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ON THE SYNTHESIS OF SPATIAL RACK MECHANISMS

M. Sc. Emilia Abadjieva, Assoc. Prof. Valentin Abadjiev[1]

Abstract: Two relatively independent areas of the gearing science have been distinctively outlined and considerably progressed in the last decade. The "Theory of gearing" could be considered essentially as a kinematic theory of mechanisms for motion transformation with high kinematic joints by a defined law. This theory treats the common regularity and research methods of gear sets. The area "Geometrical theory of gears" observes the mathematical modelling of the concrete gear sets. This area also develops the methods for synthesis and generation of the active tooth surfaces. This study belongs to the second area of gear theory. It is dedicated to the analytical defining of conjugate tooth surfaces of a class of the spatial gear mechanisms, called spatial rack sets. They are designed to transform rotation into translation motion by a defined law. The rotation is realized by gear with helical teeth. In the most common case this gear has a conic form. And the translation is realized by rack with helical teeth. In this study the active tooth surfaces of the first link are linear conic helicoid, and the teeth surfaces of the rack are kinematically conjugate of the conic helicoid. The obtained equations are the basis for creation of the algorithms for synthesis and design of the spatial rack mechanisms.

Key words: spatial rack set, synthesis, design, mathematical model

1. Introduction

The studies incompleteness of spatial rack mechanisms motivates authors of this work to do systematical investigations on these type mechanisms. The rack mechanisms are purposed for transformation of rotation into translation (R-T) by defined law of motion [1, 2, 3]. The spatial rack mechanism can be considered as a special case of a spatial three-link gear mechanism transforming rotation between crossed axes. In other word we can consider that the spatial rack mechanism is obtained from the skew-axes gear mechanism by increasing the teeth number of one gear to infinity without increasing the number of meshed tooth (high kinematic joints).

The tooth link with a theoretically endless tooth number we call **helical rack**, and the link with a limited number of teeth is called **helical wheel**. In a common case the helical wheel has a conic form and the number of teeth is more then six. If the number of teeth of the helical wheel is less than six it is transformed into worm (conic or cylindrical). As a result of this change of the spatial three-link gear mechanism "rotation axis" of the link "helical rack" is placed to infinity and its rotation is transformed into translation.

[1] E-mails: abadjieva@yahoo.com, abadjiev@imbm.bas.bg,
Institute of Mechanics and Biomechanics, Bulgarian Academy of Sciences, Acad. G. Bonchev Str., block 4, 1113 Sofia, Bulgaria

2. Mathematical model of kinematic transformation of motion (R-T), by spatial rack mechanisms

In Fig. 1 is shown the kinematic scheme of spatial rack set realizing a definite law of transformation of rotation into translation by contacting in line D_{12} tooth surfaces Σ_1 and Σ_2 [4]. The tooth surfaces Σ_1 belonging to the link 1 rotates around axis 1-1 with angular velocity ϖ_1, and Σ_2 belonging to the link 2-translates on the direction 2-2 with velocity $\overline{V}_2$. The axis of rotation 1-1 and the translation direction 2-2 are fixed, i.e. $\delta = \angle(\varpi_1, \overline{V}_2) = const$. The realized transformation of the type (R-T) from this mechanism is characterized by the following kinematic regularity:

$$j_{12} = \omega_1 / V_2 = const., \qquad (1)$$

where ω_1 is absolute value of the angular velocity vector ϖ_1; V_2 is absolute value of the translation velocity vector $\overline{V}_2$.

