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OSCILLATOR WITH HERTZ'S MODEL OF SOFT IMPACTS

František PETERKA*

Abstract: *Dynamic properties of the mechanical system with one degree of freedom containing soft stop are investigated. Dynamical impact model respects the non-linearity of the restoring contact force between solid bodies as function of deformation according to the Hertz theory. The explanation of the system motion behaviour, when static clearance decreases up to negative values, which express the static prestress, is the main aim of this paper. This process is explained in more detai using phase trajectories, time series and Poincarè maps.*

Keywords: motion with soft impacts, small and negative static clearance, numerical simulation, periodic and chaotic motions

1. Introduction

Dynamic properties of the mechanical system with one degree of freedom containing soft stop (Fig.1) are investigated. Oscillator is excited by harmonic force $F_0\cos\eta\tau$ and its mass m can impact against a soft stop situated in certain distance ρ from the mass equilibrium position. The proposed general dynamical impact model (Fig.2) respects the non-linearity of the restoring contact force between solid bodies as function of deformation (according to the Hertz theory) and velocity in the form $F(X,X')=f(X)(1+g(X'))$. It describes the real behaviour of impacting system more exact than the Kelvin-Voigt model or model based on application of constant coefficient of restitution (according to the Newton impact theory).

Various types of impact motion exist in the dependence on dimensionless parameters ρ and η (Fig.3). Main effort was concentrated on the study of transitions between individual regions corresponding to periodic and chaotic impact motions. Significant differences were ascertained in comparison with those obtained for oscillator with rigid impacts. Forced vibrations with impacts were shown [1], [2] also using phase trajectories, time histories, bifurcation diagrams, basins of attraction and Poincarè maps.

The explanation of the system motion behaviour, when clearance □ decreases up to negative values, is the main aim of this paper. It is known in this case, that system motion with rigid impacts $\left(F(\rho, X') \to \infty\right)$ is characterized by the increase of the impact number z and the motion transits into the regime with sliding impacts (Fig.4). The impacting mass m is connected with the rigid stop for a certain part or all excitation period (dead zone). Similar behaviour appears firstly also for the motion with soft impacts (e.g. z=5/1, Fig.5(a)), but when negative clearance (the static prestress of mass

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* Ing. František Peterka, DrSc., Institute of Thermomechanics, AS CR, Dolejškova 5, 182 00 Prague 8, Phone:+420/2/66053083, FAX:+420/2/8584695, E-mail: peterka@it.cas.cz

m to the stop) increases, then impact number *z* decreases (e.g. *z*=4/1, Fig.5(b)). The system motion transits finally into the impactless motion (*z*=0), when mass *m* vibrates all the time in the connection with the soft stop. This process is explained in more detail along vertical line a (η=0.6) in Fig.3 using phase trajectories, time series and Poincarè maps.

2. Transition into impactless motion with decreasing clearance

Region labelled by $z = 3/1$ in Fig.3 contains all periodic and chaotic impact motions beginning $z = 3/1$. Their subregions are very narrow and therefore the behaviour of the system will be explained by the decrease of clearance ρ along line **a**. There exists series of fundamental motions $z = p/1$ (*p* = 0, 1, 2, 3, 4; see Figs.6(a),(b), (c),(d),10(g),5(a)). Transition hysteresis and beat-motion regions exist between neighbour regions of fundamental impacts motion [3]. Motions which exist in beat-motion region between regions of $z = 3/1$ and $z = 4/1$ motions are shown in Figs. 6-10, where are also phase trajectories (X, X'), e.g. Fig.7(a) and Poincarè maps (φ, X'_-), e.g. Fig.7(c). Poincarè maps indicate the state system motion at instant just before impact ($X = \rho$, X'_- - before impact velocity, φ-phase of excitation force by green circles. For example, the state of system at instant just after impact is indicated by red circles in Fig.7(d).

Figures are labelled by impact number *z*, value of which lies in interval 3<*z*<4 and it increases with decreasing ρ. Fractional values correspond to periodic subharmonic impact motion and decimal values are mean values of chaotic impact motion. Many periodic motions were ascertained by the change of ρ with gradual steps of order 0.0001.

