)\left[2.\dot{e}.\cos\vartheta + e.(2\dot{\gamma} - \Omega_B).\sin\vartheta\right] - \frac{Y^2}{2.h^2}.e.\Omega_B.\sin\vartheta \qquad (9)$$

u, v, w - circumferential, radial, axial velocity components of the oil flow in the gap.

3. FLUID INERTIA FORCES IN SHORT SQUEEZE-FILM DAMPERS AND BEARINGS

A classical assumption in squeeze-film dampers and hydrodynamical bearings theory which is supported by considerable experimental evidence [4], [5] is that the velocity profiles calculated from the solution of the Reynolds equation are valid also in the presence of significant fluid-inertia effects.

One approach to taking into account the fluid-film inertia forces starts from the Navier-Stokes equation related to the direction of the prevailing flow (at short dampers and bearings in the axial direction)

$$\rho.\left(\frac{\partial w}{\partial t} + \frac{u}{R}.\frac{\partial w}{\partial \vartheta} + v.\frac{\partial w}{\partial Y} + w.\frac{\partial w}{\partial x}\right) = -\frac{\partial p}{\partial x} + \eta.\frac{\partial^2 w}{\partial Y^2} \qquad (10)$$

ρ - denstity of the lubricant,
t - time.

The inertia effects are incorporated into the pressure function by means of averaging the terms proportional to the fluid density over the film thickness [2]. Then it holds for the pressure gradient

$$\frac{\partial p}{\partial x} = -\frac{1}{h}\int_0^h \rho.\left(\frac{\partial w}{\partial t} + \frac{u}{R}.\frac{\partial w}{\partial \vartheta} + v.\frac{\partial w}{\partial Y} + w.\frac{\partial w}{\partial x}\right)dY + \eta.\frac{\partial^2 w}{\partial Y^2} \qquad (11)$$

The pressure distribution in the gap between the inner and outer rings is obtained by integration of (11) and by taking into account the boundary conditions (4). The performed manipulations result in the expression for the pressure function

$$p = \frac{1}{2}.\left(\frac{L^2}{4} - x^2\right).A_2 \qquad (12)$$

$$\begin{aligned} A_2 = {} & \frac{6.\eta}{h^3}.\left[e.(2\dot{\gamma} - \Omega_B).\sin\vartheta + 2\dot{e}\cos\vartheta\right] + \\ & + \frac{\rho}{10.h^2}.\left[e^2.(\Omega_B^2 - 14.\Omega_B\dot{\gamma} + 24.\dot{\gamma}^2).\sin^2\vartheta + 24.\dot{e}^2.\cos^2\vartheta\right] + \\ & + \frac{\rho}{10.h^2}.e.\dot{e}.(-14.\Omega_B + 48.\dot{\gamma}).\sin\vartheta.\cos\vartheta + \\ & + \frac{\rho}{4h}.\left[(4\ddot{e} - 4.e\dot{\gamma}^2 + 4e\dot{\gamma}\Omega_B - e\Omega_B^2).\cos\vartheta + (4e\ddot{\gamma} - 4\dot{e}\Omega_B - 2e\dot{\Omega}_B + 8\dot{e}\dot{\gamma}).\sin\vartheta\right] \end{aligned} \qquad (13)$$

A similar approach has been imployed by El-Shafei. He multiplies [3] the Navier-Stokes equation (10) by the velocity component corresponding to the direction of prevailing flow and integrates all its terms modified in this way across the film thickness. After some operations the pressure function acquires the following form

$$p = \frac{1}{2}.\left(\frac{L^2}{4} - x^2\right).A_3 \tag{14}$$

$$\begin{aligned} A_3 = {} & \frac{6.\eta}{h^3}.\left[e.(2\dot{\gamma} - \Omega_B).\sin\vartheta + 2\dot{e}\cos\vartheta\right] + \\ & + \frac{3\rho}{35h^2}.\left[e^2(34\dot{\gamma}^2 - 27.\Omega_B\dot{\gamma} + \frac{17}{2}.\Omega_B^2).\sin^2\vartheta + 34\dot{e}^2.\cos^2\vartheta\right] + \\ & + \frac{3\rho}{35.h^2}.e.\dot{e}.(-27.\Omega_B + 68.\dot{\gamma}).\sin\vartheta.\cos\vartheta + \\ & + \frac{3\rho}{10h}.\left[(4\ddot{e} - 4.e\dot{\gamma}^2 + 4e\dot{\gamma}\Omega_B - e\Omega_B^2).\cos\vartheta + (4e\ddot{\gamma} - 4\dot{e}\Omega_B - 2e\dot{\Omega}_B + 8\dot{e}\dot{\gamma}).\sin\vartheta\right] \end{aligned} \tag{15}$$

From the physical point of view El-Shafei's approach is based on energy averaging of the fluid flow in dominant direction.