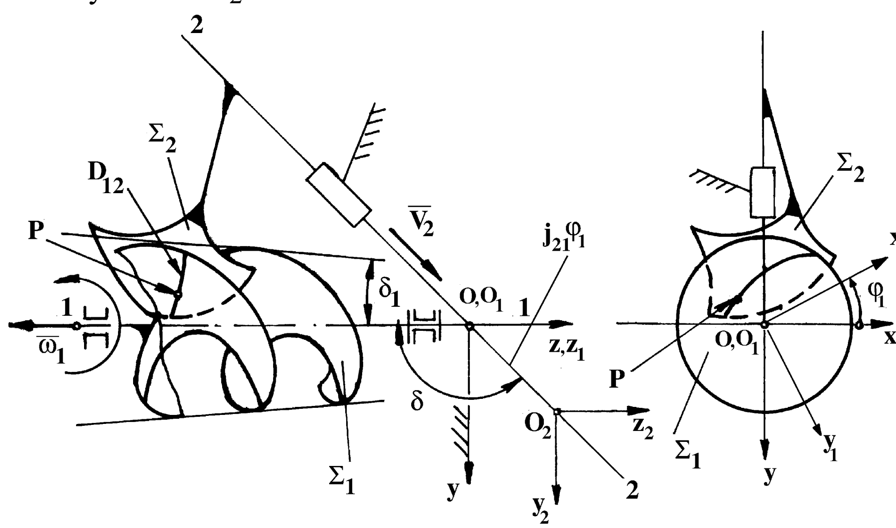

Fig. 1 Kinematic scheme of spatial rack set

The spatial rack mechanism studied in this article has a helical wheel of a type conic worm which active flanks Σ_1 are conic linear helicoid [5] and the tooth surfaces Σ_2 of helical rack are envelopes of the flanks Σ_1. The kinematic scheme shown in Fig. 1 of the spatial rack mechanism is related to the considered mathematical model. The equations of tooth surfaces Σ_1 and Σ_2 are written by means of following the right-hand orthogonal co-ordinate systems: $S(O,x,y,z)$ – firmly connected with a mechanism posture; $S_1(O_1,x_1,y_1,z_1)$– firmly connected with link 1 (a conic worm) and $S_2(O_2,x_2,y_2,z_2)$ – firmly connected with link 2 (a helical rack).

The kinematic conjugation of the joint $\Sigma_1{:}\Sigma_2$ requires the geometric elements (i.e. the tooth surfaces Σ_1 and Σ_2) to be analytically described by continuous functions and have continuous derivatives in first order [6, 7]. Additionally, it is required Σ_1 and Σ_2 to have one conjugate point at least. The kinematic conjugation of the joints $\Sigma_1{:}\Sigma_2$ suppose the motion transformation from the geometric element Σ_1 to the geometric element Σ_2. In the concrete case of spatial rack mechanism the following specific restrictions concerning tooth surfaces Σ_1 and Σ_2 are imposed: representing the contact at one conjugate point P at a given moment as a contact between two infinitely small particles of Σ_1 and Σ_2, which lie in a common tangent plane to the Σ_1 and Σ_2 (Fig. 1). It is evident that these areas have a relative sliding which is performed with a velocity vector $\overline{V}_{12} = \overline{V}_1 - \overline{V}_2$. $\overline{V}_{12}$ lies in the tangent plane. The relative velocity vector $\overline{V}_{12}$ in every common contact point P from contact line D_{12} of the Σ_1 and Σ_2 in the co-ordinate system $S(O,x,y,z)$ is:

$$\overline{V}_{12} = y\bar{i} - (j_{21} \sin\delta + x)\bar{j} + j_{21} \cos\delta \bar{k}. \qquad (2)$$

The vector equation (2) is written when $\omega_1 = 1$ rad/s and $V_2 = 1/j_{12} = j_{21}$ respectively.

3. Synthesis of tooth surfaces Σ_1 and Σ_2

3.1. Geometry of conic convolute and involute helicoids

In Fig. 2 is shown one possible case of the generated right-hand conic convolute helicoids $\Sigma^{(j)}$ $(j = 1, \ 2)$. The process of generating of these helical surfaces is examined in the static co-ordinate system $S_1^{(j)}(O_1^{(j)}, x_1^{(j)}, y_1^{(j)}, z_1^{(j)})$.

