Národní konference s mezinárodní účastí

INŽENÝRSKÁ MECHANIKA 2002

13. – 16. 5. 2002, Svratka, Česká republika

# OPTIMIZATION OF A VIBRATING BLOCK FOUNDATION

Vít Šmilauer[1],
Jiří Máca[2]

**Abstract**

The procedure based on a traditional genetic algorithm (GA) is demonstrated on the price optimization of a vibrating block foundation. Unbalanced rotating mass in a machine produces harmonic dynamic load that is reduced by a set of dashpots supporting a foundation block with a machine. Six parameters have to be optimized: 3 dimensions of the block, number of rows for two directions of dashpots and the type of a dashpot in a database. Restrictions on a maximal displacement and on dashpot configuration are introduced. Fitness function represents the price for materials plus penalty function due to unsatisfied restrictions. GA with specified additional parameters searches for a minimal value of a fitness function.

Keywords: *genetic algorithm, penalty function, block foundation, optimization, damping, vibration, design.*

## 1 Introduction

A design procedure can be accomplished is two ways: a classical approach, i.e. design and evaluation procedure or using appropriate optimization method. Genetic algorithms (GAs) are stochastic methods able to find an optimal solution in case of a discontinuous functions, varying conditions during calculation and among large sets of variables. The principles of GAs are known from 1960's and since that time were largely improved and applied in numerical function optimization, photo merging tests, combinatorial optimization etc. In the civil engineering problems were applied in optimization of strength/weight ratio, bridge design, in concrete frames and their reinforcement [3], for example. In this case a genetic algorithm is applied to the price optimization of a vibrating block foundation.

[1]Vít Šmilauer, Czech Technical University in Prague, Faculty of Civil Engineering, smilauer@post.cz

[2]Doc. Ing. Jiří Máca, CSc., Czech Technical University in Prague, Faculty of Civil Engineering, Department of Structural Mechanics, maca@fsv.cvut.cz

## 2 Principles of GA

Solving any problem requires knowledge of the solving algorithm step by step. A solution results from input values. Such solution is correct but the question is if there exists a better one. In case of a simple task the analytic solution can be found but this is not the universal approach to find an optimum. The universal and robust procedure is to search for optimal solution within the whole range of variables. GAs are more flexible on the contrary to analytical methods and they do not finish necessary in local extremes. GAs do not guarantee to find an optimum, they will find solution good enough appropriate to the elapsed time; longer time means searching bigger areas for optimum so the probability to find the optimum increases.

GAs are based on genetic processes of organisms. Charles Darwin's theory of evolution states that population evolves according to the principles of natural selection and the strongest organisms survive. The life is a struggle among organisms for resources and mates attracting. The most successful individuals have the biggest chance to have more offspring in the next generation. Good characters are easily spread among most successful individuals and can produce the better offspring. Over the number of generations poor individuals die out and the best multiply.

Genetic information of organism is stored in DNA. Information form *gene*, genes *chromosome*. The set of parameters represented by specific chromosome is *genotype* in genetic terms. The information stored in genotype creates an *individual* (organism), referred to as *phenotype*. The set of organisms makes *population*. The set of approximately equally old individuals creates *generation*. GAs are the analogy to nature. Genes are specific variables, chromosome is a set of variables and individual represents the solution.

The representation of a number in a gene is called coding and is crucial to a successful outcome. The first and most simple coding similar to nature is binary, represented only by 0's and 1's. It's simple and can be improved using Gray's coding. The main disadvantages are limited precision and handling negative numbers. The real coding is more difficult but reasonable where real numbers are used.

The *fitness function* assigns a number to the individual proportional to his utility. In case of function optimization it can be function value, in GAs' application price, strength/weight ratio, deformation, time consumption etc. The value of fitness function states how individual is involved in reproduction. If an individual does not satisfy given restrictions, e.g. exceeded deformation, he is not disqualified but charged with penalty function, figure 1. Such individual could namely contain another desirable useful gen.