4. CALCULATION OF THE RESPONSE AND EIGENVALUES OF THE ROTOR SYSTEM

Lateral vibration of rotors supported by rolling-element bearings with squeeze-film dampers or by fluid-film bearings is described by the equation of motion

$$\mathbf{M}.\ddot{\mathbf{x}} + (\mathbf{B} + \eta_V.\mathbf{K}_{SH} + \Omega.\mathbf{G}).\dot{\mathbf{x}} + (\mathbf{K} + \Omega.\mathbf{K}_C).\mathbf{x} = \mathbf{f}_A + \mathbf{f}_V + \mathbf{f}_H(\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}) \tag{16}$$

and by relationships for boundary conditions

$$\mathbf{x}_{BC} = \mathbf{o} \tag{17}$$

$\mathbf{M}$, $\mathbf{G}$, $\mathbf{K}$ - mass, gyroscopic, stiffness matrices of the rotor system,
$\mathbf{B}$, $\mathbf{K}_C$ - (external) damping, circulation matrices of the rotor system,
$\mathbf{K}_{SH}$ - stiffness matrix of the shaft,
$\mathbf{f}_A$, $\mathbf{f}_V$, $\mathbf{f}_H$ - vectors of applied, constraint, hydraulical forces acting on the rotor system,
$\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}$ - vectors of generalized displacements, velocities, accelerations of the rotor system,
$\mathbf{x}_{BC}$, $\mathbf{o}$ - vector of boundary conditions, zero vector,
Ω - angular speed of the rotor rotation,
η_V - coefficient of viscous damping (material of the shaft).

The hydrodynamical bearings or squeeze-film dampers are considered in the model system by means of nonlinear force couplings. Elements of vector $\mathbf{f}_H$ represent forces through which the layers of lubricant act on the rotor journal centres.

Realationships for the oil pressure distibution (5), (12) and (14) are valid only in uncavitated regions. If the pressure drops under a certain limit, the oil starts to libarate gases and to boil. Then an acceptable assumption is that the pressure in such regions remains constant.

The mean value of the pressure profile in axial direction is given by the following relations

$$p_m(\vartheta) = \frac{1}{12}A_j.L^2 \qquad \text{for} \quad \frac{1}{8}A_j.L^2 \geq p_{cav} \tag{18}$$

$$p_m(\vartheta) = \frac{A_j}{12.L}.(L^3 - 3.L^2x_{cav} + 4.x_{cav}^3) + \frac{2.p_{cav}x_{cav}}{L} \qquad \text{for} \quad \frac{1}{8}A_j.L^2 < p_{cav} \tag{19}$$

$$x_{cav} = +\sqrt{\frac{L^2}{4} - \frac{2.p_{cav}}{A_j}} \tag{20}$$

Parameters A_j for j = 1, 2, 3 are obtained from relationships (6), (13) and (15). Radial and tangential components of the hydraulical forces are calculated by integration of the mean value of the pressure over the circumference of the damper or bearing

$$F_r = L.R.\int_0^{2\pi} p_m(\vartheta).\cos\vartheta.d\vartheta \tag{21}$$

$$F_t = L.R.\int_0^{2\pi} p_m(\vartheta).\sin\vartheta.d\vartheta \tag{22}$$

After transformation of (21) and (22) into the global frame of reference they are substituted into appropriate elements of vector $\mathbf{f}_H$.

The equation of motion (16) is nonlinear. One possibility of its solution is based on modification of a Newmark method. The vector of hydraulical forces $\mathbf{f}_H$ at time t+Δt is calculated by means of its expansion into a Taylor series in the neighbourhood of time t neglecting terms of the second and higher orders.

$$\mathbf{f}_{H,t+\Delta t} = \mathbf{f}_{H,t} + \mathbf{D}_{K,t}.(\mathbf{x}_{t+\Delta t} - \mathbf{x}_t) + \mathbf{D}_{B,t}.(\dot{\mathbf{x}}_{t+\Delta t} - \dot{\mathbf{x}}_t) + \mathbf{D}_{M,t}.(\ddot{\mathbf{x}}_{t+\Delta t} - \ddot{\mathbf{x}}_t) + \ldots \tag{23}$$

