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STABILITY INVESTIGATION OF ROTORS EXCITED BY UNBALANCE OF THE ROTATING PARTS AND SUPPORTED BY SHORT NON-CIRCULAR FLUID FILM BEARINGS

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Summary : *Properties of fluid film bearings are significantly influenced by the shape of the cross section of the holes in the bearing shells. In computational models they are usually incorporated by means of nonlinear force couplings. To determine components of the hydraulical forces it is necessary to solve a Reynolds' equation to obtain a pressure function that describes a pressure distribution in the oil layer. In the case of short bearings the pressure function can be expressed in a closed form. If at some location in the bearing gap the pressure drops below a certain limit, a cavitation takes place. Accommodation of this phenomenon in the computational procedure assumes that pressure of the medium in cavitated areas remains approximately constant. Components of the bearing forces are then calculated by means of integration of the pressure distribution around the circumference of the bearings. The considered model rotor systems are able to cover all significant properties of the real ones. Their steady state response on excitation produced by centrifugal forces due to unbalance of the rotating parts can be determined for a certain class of problems by application of a trigonometric collocation method. To perform stability and bifurcation analysis a perturbation technique based on utilization of a Floquet theory has been used. Principal steps of this procedure consist in setting up a transition matrix over the span of time of one period and in calculation of its eigenvalues.*

Keywords : non-circular fluid film bearings, short bearings, Reynolds' equation, stability and bifurcation analysis, a trigonometric collocation method, Floquet theory

1. INTRODUCTION

Vibration of rotors working in industrial enterprises can be considerably attenuated if they are coupled with the stationary part through fluid film bearings. On the other hand incorrect design of these constraint elements can produce operating conditions that are undesirable from the point of view of a limit state of deformation or of a control.

Properties of fluid film bearings strongly depend on the shape of the cross section of the hole in the bearing shell. Circular bearings especially if they are lightly loaded are prone to produce self-excited vibration known as oil-whirl and oil-whip which is marked for large amplitudes. The stability limit (from the point of view of the speed of the rotor rotation) can be significantly increased if the bearings of non-circular cross section are applied (e.g. elliptical, lemon, with pressure dams, etc.).

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2. CALCULATION OF THE BEARING FORCES

Hydrodynamical bearings are usually incorporated into the computational models by means of nonlinear force couplings. To determine components of the hydraulical force through which the layer of lubricant acts on the rotor journal and bearing shell it is necessary to know a pressure function that describes a pressure distribution in the bearing gap. After introduction of several assumptions given e.g. in [7] : (i) the bearing surfaces are absolutely smooth and rigid, (ii) the cross section in the bearing shell is constant in the axial direction, (iii) the oil film thickness is small compared to the journal radius, (iv) the lubricant is incompresible Newtonian liquid of constant viscosity in the whole oil film that perfectly adheres to the bearing surfaces (v) the flow is laminar, (vi) the pressure of the lubricant is constant in the radial direction and (vii) the velocity gradient in the radial direction is large in relation to those in the tangential and axial ones calculation of the pressure function arrives at solving a Reynolds‘ equation

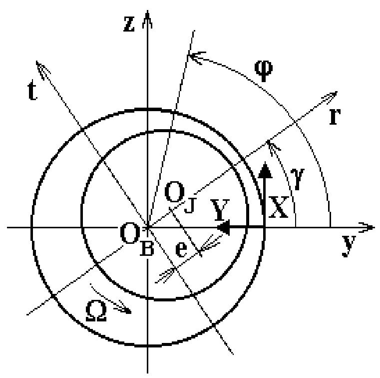

Fig.1 Scheme of the bearing

$$\frac{1}{R^2}\frac{\partial}{\partial\varphi}\left(h^3\frac{\partial p}{\partial\varphi}\right)+\frac{\partial}{\partial Z}\left(h^3\frac{\partial p}{\partial Z}\right)=\frac{6\eta}{R}\frac{\partial}{\partial\varphi}\left[h\,(u_1+u_2)\right]+12\eta\frac{\partial h}{\partial t} \qquad (1)$$

where

$$h = h_0 - e.\cos(\varphi-\gamma) \qquad (2)$$

$$\frac{\partial h}{\partial\varphi}=\frac{\partial h_0}{\partial\varphi}+e.\sin(\varphi-\gamma) \qquad (3)$$

$$\frac{\partial h}{\partial t}=-\dot{e}.\cos(\varphi-\gamma)-e\dot{\gamma}\sin(\varphi-\gamma) \qquad (4)$$