New genes appear as the result of reproduction. The most simple kind of binary reproduction is *crossover*; swapping two parts of chromosomes at random position. In a few pro miles of cases *mutation* of genes is applied. It randomly alters gene helping to obtain new information that would appear rarely and may be rescue from a local extreme. There is a lot of possibilities how the individual evolves, the most simple one is introduced. Suppose knowing the number, range and precision of variables, number of individuals in population, fitness function

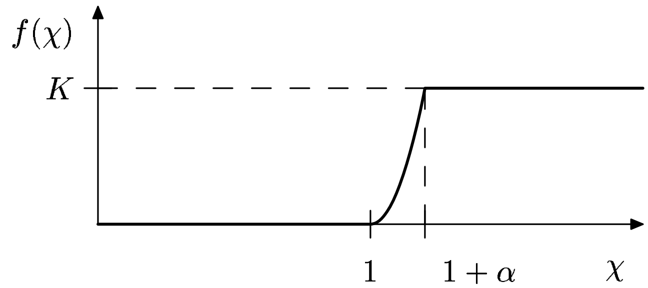


Figure 1: Penalty function. $\chi = \phi/\phi_{max}$, where $\phi$ is a variable. The part of penalty function follows $f(\chi) = L((\chi - 1)/\gamma)^{\beta}$.

and the number of generations. The variables are coded into genes as binary numbers so it leads to the discretization of the variable space. At the beginning the number of individuals is randomly created, evaluated by fitness function and selected for reproduction based on fitness function, e.g. roulette wheel selection. Two offspring from two parents are created and some genes are mutated. There is a new population of individuals and the process is repeated. The repetition is given either by the number of generations or by required precision. At the end is found the best individual of all generations.

The GA used in a foundation block optimization is taken from MIT [4] developed in years 1995-1996. The coding most appropriate to the problem is the real one with certain precision. The fitness function is the price calculated as the sum from the price of dashpots, concrete and the penalty function, figure 1. The algorithm uses searching for the maximum, the minimum is the opposite task: subtraction from any big value or negative numbers used in this case yield the same result. GA is a "simple" algorithm by Goldberg [2] with a selection for reproduction based on the variation from the average. It provides better variability in population.

## 3 Task formulation

Idealization and simplification of objects is introduced into modeling of a block foundation: the concrete block is a rigid body, driving force is harmonic, stiffness of dashpots is constant, viscous damping is introduced, machine with the block foundation is symmetric by two vertical planes, fatigue of materials is neglected. The task is solved as a steady-state vibration of a rigid block with 3 degrees of freedom in 2D. Damping plays an important role in high frequencies and around resonance, for a particular case the role of damping is shown on figure 3.

The basic scheme of the block foundation and the machine is on figure 2. The 6 unknown parameters for optimizations are: dimensions $a$, $b$, $c$, number of rows of dashpots in $a$, $b$ directions, the number of the dashpot is a database - dimensions, stiffness, maximal static load, price. The range of variables is set up in an input file. Other parameters are input values: rotating unbalanced mass $m_0$ with eccentricity $e$, the mass of machine $m_s$, the exciting frequency $f$, mass-moment of inertia of the machine $I_s$ to own axis in the center of gravity parallel with the side $b$, the position of the machine center of gravity, the maximal displacement of machine in the center of gravity $w_{max}$, the specific mass and the

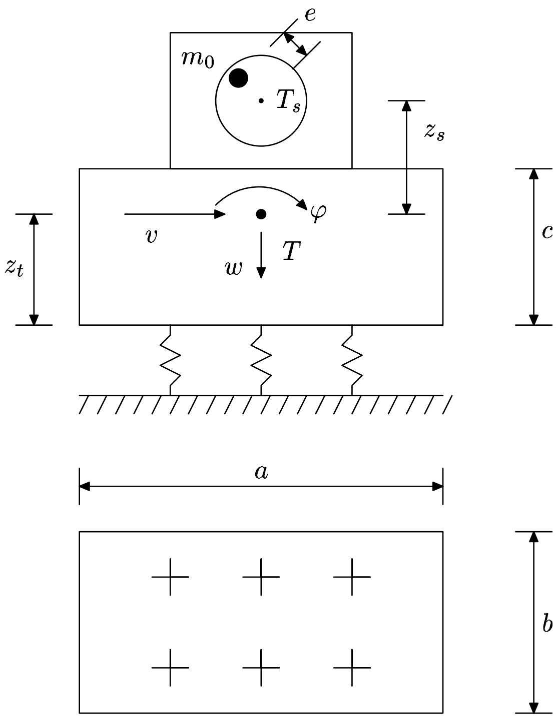


Figure 2: The scheme of a block foundation and machine with variable description.