Taking into account only the linear part of the Taylor series (23) and after performing some manipulations the equation of motion related to the instant of time t+Δt has the form

$$\mathbf{M}.\ddot{\mathbf{x}}_{t+\Delta t} + (\mathbf{B} + \eta_V.\mathbf{K}_{SH} + \Omega.\mathbf{G}).\dot{\mathbf{x}}_{t+\Delta t} + (\mathbf{K} + \Omega.\mathbf{K}_C).\mathbf{x}_{t+\Delta t} = \mathbf{f}_{t+\Delta t} \tag{24}$$

where

$$\mathbf{f}_{t+\Delta t} = \mathbf{f}_{\mathbf{A},t+\Delta t} + \mathbf{f}_{V,t+\Delta t} + \mathbf{f}_{H,t} + \mathbf{D}_{K,t}(\mathbf{x}_{t+\Delta t} - \mathbf{x}_t) + \mathbf{D}_{B,t}(\dot{\mathbf{x}}_{t+\Delta t} - \dot{\mathbf{x}}_t) + \mathbf{D}_{M,t}(\ddot{\mathbf{x}}_{t+\Delta t} - \ddot{\mathbf{x}}_t) \tag{25}$$

$$\mathbf{D}_{K,t} = \left[\frac{\partial \mathbf{f}_H(\mathbf{x},\dot{\mathbf{x}},\ddot{\mathbf{x}})}{\partial \mathbf{x}}\right]_{\mathbf{x}=\mathbf{x}_t,\dot{\mathbf{x}}=\dot{\mathbf{x}}_t,\ddot{\mathbf{x}}=\ddot{\mathbf{x}}_t} \tag{26}$$

$$\mathbf{D}_{B,t} = \left[\frac{\partial \mathbf{f}_H(\mathbf{x},\dot{\mathbf{x}},\ddot{\mathbf{x}})}{\partial \dot{\mathbf{x}}}\right]_{\mathbf{x}=\mathbf{x}_t,\dot{\mathbf{x}}=\dot{\mathbf{x}}_t,\ddot{\mathbf{x}}=\ddot{\mathbf{x}}_t} \tag{27}$$

$$\mathbf{D}_{M,t} = \left[\frac{\partial \mathbf{f}_H(\mathbf{x},\dot{\mathbf{x}},\ddot{\mathbf{x}})}{\partial \ddot{\mathbf{x}}}\right]_{\mathbf{x}=\mathbf{x}_t,\dot{\mathbf{x}}=\dot{\mathbf{x}}_t,\ddot{\mathbf{x}}=\ddot{\mathbf{x}}_t} \tag{28}$$

$\mathbf{D}_{K,t}$, $\mathbf{D}_{B,t}$, $\mathbf{D}_{M,t}$ - square matrices of partial derivatives.

Solution of the equation of motion must satisfy the boundary conditions. Therefore equation (24) is transformed to the form

$$\mathbf{A}_{2,t+\Delta t}.\ddot{\mathbf{y}}_{t+\Delta} + \mathbf{A}_{1,t+\Delta t}.\dot{\mathbf{y}}_{t+\Delta} + \mathbf{A}_{0,t+\Delta t}.\mathbf{y}_{t+\Delta} = \mathbf{b}_{t+\Delta} \tag{29}$$

Matrices $\mathbf{A}_{2,t+\Delta t}$, $\mathbf{A}_{1,t+\Delta t}$, $\mathbf{A}_{0,t+\Delta t}$ and vector $\mathbf{b}_{t+\Delta t}$ are obtained from the ones given by (30), (31), (32) and (33) respectively by omitting their rows and columns corresponding to the degrees of freedom for which the boundary conditions are prescribed.

$$\mathbf{A}^*_{2,t+\Delta t} = \mathbf{M} - \mathbf{D}_{M,t} \tag{30}$$

$$\mathbf{A}^*_{1,t+\Delta t} = \mathbf{B} + \eta_V.\mathbf{K}_{SH} + \Omega.\mathbf{G} - \mathbf{D}_{B,t} \tag{31}$$

$$\mathbf{A}^{*}_{0,t+\Delta t} = \mathbf{K} + \Omega.\mathbf{K}_{C} - \mathbf{D}_{K,t} \tag{32}$$

$$\mathbf{f}^{*}_{t+\Delta t} = \mathbf{f}_{A,t+\Delta t} + \mathbf{f}_{H,t} - \mathbf{D}_{M,t}.\ddot{\mathbf{x}}_{t} - \mathbf{D}_{B,t}.\dot{\mathbf{x}}_{t} - \mathbf{D}_{K,t}.\mathbf{x}_{t} - \mathbf{A}^{*}_{0,t+\Delta t}.\mathbf{x}_{BC} \tag{33}$$

In addition the mentioned modification eliminates unknown values of the vector of constraint forces $\mathbf{f}_V$.