The generatrix $L^{(j)}$ doesn't intersect axis $O_1^{(j)} z_1^{(j)}$, which coincide with the geometric axis of the conic worm. The angle $0{,}5\pi < \xi^{(j)} < \pi$ is between $L^{(j)}$ and the direction of the axis $O_1^{(j)} z_1^{(j)}$ (geometric axis of the worm). Also $L^{(j)}$ belongs to the plane $T^{(j)}$, which is tangential to the directed circle cylinder $C^{(j)}$. The conic convolute helicoid $\Sigma^{(j)}$ $(j = 1, \ 2)$ generates form $L^{(j)}$, which performs complex motions consisting of [5]: a) helical motion along the axis $O_1^{(j)} z_1^{(j)}$ with a parameter $p_s^{(j)} = const.$; b) translation perpendicular to the axis $O_1^{(j)} z_1^{(j)}$ with a parameter $p_t^{(j)}$ in the plane $T^{(j)}$. The value of $p_t^{(j)}$ is in relation with the conic form of the conic worm. $\Sigma_1^{(j)}$ $(j = 1)$ is a conic convolute helical surface, which is turned to the positive direction of the axis $O_1^{(1)} z_1^{(1)}$, and $\Sigma_1^{(j)}$ $(j = 2)$ is a helicoid, which is turned to the negative direction of the axis $O_1^{(2)} z_1^{(2)}$. Particles of $\Sigma_1^{(1)}$ and $\Sigma_1^{(2)}$ are used as a active surfaces of the conic worm teeth.

The vector equation of the conic convolute helical surface $\Sigma_1^{(j)}$, according to Fig 2 is

$$\overline{\rho}_1^{(j)} = \overline{r}_0^{(j)} + \overline{s}^{(j)} + \overline{t}^{(j)} + \overline{u}^{(j)}, \qquad (3)$$

where $\overline{\rho}_1^{(j)}$ is radius - vector of point $N^{(j)}$ belonging to conic convolute helicoid; $\overline{r}_0^{(j)}$ - radius-vector of the directed cylinder; $u^{(j)}, \ \vartheta^{(j)}$ - curvilinear co-ordinates of helical surface; $s^{(j)} = p_s^{(j)} \vartheta^{(j)}$ - axial location moving of generatrix $L^{(j)}$; $t^{(j)} = p_t^{(j)} \vartheta^{(j)}$ - tangential moving of generatrix $L^{(j)}$.

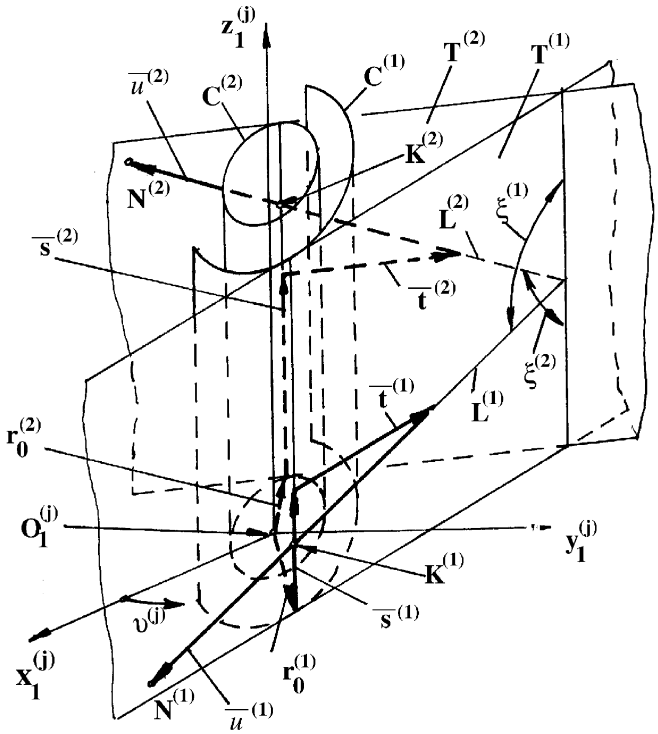

Fig. 2 Scheme of conic convolute (involute) helicoid generation

In the co-ordinate system $S_1^{(j)}(O_1^{(j)}, x_1^{(j)}, y_1^{(j)}, z_1^{(j)})$ for (1) it is obtained:

$$\begin{aligned}
x_1^{(j)} &= r_0^{(j)} \cos\vartheta^{(j)} \pm (u^{(j)} \sin\xi^{(j)} - p_t^{(j)} \vartheta^{(j)}) \sin\vartheta^{(j)}, \\
y_1^{(j)} &= r_0^{(j)} \sin\vartheta^{(j)} \mp (u^{(j)} \sin\xi^{(j)} - p_t^{(j)} \vartheta^{(j)}) \cos\vartheta^{(j)}, \\
z_1^{(j)} &= p_s^{(j)} \vartheta^{(j)} \pm u^{(j)} \cos\xi^{(j)}.
\end{aligned} \tag{4}$$