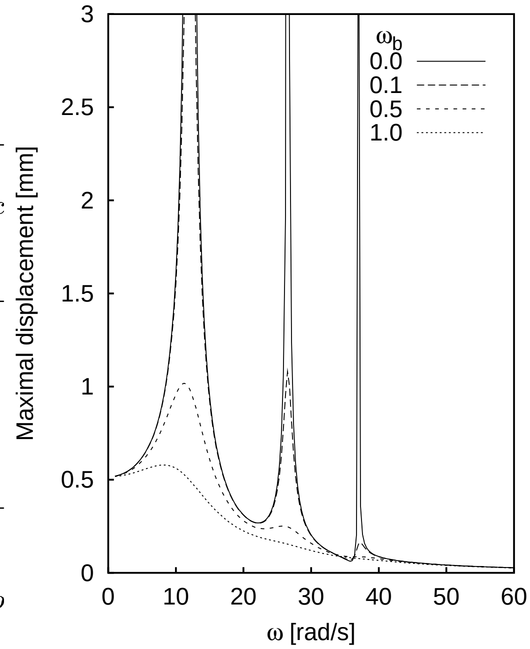


Figure 3: Damping with different $\omega_b$ and corresponding maximal displacement of machine at the center of gravity $T_s$.

price of concrete for 1 m$^3$, minimal distances of dashpots from the block edges, vertical to horizontal ratio of stiffness $q$, the relative damping $b_r$. The GA needs additional parameters: the number of individuals in one generation, the number of generations, the mutation and cross-over probability.
GA generates individuals with 6 genes resulting into different fitness values using following algorithm:

1. the exciting frequency $\omega = 2\pi f$ and amplitude of the driving force $F_b = m_0 e \omega^2$
2. the mass of the concrete block $m_z$
3. the common center of inertia of the block and the machine $T$
4. the mass-moment of inertia to the block axis $I_z$ intersecting the center of gravity of the block and parallel to side $b$ and the total mass-moment of inertia $I_{tot}$ similarly defined to the $T$ axis.
5. the lumped mass matrix

$$\mathbf{M} = \begin{pmatrix} m_z + m_s & 0 & 0 \\ 0 & m_z + m_s & 0 \\ 0 & 0 & I_{tot} \end{pmatrix}$$

6. the stiffness matrix of the structure

$$\mathbf{K} = \begin{pmatrix} \sum_{i=1}^{n} k_i & 0 & 0 \\ 0 & \sum_{i=1}^{n} qk_i & -\sum_{i=1}^{n} z_t qk_i \\ 0 & -\sum_{i=1}^{n} z_t qk_i & \sum_{i=1}^{n} (y_i^2 k_i + z_t^2 qk_i) \end{pmatrix}$$

n is number of dashpots,
$z_t$ see figure 2,
$y_i$ is the distance from the axis of dashpot to the vertical plane of symmetry

7. solving equation of steady-state vibrations with damping

$$\mathbf{M}\ddot{\mathbf{r}} + \mathbf{B}\dot{\mathbf{r}} + \mathbf{K}\mathbf{r} = \mathbf{F} \tag{1}$$

where

$$\mathbf{F_c} = (0, F_b, F_b z_s)^T \tag{2}$$
$$\mathbf{F_s} = (F_b, 0, 0)^T \tag{3}$$
$$\mathbf{F} = \mathbf{F_c} \cos \omega t + \mathbf{F_s} \sin \omega t \tag{4}$$
$$\mathbf{r} = \mathbf{r_c} \cos \omega t + \mathbf{r_s} \sin \omega t \tag{5}$$

and with considered Rayleigh's damping with the fundamental frequency $\omega_{(1)}$

$$\alpha = \frac{b_r}{\omega_{(1)}} \tag{6}$$
$$\beta = b_r \omega_{(1)} \tag{7}$$
$$\mathbf{B} = \alpha \mathbf{M} + \beta \mathbf{K} \tag{8}$$

The backsubstitution from equation 5 and 4 to 1 yields

$$\begin{aligned} \left(\mathbf{K} - \omega^2 \mathbf{M}\right) \mathbf{r_c} + \omega \mathbf{B} \mathbf{r_s} &= \mathbf{F_c} \\ -\omega \mathbf{B} \mathbf{r_c} + \left(\mathbf{K} - \omega^2 \mathbf{M}\right) \mathbf{r_s} &= \mathbf{F_s} \end{aligned} \tag{9}$$