Advantage of the described procedure is that it avoids repeated solving a set of nonlinear algebraic equations. On the other hand it requires to perform linearization of the vector of hydraulical forces and modification of the coefficient matrices of the system at each integration step.

Calculation of eigenvalues of the rotor system starts from equation (29). The vector of hydrodynamical forces is expanded into a Taylor series in the neighbourhood of the equilibrium position and the right-hand side of (29) is set equal to zero.

The task results in solving a quadratic eigenvalue problem

$$\det\left(\lambda^{2}.\mathbf{A}_{2} + \lambda.\mathbf{A}_{1} + \mathbf{A}_{0}\right) = 0 \tag{34}$$

The coefficient matrices in (34) are obtained from $\mathbf{A}^{*}_{2,t+\Delta t}$, $\mathbf{A}^{*}_{1,t+\Delta t}$, $\mathbf{A}^{*}_{1,t+\Delta t}$ given by relationships (30), (31) and (32) by omitting their rows and columns corresponding to the degrees of freedom to which the boundary conditions are imposed.

5. EXAMPLE

The mentioned approach to analysis of behaviour of rotors supported by squeeze-film dampers or fluid-film bearings was tested by means of computer simulations.

The rotor of rotor system ROT21 consists of a shaft (SH) and of two discs (D1, D2) attached to its overhung end (Fig.2). The rotor is coupled with a rigid foundation plate through two rolling-element bearings with squeeze-film dampers and rotates at constant angular speed.

The shaft was loaded by concentrated forces of constant magnitude acting on it at the discs locations in radial direction. In addition the system was excited by centrifugal forces caused by the discs unbalances.

The task was to study influence of the dampers on dynamical properties and behaviour of the system.

The shaft was represented by a beam-like body that was discretized into 5 finite elements. Both dampers were considered as short. Pressure in cavitated regions was considered to be zero.

The response was calculated using the adapted Newmark method. Calculation of integrals (21) and (22) was performed numerically by application of a Simpson's rule.

The principal results are summarized in the following figures. The analyses are mean : A - rotor system with no dampers, B - determination of inertia effects of the lubricant through averaging inertia terms in the Navier-Stokes equation and C - determination of inertia effects of the lubricant through averaging the fluid flow energy in the prevailing direction.

It is evident that presence of the dampers (i) changes spectrum of the natural frequencies of the rotor system (Tab.1) and (ii) significantly decreases amplitudes of the forced vibration (Fig.3). Results obtained in variants B and C are close one to another.

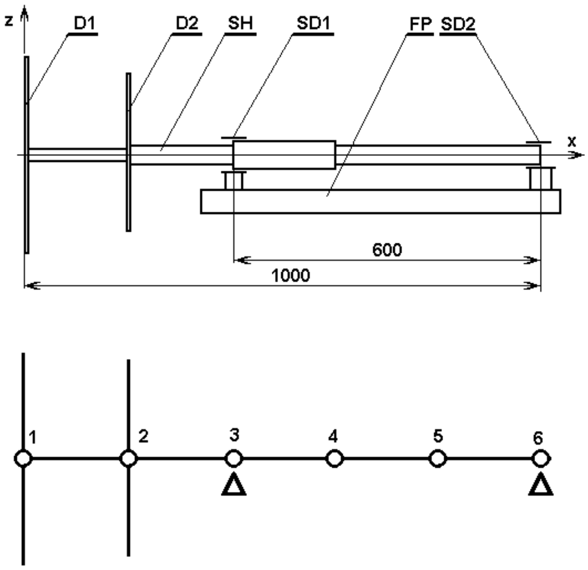

Fig.2 Scheme of the rotor system ROT21

	A [rad/s]	B [rad/s]	C [rad/s]
1.	127.3	158.9	156.5
2.	212.6	268.9	249.7
3.	374.9	331.1	310.1
4.	835.7	566.1	558.7
5.	851.3	750.0	746.6
6.	1390.4	1651.3	1650.8
7.	1747.9	3034.8	3031.6
8.	2329.9	3361.8	3356.7
9.	3537.1	3443.5	3442.0
10.	4031.3	4099.0	4098.9