For the system of equation (4) upper signs and j=1 are referred to the $\Sigma_1^{(1)}$, j=2 and the following signs are referred to $\Sigma_1^{(2)}$. Substituting bellow expressions (5) into equations (4)

$$R_0^{(j)} = u^{(j)} - p_t^{(j)} \vartheta^{(j)} \Big/ \sin\xi^{(j)}, \quad p_1^{(j)} = p_s^{(j)} \pm p_t^{(j)} \cot\xi^{(j)}. \tag{5}$$

we obtained:

$$\begin{aligned}
x_1^{(j)} &= r_0^{(j)} \cos\vartheta^{(j)} \pm R_0^{(j)} \sin\xi^{(j)} \sin\vartheta^{(j)}, \\
y_1^{(j)} &= r_0^{(j)} \sin\vartheta^{(j)} \mp R_0^{(j)} \sin\xi^{(j)} \cos\vartheta^{(j)}, \\
z_1^{(j)} &= p_1^{(j)} \vartheta^{(j)} \pm R_0^{(j)} \cos\xi^{(j)}.
\end{aligned} \tag{6}$$

Equations (6) represent a conic convolute surface $\Sigma_1^{(j)}$ as a cylindrical convolute surface, with helical parameter $p_1^{(j)}$ and curvilinear co-ordinates $\vartheta^{(j)}$ and $R_0^{(j)}$. The point $K^{(j)}$ is the accounting origin of co-ordinate $R_0^{(j)}$ ($K^{(j)}$ is a point of intersecting of the $L^{(j)}$ and the generatrix of the directed circle cylinder $C^{(j)}$; $C^{(j)}$ and plane $T^{(j)}$ are contacting in this generatrix). This point $K^{(j)}$ is considered as a point from the directed helical line $\overline{\rho}_0^{(j)} = \overline{\rho}_0^{(j)}(\vartheta^{(j)})$ on the $C^{(j)}$. Let we present equation (6) in the following type [5]:

$$\overline{\rho}_1^{(j)} = \overline{\rho}_0^{(j)}(\vartheta^{(j)}) + R_0^{(j)}\bar{l}^{(j)},$$

$$\left\| \begin{matrix} \rho_{0,x_1}^{(j)} \\ \rho_{0,y_1}^{(j)} \\ \rho_{0,z_1}^{(j)} \end{matrix} \right\| = \left\| \begin{matrix} r_0^{(j)} \cos\vartheta^{(j)} \\ r_0^{(j)} \sin\vartheta^{(j)} \\ p_1^{(j)}\vartheta^{(j)} \end{matrix} \right\|, \quad \left\| \begin{matrix} l_{x_1}^{(j)} \\ l_{y_1}^{(j)} \\ l_{z_1}^{(j)} \end{matrix} \right\| = \left\| \begin{matrix} \pm\sin\xi^{(j)} \sin\vartheta^{(j)} \\ \mp\cos\xi^{(j)} \cos\vartheta^{(j)} \\ \pm\cos\xi^{(j)} \end{matrix} \right\|, \tag{7}$$

where $\bar{l}^{(j)}$ is the vector of direction of $L^{(j)}$.
Then for the parameter of distribution of the conic convolute helicoid $\Sigma_1^{(j)}$ we can write:

$$h_1^{(j)} = [d\overline{\rho}_0^{(j)},\ \bar{l}^{(j)},\ d\bar{l}^{(j)}] \big/ d\bar{l}^{(j)2}. \tag{8}$$

From (7) and (8) after transformation is obtained:

$$h_1^{(j)} = p_1^{(j)} + r_0^{(j)} \cot\xi^{(j)}. \tag{9}$$

Equations (4) and (6) describe the geometry of the conic convolute helicoid in the most common case, when $h_1^{(j)} \neq 0$. If $h_1^{(j)} = 0$, i.e.

$$\cot\xi^{(j)} = -p_1^{(j)} \big/ r_0^{(j)}. \tag{10}$$

$\Sigma_1^{(j)}$ transforms in conic involute helicoid. $j = 1$, and upper signs in all equations and in further equations are referred to the active surface $\Sigma_1^{(1)}$ which rotates with the angular velocity ϖ_1 and the corresponding translation velocity of $\Sigma_2^{(1)}$ is $\overline{V}_2$ (see Fig. 1); $j = 2$ and the lower signs are referred to those meshed surfaces $\Sigma_1^{(2)}$ and $\Sigma_2^{(2)}$ which velocities are (-ϖ_1) and (-$\overline{V}_2$).