The maximal values of $\mathbf{r}$ are in a phase shift with the machine frequency. The equation 5 for the vertical movement gives the solution to the maximal values of the vertical displacement:

$$\mathbf{r}_1(t) = \mathbf{r_{c,1}} \cos \omega t + \mathbf{r_{s,1}} \sin \omega t \tag{10}$$

Finding extreme value after deriving equation 10 yields:

$$\mathbf{r_{c,1}} \omega \cos \omega t - \mathbf{r_{s,1}} \omega \sin \omega t = 0 \tag{11}$$
$$\varphi = \arctan \frac{\mathbf{r_{c,1}}}{\mathbf{r_{s,1}}} \tag{12}$$

The same phase shift is valid for horizontal displacement $\mathbf{r_2}$ and rotation $\mathbf{r_3}$. Horizontal plus rotational displacement is added as vector to the vertical displacement resulting in total displacement at the center of machine gravity.

8. penalty function, figure 1, expresses restrictions: displacement bigger than $w_{max}$ or small distance among dashpots. Total cost is the sum of the penalty function and the price for the concrete and the dashpots.

9. all individuals are evaluated and is found the one with the lowest fitness function. If the fitness function includes penalty it means insufficient number of individuals or too strict restrictions that are not able to be satisfied.

## 4 Optimization results

As an example a vibrating block foundation with following parameters was chosen:
a $\in \langle 0.5; 3\rangle$ m
b $\in \langle 0.5; 5\rangle$ m
c $\in \langle 0.5; 2\rangle$ m
the machine mass = 6 t
the unbalanced mass $m_0 = 200$ kg
the eccentricity of unbalanced mass $e = 2e^{-4}$ m
the exciting frequency $f = 8$ Hz
the specific mass of concrete $\rho = 2500$ kg/m$^3$
the price for m$^3$ of concrete with labor and reinforcement = 3000 Kč
the center of gravity of machine above top side of a block = 0.7 m
vertical to horizontal ratio of stiffness $q = 0.7$
the minimal distance of a dashpot from a block edge = 0.2 m
the maximal displacement of the machine at $T_s = 4e^{-6}$ m
the number of generations = 10000
maximal number of rows of in direction $a$, $b$ = 10, 10
number of individuals in one generation = 30
relative damping $b_r = 0.12$
used dashpots are in table 1, side swapping $a$ and $b$ is considered.

| max. [kg] | k [kN/m] | a [mm] | b [mm] | c [mm] | price [Kč] | number |
|---|---|---|---|---|---|---|
| 30 | 120 | 92 | 55 | 40 | 310 | 0 |
| 50 | 160 | 92 | 55 | 40 | 438 | 1 |
| 50 | 160 | 122 | 77 | 50 | 542 | 2 |
| 100 | 200 | 122 | 77 | 50 | 690 | 3 |
| 100 | 250 | 166 | 95 | 75 | 712 | 4 |
| 200 | 290 | 166 | 95 | 75 | 968 | 5 |
| 200 | 400 | 185 | 115 | 85 | 932 | 6 |
| 350 | 510 | 185 | 115 | 85 | 1284 | 7 |
| 350 | 690 | 150 | 150 | 83 | 1232 | 8 |
| 600 | 850 | 150 | 150 | 83 | 1710 | 9 |

Table 1: Parameters of dashpots used in optimization

The penalty function, figure 1, for displacement has parameters $L = 10^7$, $\alpha = 1$, $\beta = 2.0$, $\gamma = 0.8$ and for dashpot position $L = 10^4$, $\alpha = \infty$, $\beta = 2.0$, $\gamma = 1$. Optimal values were found: $a = 3$ m, $b = 4.965$ m, $c = 0.62$ m, 7 x

7 rows with dashpot number 9, price for concrete 27 941 Kč, for dashpots 83 790 Kč, total price 111 731 Kč. Fitness function expressed in price while varying dimensions $a$, $b$ and the number of rows of rows in direction $a$, $b$ is on figure 4. The other parameters and variables are fixed at their optimal position. The maximal displacement at $T_s$ for non-constant $b$ and $c$ is shown on figure 5.