φ, Z - circumferential, axial coordinates (Fig.1),
e, γ - eccentricity of the rotor journal centre, position angle of the line of centres (Fig.1),
h_0, h - width of the gap at centric, eccentric position of the journal,
R, η, t - radius of the rotor journal, oil dynamical viscosity, time,
p - pressure, pressure function,
u_1, u_2 - circumferential velocity of the bearing housing and rotor journal surfaces,
$\dot{\gamma}$ - derivative of the position angle of the line of centres with respect to time,
$\dot{e}$ - derivative of the eccentricity with respect to time.

If geometry and design parameters of a bearing make possible to consider it as short (length to diameter ratio less than approximately 0.25, insufficient sealing at the bearing faces), then the pressure gradient in the axial direction is considerably greater than those in the circumferential one and therefore the first term on the left hand side of (1) can be neglected

$$\frac{\partial}{\partial Z}\left(h^3\frac{\partial p}{\partial Z}\right)=\frac{6\eta}{R}(u_1+u_2)\frac{\partial h}{\partial \varphi}+\frac{6\eta}{R}h\left(\frac{\partial u_1}{\partial \varphi}+\frac{\partial u_2}{\partial \varphi}\right)+12\eta\frac{\partial h}{\partial t} \quad (5)$$

Solution of the Reynolds' equation (5) utilizing two additional pressure conditions

$$p=p_a \qquad \text{for} \qquad Z=+\frac{L}{2} \quad (6)$$

$$p=p_a \qquad \text{for} \qquad Z=-\frac{L}{2} \quad (7)$$

p_a - pressure of the outer environment at the bearing faces ,
L - length of the bearing,

can be expressed in a closed form

$$p=p_a+\frac{1}{2}.\left(\frac{L^2}{4}-Z^2\right)A \quad (8)$$

$$A=-\frac{6\eta}{Rh^3}(u_1+u_2)\frac{\partial h}{\partial \varphi}-\frac{6\eta}{Rh^2}\left(\frac{\partial u_1}{\partial \varphi}+\frac{\partial u_2}{\partial \varphi}\right)-\frac{12\eta}{h^3}\frac{\partial h}{\partial t} \quad (9)$$

Usually it is accepted for the boundary conditions of the velocity component of the oil flow in the circumferential direction [7]

$$u_1=0 \quad (10)$$

$$u_2=R.\Omega \quad (11)$$

Ω - angular speed of the rotor rotation,

but in some special cases (low speed rotors, large velocities of the rotor and stationary part vibration, etc.) they can be expressed by more accurate relationships.

If pressure of the oil in the bearing gap drops under a certain limit, a cavitation takes place. It is a complex phenomenon when air is sucked into the bearing, the oil starts to boil, and gasses dissolved in it are liberated. In cavitated regions a Reynolds' equation does not hold and the flow is governed by different rules. A cavitation has been studied by many researchers. Zeidan and Vance [8], [9] revealed five regimes that can occur during the bearing operation. Results of their work shows that pressure of the medium in cavitated areas remains approximately constant. That's why the pressure distribution in the oil layer can be describe by the following relationships

$$p_t=p \qquad \text{for} \qquad p\geq p_{cav} \quad (12)$$

$$p_t=p_{cav} \qquad \text{for} \qquad p<p_{cav} \quad (13)$$

p_t - pressure distribution in the bearing gap,
p_{cav} - pressure of the medium in a cavitated region.