3.2. Geometry of the conic Archimedean helicoid

We will examine the synthesis of the active flanks $\Sigma_1^{(j)}$ (j=1, 2) of an Archimedean conic worm with right-hand threads. The process of generation of $\Sigma_1^{(j)}$ is shown in Fig. 3. In the case $\Sigma_1^{(j)}$ is an Archimedean conic helicoid which generatix $L^{(j)}$ is a straight line intersecting the axis $O_1^{(j)}z_1^{(j)}$, i.e. $r_0^{(j)} = 0$ ($L^{(j)}$lies in the plane $A^{(j)}$). $\pi/2 < \xi^{(j)} < \pi$ is the angle between $L^{(j)}$ and the direction of the axis $O_1^{(j)}z_1^{(j)}$. The generatix $L^{(j)}$ performs a complex motion, as it is in generation of the conic convolute helicoid.

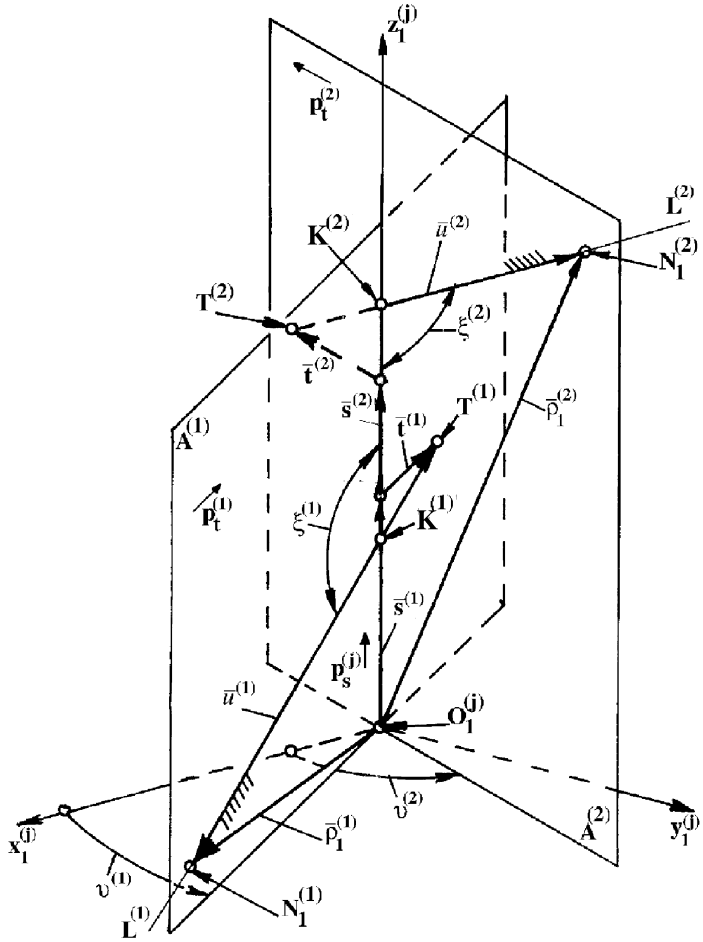

Fig 3 Scheme of conic Archimedean helicoid generation

Then the vector equations of the conic Archimedean helicoid $\Sigma_1^{(j)}$ and their analytical form in the system $S_1(O_1, x_1, y_1, z_1)$ firmly connected with the conic worm, respectively are:

$$\bar{\rho}_1^{(j)} = \bar{s}^{(j)} + \bar{t}^{(j)} + \bar{u}^{(j)}, \tag{11}$$

$$\begin{aligned} x_1^{(j)} &= (u^{(j)} \sin\xi^{(j)} - p_t^{(j)}\vartheta^{(j)})\cos\vartheta^{(j)}, \\ y_1^{(j)} &= (u^{(j)} \sin\xi^{(j)} - p_t^{(j)}\vartheta^{(j)})\sin\vartheta^{(j)}, \\ z_1^{(j)} &= p_s^{(j)}\vartheta^{(j)} \pm u^{(j)}\cos\xi^{(j)}. \end{aligned} \tag{12}$$