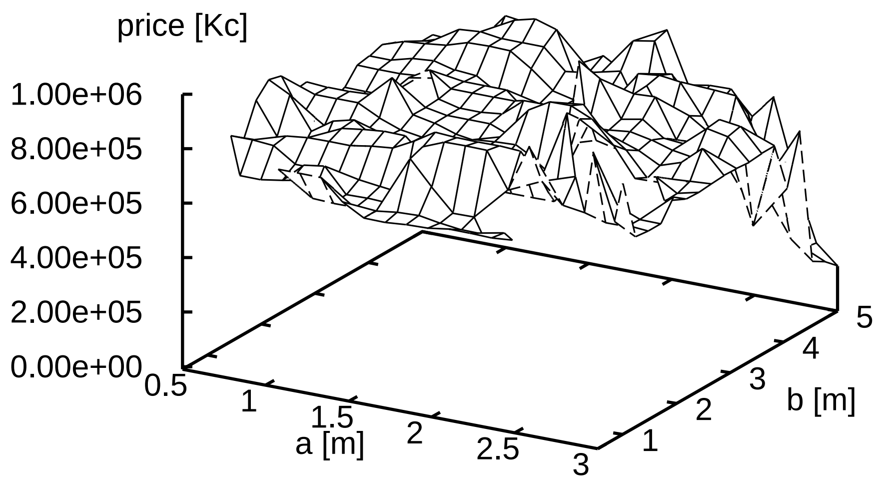


Figure 4: The fitness function with non-constant $a$, $b$ and the number of rows.

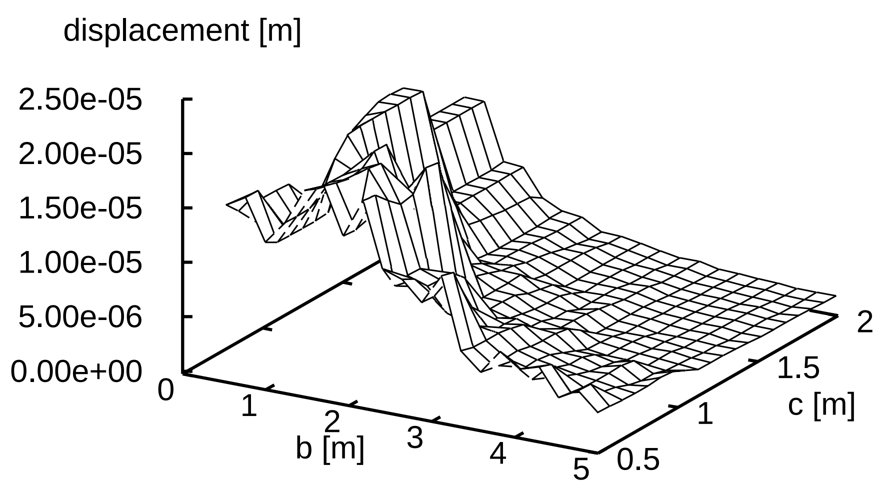


Figure 5: Maximal displacement at $T_s$ while changing dimensions $b$ and $c$.

If restrictions on this task were not too strict, GA has converged after about 500 generations, i.e. 15 000 individuals (5s/400 MHz CPU). The damping played the important role in the regions close to resonance and convergence has slowed down but even there the results were not far from the optimum. 500 000 individuals was always satisfactory for optimum finding (80s/400 MHz CPU). In case of strict restrictions GA found the lowest cost but it was high due to the penalty function.

## 5 Conclusion

The biggest advantage of GAs is the speed and easy problem formulation. GAs are universal in tasks with many variables where classical methods fail and release the space for evolution methods. In the task with “incorrect” boundary conditions GAs converge slower but approaches toward an optimum. It shows the variability of not only biological organisms but also of GAs. GAs were found to be the best method in variety of tasks and GAs will be likely used more frequently in the future.


## 6 Acknowledgements

This paper was supported by the research grant of the Czech Grant Agency No. 103/01/1528 GA ČR.


## 7 References


[1] Baťa, M., Plachý, V., Trávniček, F. **Dynamika stavebních konstrukcí.** Bratislava, SNTL, 1987.

[2] Goldberg, D. **Genetic Algorithms in Search, Optimization and Machine Learning.** Addison-Wesley, 1989.

[3] Lepš, M. **Optimalizace železobetonového spojitého nosníku.** Diplomová práce, ČVUT stavební Praha, 2000.

[4] Massachusetts Institute of Technology. **A C++ Genetic Algorithms Library, version 2.4.** http://lancet.mit.edu/ga, 1995-1996.

[5] Peterka, I. **Genetické algoritmy.** Diplomová práce, Matematicko-fyzikální fakulta UK Praha, http://ipeterka.misto.cz, 1999.