Assuming that

$$p_a\geq p_{cav} \quad (14)$$

the cavitation takes place if magnitude of the pressure function in the middle of the bearing length drops under a limit value of p_{cav} . Coordinates Z_{cav} of the extent of the cavitation area in the axial direction result from solving the equation

$$p_a + \frac{1}{2}\left(\frac{L^2}{4} - Z_{cav}^2\right)A = p_{cav} \tag{15}$$

From (8) it is evident that the pressure profile in the axial direction is parabolic. Its mean value p_m is defined as follows

$$p_m = \frac{1}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} p_t \, dZ \tag{16}$$

After performing the integration making use of relationships (12) and (13) it holds

$$p_m = \frac{1}{12}AL^2 + p_a \qquad \text{for} \quad \frac{1}{8}AL^2 + p_a \geq p_{cav} \tag{17}$$

$$p_m = \frac{A}{12L}\left(L^3 - 3L^2|Z_{cav}| + 4|Z_{cav}|^3\right) + \frac{2(p_{cav} - p_a)|Z_{cav}|}{L} + p_a \qquad \text{for} \quad \frac{1}{8}AL^2 + p_a < p_{cav} \tag{18}$$

Components of the hydraulical force through which the oil film acts on the rotor journal are calculated by integration of the mean pressure value p_m around the circumference of the bearing

$$F_y = -RL\int_0^{2\pi} p_m \cos\varphi \, d\varphi \tag{19}$$

$$F_z = -RL\int_0^{2\pi} p_m \sin\varphi \, d\varphi \tag{20}$$

F_y, F_z - y, z components of the hydraulical force.

3. STEADY-STATE RESPONSE OF A ROTOR SYSTEM ON PERIODIC EXCITATION

An important instrument for investigation of rotor systems is a computer modelling method. The model rotors are assumed to have the following properties : (i) the shaft is represented by a beam-like body that is discretized into finite elements, (ii) the stationary part is flexible, (iii) the discs are axisymmetric rigid bodies, (iv) inertia and gyroscopic effects of the rotating parts are taken into account, (v) material damping of the shaft is viscous, other kinds of damping (except the bearings) are linear, (vi) the bearings are hydrodynamical, (vii) the rotor rotates at constant angular speed and (viii) is loaded by forces and kinematic effects of constant and periodical time histories.

Lateral vibration of such rotor systems is described by the equation of motion

$$\mathbf{M}.\ddot{\mathbf{x}} + (\mathbf{B} + \eta_V.\mathbf{K}_{SH} + \Omega.\mathbf{G}).\dot{\mathbf{x}} + (\mathbf{K} + \Omega.\mathbf{K}_C).\mathbf{x} = \mathbf{f}_A + \mathbf{f}_V + \mathbf{f}_H(\mathbf{x}, \dot{\mathbf{x}}) \tag{21}$$

and by relationships for boundary conditions

$$\mathbf{x}_{BC} = \mathbf{x}_{BC}(t) \tag{22}$$

M, **G**, **K** - mass, gyroscopic, stiffness matrices of the rotor system,
B, $\mathbf{K}_C$ - (external) damping, circulation matrices of the rotor system,
$\mathbf{K}_{SH}$ - stiffness matrix of the shaft,
$\mathbf{f}_A$, $\mathbf{f}_V$, $\mathbf{f}_H$ - vectors of applied, constraint, hydraulical forces acting on the rotor system,

$\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}$ - vectors of generalized displacements, velocities, accelerations of the rotor system,
$\mathbf{x}_{BC}$ - vector of boundary conditions,
η_V - coefficient of viscous damping (material of the shaft).

Solution of the equation of motion (21) after dying out the initial transient component can be obtained for a certain class of problems by application of a trigonometric collocation method. This approach assumes that : (i) the steady-state vibration is a periodic function of time, (ii) its preriod T is a real multiple of the period of excitation and (iii) the response can be approximated by a finite number of terms of a Fourier series.

To be satisfied the boundary conditions (22) the equation of motion (21) is transformed into the following form

$$\mathbf{A}_2.\ddot{\mathbf{y}} + \mathbf{A}_1.\dot{\mathbf{y}} + \mathbf{A}_0.\mathbf{y} = \mathbf{b} \tag{23}$$