$$x_1^{(j)} = R_0^{(j)} \boldsymbol{sin}\, \xi^{(j)} \boldsymbol{cos}\, \vartheta^{(j)},$$
$$y_1^{(j)} = R_0^{(j)} \boldsymbol{sin}\, \xi^{(j)} \boldsymbol{sin}\, \vartheta^{(j)}, \tag{13}$$
$$z_1^{(j)} = p_1^{(j)} \vartheta^{(j)} \pm R_0^{(j)} \boldsymbol{cos}\, \xi^{(j)}.$$

3.3. Geometry of the helical rack tooth surfaces

In the case of spatial rack set the equations of the helical rack tooth surfaces $\Sigma_2^{(j)}$ (j=1, 2) are written by using the equations of their kinematic conjugate conic linear helicoids $\Sigma_1^{(j)}$ (j=1, 2) belonging to the conic worm. When synthesizing conjugate active flanks of spatial gear mechanism the well known in the gearing theory a basic equation of meshing is applied [7]. This method is based on the regularity that the relative velocity in every pitch contact point of conjugate tooth surfaces $\Sigma_1^{(j)}$ and $\Sigma_2^{(j)}$ in every mesh moment lies in their common tangent plane. Analytical description of this statement is defined as the equation:

$$\bar{n}_1^{(j)} . \bar{V}_{12} = n_{1,x}^{(j)} V_{12,x} + n_{1,y}^{(j)} V_{12,y} + n_{1,z}^{(j)} V_{12,z} = f(u^{(j)}, \vartheta^{(j)}, \varphi_1^{(j)}) = 0. \tag{14}$$

The relation (14) is known as the **basic equation of meshing**. Here $\bar{n}_1^{(j)}$ is the normal vector to $\Sigma_1^{(j)}$; $\bar{V}_{12}$ is the defined with equation (2) relative velocity between $\Sigma_1^{(j)}$ and $\Sigma_2^{(j)}$ in the random contact point; $\varphi_1^{(j)}$ is the parameter of meshing.

Following the kinematic method, we determine the equations of the mesh region, as a locus of the contact lines of $\Sigma_1^{(j)}$ and $\Sigma_2^{(j)}$ in the static space.

Then

$$\bar{n}_1^{(j)} = \partial \bar{\rho}_1^{(j)} \Big/ \partial u_1^{(j)} \times \partial \bar{\rho}_1^{(j)} \Big/ \partial \vartheta_1^{(j)}. \tag{15}$$

Let we write (15) in co-ordinate system $S_1^{(j)}(O_1^{(j)}, x_1^{(j)}, y_1^{(j)}, z_1^{(j)})$ for cases of the conic convolute helicoid, conic involute helicoid, and conic Archimedean helicoid. Then from the systems of equations (4) and (5) for the normal vectors of these surfaces we derived respectively:

- for conic convolute helicoid ($h_1^{(j)} \neq 0$)

$$n_{1,x_1}^{(j)} = \mp h_1^{(j)} \boldsymbol{sin}\, \xi^{(j)} \boldsymbol{cos}\, \vartheta^{(j)} - U^{(j)} \boldsymbol{cos}\, \xi^{(j)} \boldsymbol{sin}\, \vartheta^{(j)},$$
$$n_{1,y_1}^{(j)} = \mp h_1^{(j)} \boldsymbol{sin}\, \xi^{(j)} \boldsymbol{sin}\, \vartheta^{(j)} + U^{(j)} \boldsymbol{cos}\, \xi^{(j)} \boldsymbol{cos}\, \vartheta^{(j)}, \tag{16}$$
$$n_{1,z_1}^{(j)} = U^{(j)} \boldsymbol{sin}\, \xi^{(j)},$$