Tab.1 Damped natural frequencies of ROT21

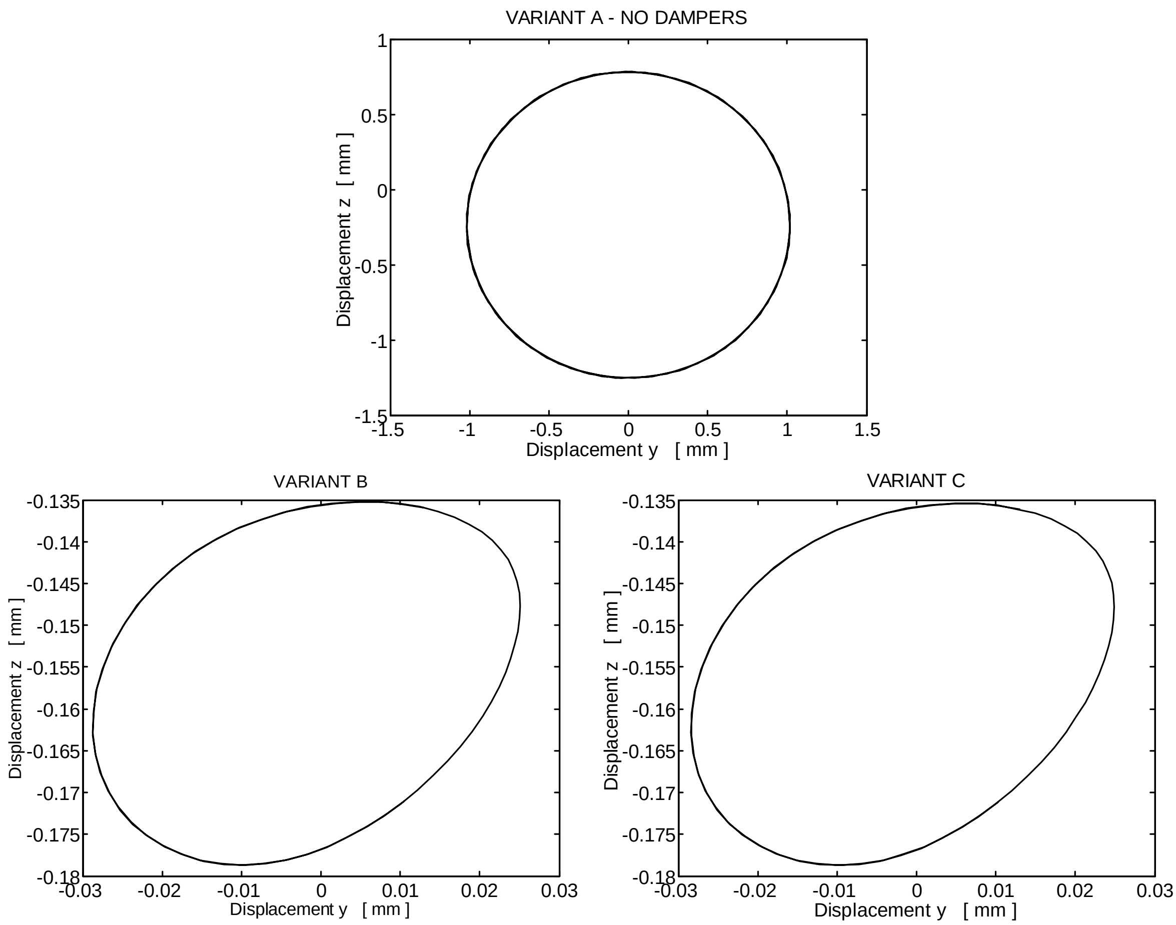

Fig.3 Trajectories of the disc D1 centre (variants A, B, C)

6. CONCLUSION

The modification of a Newmark method utilizing repeated expansion of the vector of hydrodynamical forces into a Taylor series in the neighbourhood of the current position is marked for good numerical stability and makes possible to apply reasonably long integration steps.

Investigation of behaviour of rotors supported by squeeze-film dampers or fluid-film bearings is an important but also rather complicated technical problem. For its solution a computer modelling method can be used. The computer simulations are valuable especially if they are performed for different operating or design parameters of the rotor system.

7. ACKNOWLEDGEMENT

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