$\mathbf{A}_2$, $\mathbf{A}_1$, $\mathbf{A}_0$, $\mathbf{y}$, $\dot{\mathbf{y}}$, $\ddot{\mathbf{y}}$ and $\mathbf{b}$ are obtained from matrices $\mathbf{A}_2^*, \mathbf{A}_1^*, \mathbf{A}_0^*$ and vectors $\mathbf{x}$, $\dot{\mathbf{x}}$, $\ddot{\mathbf{X}}$, $\mathbf{b}^*$ by omitting their rows and columns that correspond to the degrees of freedom to which the boundary conditions are assigned

$$\mathbf{A}_2^* = \mathbf{M} \tag{24}$$

$$\mathbf{A}_1^* = \mathbf{B} + \eta_V.\mathbf{K}_{SH} + \Omega.\mathbf{G} \tag{25}$$

$$\mathbf{A}_0^* = \mathbf{K} + \Omega.\mathbf{K}_C \tag{26}$$

$$\mathbf{b}^* = \mathbf{f}_A + \mathbf{f}_H - \mathbf{A}_2^*.\ddot{\mathbf{x}}_{BC} - \mathbf{A}_1^*.\dot{\mathbf{x}}_{BC} - \mathbf{A}_0^*.\mathbf{x}_{BC} \tag{27}$$

Approximation of the modified solution $\mathbf{y}$ is expressed by relationship (28)

$$\mathbf{y} = \mathbf{a}_0 + \sum_{j=1}^{N_H} \mathbf{a}_j.\cos\left(j\frac{2\pi}{T}t\right) + \mathbf{b}_j.\sin\left(j\frac{2\pi}{T}t\right) \tag{28}$$

N_H - number of the considered harmonical terms,
$\mathbf{y}_0$, $\mathbf{y}_{cj}$, $\mathbf{y}_{sj}$ - vectors of Fourier coefficients (j = 1, 2, ... , N_H).

A trigonometric collocation method requires to specify N_C collocation points (collocation instants of time t_k) e.g.

$$t_k = \frac{T}{N_C}(k-1) \tag{29}$$

Substitution of the assumed solution (28) and its first and second derivatives with respect to time into the modified equation of motion (23) for all collocation points of time t_k arrives at a set of nonlinear algebraic equations.

$$\begin{aligned} \mathbf{A}_0.\mathbf{y}_0 &+ \sum_{j=1}^{N_H}\left[\cos\left(j\frac{2\pi}{T}t_k\right).\left(\mathbf{A}_0 - j^2\frac{4\pi^2}{T^2}.\mathbf{A}_2\right) - j\frac{2\pi}{T}\sin\left(j\frac{2\pi}{T}t_k\right).\mathbf{A}_1\right].\mathbf{a}_j + \\ &+ \sum_{j=1}^{N_H}\left[\sin\left(j\frac{2\pi}{T}t_k\right).\left(\mathbf{A}_0 - j^2\frac{4\pi^2}{T^2}.\mathbf{A}_2\right) + j\frac{2\pi}{T}\cos\left(j\frac{2\pi}{T}t_k\right).\mathbf{A}_1\right].\mathbf{b}_j = \\ &= \mathbf{r}(\mathbf{y}_0, \mathbf{y}_{C1}, \mathbf{y}_{S1}, \mathbf{y}_{C2}, \mathbf{y}_{S2}, \dots, \mathbf{y}_{CN_H}, \mathbf{y}_{SN_H}) \end{aligned} \tag{30}$$

for k = 1,2, ..., N_C.

which be also expressed in a matrix form

$$\mathbf{S}.\mathbf{g} = \mathbf{r}(\mathbf{g}) \tag{31}$$

S - coefficient matrix,
r - right-hand side vector,
g - vector of unknowns.

The unknowns are Fourier coefficients of all displacements of the rotor system to which no boundary conditions are prescribed.

4. STABILITY INVESTIGATION OF THE PERIODIC VIBRATION

For stability and bifurcation analysis of a rotor system periodically excited by centrifugal forces of unbalanced rotating parts a perturbation technique has been adopted. The steady-state component of the forced vibration is slightly disturbed at the beginning of the investigated period but time histories of the applied forces and the boundary conditions must remain unchanged. To determine bearing forces corresponding to the disturbed motion the vector of hydraulical forces $\mathbf{f}_H$ is expanded into a Taylor series in the neighbourhood of the current state

$$\mathbf{f}_H(\mathbf{x} + \Delta\mathbf{x}, \dot{\mathbf{x}} + \Delta\dot{\mathbf{x}}) = \mathbf{f}_H(\mathbf{x}, \dot{\mathbf{x}}) + \mathbf{D}_B.\Delta\dot{\mathbf{x}} + \mathbf{D}_K.\Delta\mathbf{x} + ... \tag{32}$$