Main series of different subharmonic motions is:
$z = 6/2$ (Fig.7(b)), $z = 7/2$ (Fig.7(d)), $z = 20/6$ (Figs.8(a),(c)), $z = 40/12$ (Figs.8(e),(g)) as an example of period doubling of $z = 20/6$ motion, $z = 33/10$ (Fig.9(a)), $z = 13/4$ (Fig.9(c)), $z = 22/7$ (Fig.9(d)), $z = 17/5$ (Fig.9(e)), $z = 16/5$ (Fig.9(f)) as an example of the loss of impact from $z = 17/5$ impact motion on grazing boundary, $z = 10/3$ (Fig.9(g)), $z = 11/3$ (Fig.9(h)) as an example of additional impact in $z = 10/3$ motion appearing on grazing boundary, $z = 18/5$ (Fig.10(a)), $z = 15/4$ (Fig.10(b)), $z = 16/4$ (Fig.10(d)) and $z = 8/2$ (Fig.10(e)) as examples of period doublings of $z = 4/1$ motion (Figs.10(f),(g),(h)).

There exist also many other periodic subharmonic motions developed from mentioned above motions by periodic doublings. Periodic motions are immersed into chaotic motion (periodic windows in the chaos) shown for example in Figs.8(d),(f),(h), 9(b), 10(c). All four ways from periodic into chaotic impact motion, explained for motion with rigid impacts [4], [5] were ascertained also for this system with soft impacts. For example, the intermittency chaos (Figs.8(b),(d)) arises with increasing ρ from $z = 20/6$ (Fig.8(a)) on its saddle-node stability boundary ρ = - 0.272092). This chaotic motion jumps then into periodic motion $z = 6/2$ (Fig.7(d)), when ρ = - 02699. It means, that hysteresis of chaotic motion into $z = 2/6$ motion exists, which is characteristic feature of this way into chaos connected with the jump from chaotic into periodic impact motion.

This wide variety of periodic subharmonic and chaotic motions is caused by the exclusion of viscous damping *b* of spring *k* (Fig.1) during the contact of mass *m* with the stop, while out of contact acts the damping *b*=0.1 $\sqrt{km}$.

Similar complex system behaviour was ascertained also in subregion of subharmonic periodic and chaotic motions between $z = 4$ (Fig.10(g)) and $z = 5$ (Fig.5(a)) motions. Periodic motions z = 21/5, 9/2, 23/5, 14/3, 28/6, 42/9, 34/7 and their period doubling derivatives are again immersed into the region of chaotic motion. Such motions are not shown here.

It is apparent from mentioned two series of z, that subharmonic motions of order n = 2, 3, 4, 5, 7, 9, 10 exist in transition beat motion regions between regions of fundamental motions z=3, z=4, z=5.

The increase of quantity z does not continue with next increase of negative clearance ρ. The physical explanation of this phenomenon consists in increasing static prestress of the body m to the soft stop and some oscillation of higher frequency, caused by the stop stiffness, does not lose the contact with the stop. The number of such oscillation increases (Fig.5(b)) and therefore number z decreases. Next fundamental motion $z = 3/1$ is shown in Fig.11(a). Between motions $z = 5$, $z = 4$ and $z = 4$, $z = 3$ exist again subharmonic and chaotic motions, but motion $z = 3$ transits by jump into $z = 1/1$ (Fig.11(b)) on the saddle-node stability boundary. Finally, the last impact gradually disappears and system moves without impacts in the steady connection of mass m with the stop. As the system motion turns to impactless motion, the intensity of impacts weakens and natural oscillations with high frequency die (Fig.11) and excited vibrations remain.

3. Conclusion

The main result of this paper is the explanation of the behaviour of the oscillator with soft impacts, when impacts disappear owing to the increase of static prestress, i.e. large negative clearance. Quantity z, which characterizes the impact motion, firstly increases and later decreases to $z = 0$ corresponding to impactless motion. During this process were ascertained all characteristic features of impact oscillator dynamics as different bifurcations, instabilities, ways into chaos etc.

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