- for conic involute helicoid ($h_1^{(j)} = 0$)

$$n_{1,x_1}^{(j)} = -U^{(j)} \cos\xi^{(j)} \sin\vartheta^{(j)},$$

$$n_{1,y_1}^{(j)} = U^{(j)} \cos\xi^{(j)} \cos\vartheta^{(j)}, \tag{17}$$

$$n_{1,z_1}^{(j)} = U^{(j)} \sin\xi^{(j)},$$

- for conic Archimedean helicoid ($r_0^{(j)} = 0$)

$$n_{1,x_1}^{(j)} = p_1^{(j)} \sin\xi^{(j)} \sin\vartheta^{(j)} \mp U^{(j)} \cos\xi^{(j)} \cos\vartheta^{(j)},$$

$$n_{1,y_1}^{(j)} = -p_1^{(j)} \sin\xi^{(j)} \cos\vartheta^{(j)} \mp U^{(j)} \cos\xi^{(j)} \sin\vartheta^{(j)}, \tag{18}$$

$$n_{1,z_1}^{(j)} = U^{(j)} \sin\xi^{(j)},$$

when $U^{(j)} = u^{(j)} \sin\xi^{(j)} - p_t^{(j)}\vartheta^{(j)}$.

Using the transition matrix from a system S_1 to S:

$$M_{SS_1} = \begin{Vmatrix} \cos\varphi_1^{(j)} & \sin\varphi_1^{(j)} & 0 & 0 \\ -\sin\varphi_1^{(j)} & \cos\varphi_1^{(j)} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{Vmatrix}, \quad L_{SS_1} = \begin{Vmatrix} \cos\varphi_1^{(j)} & \sin\varphi_1^{(j)} & 0 \\ -\sin\varphi_1^{(j)} & \cos\varphi_1^{(j)} & 0 \\ 0 & 0 & 1 \end{Vmatrix}.$$

For the equation of the mesh region is obtained:

-for spatial convolute rack mechanism

$$y^{(j)}.n_{1,x}^{(j)} - (x^{(j)} + j_{21} \sin\delta) n_{1,y}^{(j)} + j_{21} \cos\delta n_{1,z}^{(j)} = 0,$$

$$x^{(j)} = r_0^{(j)} \cos(\vartheta^{(j)} - \varphi_1^{(j)}) \pm U^{(j)} \sin(\vartheta^{(j)} - \varphi_1^{(j)}),$$

$$y^{(j)} = r_0^{(j)} \sin(\vartheta^{(j)} - \varphi_1^{(j)}) \mp U^{(j)} \cos(\vartheta^{(j)} - \varphi_1^{(j)}),$$

$$z^{(j)} = p_1^{(j)} \vartheta^{(j)} \pm U^{(j)} \cot\xi^{(j)}, \tag{19}$$

$$n_{1,x}^{(j)} = \mp h_1^{(j)} \sin\xi^{(j)} \sin(\vartheta^{(j)} + \varphi_1^{(j)}) - U^{(j)} \cos\xi^{(j)} \cos(\vartheta^{(j)} + \varphi_1^{(j)}),$$

$$n_{1,y}^{(j)} = \pm h_1^{(j)} \sin\xi^{(j)} \cos(\vartheta^{(j)} + \varphi_1^{(j)}) + U^{(j)} \cos\xi^{(j)} \sin(\vartheta^{(j)} + \varphi_1^{(j)}),$$

$$n_{1,z}^{(j)} = U^{(j)} \sin\xi^{(j)},$$

-for spatial involute rack mechanism

$$
\begin{aligned}
& y^{(j)}.n_{1,x}^{(j)} - (x^{(j)} + j_{21}\,\boldsymbol{sin}\,\delta)n_{1,y}^{(j)} + j_{21}\,\boldsymbol{cos}\,\delta n_{1,z}^{(j)} = 0,\\
& x^{(j)} = r_0^{(j)}\,\boldsymbol{cos}(\vartheta^{(j)} - \varphi_1^{(j)}) \pm U^{(j)}\,\boldsymbol{sin}(\vartheta^{(j)} - \varphi_1^{(j)}),\\
& y^{(j)} = r_0^{(j)}\,\boldsymbol{sin}(\vartheta^{(j)} - \varphi_1^{(j)}) \mp U^{(j)}\,\boldsymbol{cos}(\vartheta^{(j)} - \varphi_1^{(j)}),\\
& z^{(j)} = p_1^{(j)}\vartheta^{(j)} \pm U^{(j)}\,\boldsymbol{cot}\,\xi^{(j)},\\
& n_{1,x}^{(j)} = -U^{(j)}\,\boldsymbol{cos}\,\xi^{(j)}\,\boldsymbol{cos}(\vartheta^{(j)} + \varphi_1^{(j)}),\\
& n_{1,y}^{(j)} = U^{(j)}\,\boldsymbol{cos}\,\xi^{(j)}\,\boldsymbol{sin}(\vartheta^{(j)} + \varphi_1^{(j)}),\\
& n_{1,z}^{(j)} = U^{(j)}\,\boldsymbol{sin}\,\xi^{(j)},
\end{aligned}
\tag{20}
$$