where

$$\mathbf{D}_B = \left[\frac{\partial \mathbf{f}_H(\mathbf{x}, \dot{\mathbf{x}})}{\partial \dot{\mathbf{x}}}\right]_{\mathbf{x},\dot{\mathbf{x}}} \tag{33}$$

$$\mathbf{D}_K = \left[\frac{\partial \mathbf{f}_H(\mathbf{x}, \dot{\mathbf{x}})}{\partial \mathbf{x}}\right]_{\mathbf{x},\dot{\mathbf{x}}} \tag{34}$$

$\mathbf{D}_B$, $\mathbf{D}_K$ - square matrices of partial derivatives,
$\Delta\mathbf{x}$, $\Delta\dot{\mathbf{x}}$, $\Delta\ddot{\mathbf{x}}$ - vectors of deviations of displacements, velocities and accelerations of the disturbed rotor system.

Substruction of the equation of motion of the undisturbed vibration from the one of the disturbed motion and a series of consequent manipulations taking into account only the linear portion of the Taylor series (32) and the boundary conditions arrives at a set of linear differential equations of the second order describing time history of deviations of displacements of the disturbed motion

$$\mathbf{A}_2.\Delta\ddot{\mathbf{y}} + \mathbf{A}_1.\Delta\dot{\mathbf{y}} + \mathbf{A}_0.\Delta\mathbf{y} = \mathbf{o} \tag{35}$$

which is further transformed into a state space

$$\begin{bmatrix}\Delta\ddot{\mathbf{y}}\\ \Delta\dot{\mathbf{y}}\end{bmatrix} = \begin{bmatrix} -\mathbf{A}_2^{-1}.\mathbf{A}_1[\dot{\mathbf{x}}(t), \mathbf{x}(t)] & -\mathbf{A}_2^{-1}.\mathbf{A}_0[\dot{\mathbf{x}}(t), \mathbf{x}(t)] \\ \mathbf{I} & \mathbf{O}\end{bmatrix} . \begin{bmatrix}\Delta\dot{\mathbf{y}}\\ \Delta\mathbf{y}\end{bmatrix} \tag{36}$$

$\mathbf{A}_2$, $\mathbf{A}_1$, $\mathbf{A}_0$, $\Delta\mathbf{y}$, $\Delta\dot{\mathbf{y}}$ and $\Delta\ddot{\mathbf{y}}$ are determined from matrices $\mathbf{A}_2^*, \mathbf{A}_1^*, \mathbf{A}_0^*$ and vectors $\Delta\mathbf{x}, \Delta\dot{\mathbf{x}}, \Delta\ddot{\mathbf{x}}$ by omitting appropriate rows and columns that are related to the degrees of freedom to which the boundary conditions are imposed

$$\mathbf{A}_2^* = \mathbf{M} \tag{37}$$

$$\mathbf{A}_1^* = \mathbf{B} + \eta_V.\mathbf{K}_{SH} + \Omega.\mathbf{G} - \mathbf{D}_B[\dot{\mathbf{x}}(t), \mathbf{x}(t)] \tag{38}$$

$$\mathbf{A}_0^* = \mathbf{K} + \Omega.\mathbf{K}_C - \mathbf{D}_K[\dot{\mathbf{x}}(t), \mathbf{x}(t)] \tag{39}$$

I, O, o - identity matrix, zero matrix, zero vector.

Because of $\mathbf{D}_B$ and $\mathbf{D}_K$ the coefficient matrix in (36) is a periodic function of time. That's why stability judgement of the rotor system vibration can be carried out by means of a Floquet theory. Division of the period of T into N time subintervals Δt makes possible to express the transtion matrix between points of time 0 and T as a product of N partial transition matrices

$$\mathbf{H}(T,0) = \mathbf{H}(T, T-\Delta t).\mathbf{H}(T-\Delta t, T-2\Delta t).....\mathbf{H}(\Delta t, 0) \tag{40}$$

$\mathbf{H}(t_2,t_1)$ - transtition matrix between instants of time t_1 and t_2 .