-for spatial Archimedean rack mechanism

$$
\begin{aligned}
& y^{(j)}.n_{1,x}^{(j)} - (x^{(j)} + j_{21}\,\boldsymbol{sin}\,\delta)n_{1,y}^{(j)} + j_{21}\,\boldsymbol{cos}\,\delta n_{1,z}^{(j)} = 0,\\
& x^{(j)} = U^{(j)}\,\boldsymbol{cos}(\vartheta^{(j)} - \varphi_1^{(j)}),\\
& y^{(j)} = U^{(j)}\,\boldsymbol{sin}(\vartheta^{(j)} - \varphi_1^{(j)}),\\
& z^{(j)} = p_1^{(j)}\vartheta^{(j)} \pm U^{(j)}\,\boldsymbol{cot}\,\xi^{(j)},\\
& n_{1,x}^{(j)} = p_1^{(j)}\,\boldsymbol{sin}\,\xi^{(j)}\,\boldsymbol{sin}(\vartheta^{(j)} - \varphi_1^{(j)}) \mp U^{(j)}\,\boldsymbol{cos}\,\xi^{(j)}\,\boldsymbol{cos}(\vartheta^{(j)} - \varphi_1^{(j)}),\\
& n_{1,y}^{(j)} = -p_1^{(j)}\,\boldsymbol{sin}\,\xi^{(j)}\,\boldsymbol{cos}(\vartheta^{(j)} - \varphi_1) \mp U^{(j)}\,\boldsymbol{cos}\,\xi^{(j)}\,\boldsymbol{sin}(\vartheta^{(j)} - \varphi_1^{(j)}),\\
& n_{1,z}^{(j)} = U^{(j)}\,\boldsymbol{sin}\,\xi^{(j)}.
\end{aligned}
\tag{21}
$$

The sets (19), (20) and (21) describes the locus of the contact lines between $\Sigma_1^{(j)}$ and $\Sigma_2^{(j)}$ in the static space. The active flanks $\Sigma_2^{(j)}$ (j=1, 2) is received as locus of the contact line D_{12} in the co-ordinate system S_2:

$$
\begin{aligned}
& f(u^{(j)},\vartheta^{(j)},\varphi_1^{(j)}) = 0,\ \ \overline{\rho}_1^{(j)} = \overline{\rho}_1^{(j)}(u^{(j)},\vartheta^{(j)}),\\
& \rho_2^{(j)} = M_{S_2S_1}\,\rho_1^{(j)},\ \ M_{S_2S_1} = M_{S_2S}.M_{SS_1},
\end{aligned}
\tag{22}
$$

$$
M_{S_2S} = \begin{Vmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & -j_{21}\varphi_1^{(j)}\,\boldsymbol{sin}\,\delta\\ 0 & 0 & 1 & j_{21}\varphi_1^{(j)}\,\boldsymbol{cos}\,\delta\\ 0 & 0 & 0 & 1 \end{Vmatrix}.
$$

Conclusion

The equations of the active tooth surfaces of spatial convolute, involute and Archimedean rack sets obtained in the article represent the base of the algorithm for synthesis and analyze of these class mechanisms. The analytical relations are very important for the geometrical and technological synthesis of the spatial rack mechanism. They have the important place for the design of the tools for gear cutting, and for constructing of the control equipment, in the generating process of the tooth surfaces. When synthesizing gear sets of the examined type it is exclusively essential to be pointed out the singular points in their mesh region. Moreover, the proportion of $\Sigma_1^{(j)}$ and $\Sigma_2^{(j)}$ are limited in the design process so that the singularity in the mesh region to be eliminated. The written relations in this study are oriented to solve problems of elimination of the singular points of the spatial rack mesh region

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