Employing kinematic relationships of a Newmark method a relation between kinematic quantities corresponding to times t and t+Δt can be derived. The square matrix in it represents a partial transition matrix

$$\begin{bmatrix} \Delta\dot{\mathbf{y}}_{t+\Delta t} \\ \Delta\mathbf{y}_{t+\Delta t} \end{bmatrix} = \begin{bmatrix} \frac{2}{\Delta t}.\mathbf{Q}_t - \mathbf{I} & \frac{2}{\Delta t}(\mathbf{P}_t - \mathbf{I}) \\ \mathbf{Q}_t & \mathbf{P}_t \end{bmatrix} . \begin{bmatrix} \Delta\dot{\mathbf{y}}_t \\ \Delta\mathbf{y}_t \end{bmatrix} \tag{41}$$

where

$$\mathbf{P}_t = \mathbf{D}_t^{-1}.\left(\frac{4}{\Delta t^2}\mathbf{A}_2 + \frac{2}{\Delta t}\mathbf{A}_{1,t+\Delta t} - \mathbf{A}_{0,t} \right) \tag{42}$$

$$\mathbf{Q}_t = \mathbf{D}_t^{-1}.\left(\frac{4}{\Delta t}\mathbf{A}_2 + \mathbf{A}_{1,t+\Delta t} - \mathbf{A}_{1,t} \right) \tag{43}$$

$$\mathbf{D}_t = \frac{4}{\Delta t^2}\mathbf{A}_2 + \frac{2}{\Delta t}\mathbf{A}_{1,t+\Delta t} + \mathbf{A}_{0,t+\Delta t} \tag{44}$$

Vibration of the rotor system is stable if magnitudes of all eigenvalues of the transition matrix **H** are less that 1. If the leading eigenvalue whose magnitude is greater than 1 is real and negative, the periodic solution becomes unstable and another solution winding twice per the driving cycle appears (a period doubling bifurcation). A pair of complex-conjugate leading eigenvalues is a mark of a secondary Hopf bifurcation. Then the resulting motion is composed of two fundamental harmonical components whose frequencies are irrationally (quasi-periodic vibration) or rationally (subharmonic vibration) related. Real and positive leading eigenvalue greater than 1 is referred to a saddle-node instability. Any disturbance will cause the rotor to jump to a stable cycle limit of the same type.

5. EXAMPLE

The investigated rotor system (Fig. 2) consists of a shaft (SH) and of two discs (D1, D2) attached to its overhanging end. The shaft is coupled with a rigid foundation plate (FP) through two hydrodynamical bearings (B1, B2 - oil dynamical viscosity 0.006 Pa.s, pressure in cavitated areas -10 kPa) whose shells consist of two circular segments mutually shifted in the horizontal direction (radius 40 mm, length 20 mm, clearance width 0.2 mm, shift of the segments 0.05 mm).

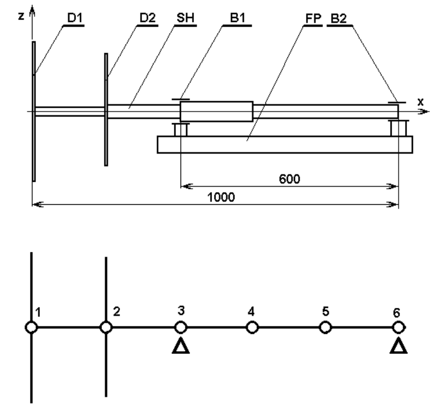

Fig.2 Scheme of the investigated rotor system

The rotor rotates at constant angular speed (380 rad/s) and is loaded by its weight. In addition the system is excited by the discs unbalance (eccentricities 0.15 mm). The task was to analyze stability of the steady-state response.

In the computational model the shaft was represented by a beam-like body that was discretized into five finite elements. The steady-state component of the response was approximated by one absolute and by the first 12 harmonical terms of a Fourier series. It was assumed that period of the vibration is equal to the period of excitation (the period of rotation). For the purpose of calculation of the transition matrix the period was divided into 500 time subintervals. Some of the results are given in Figs. 3 - 7.

In Fig.3 there are trajectories of the discs D1 and D2 centres. If the discs find themselves in some limited space, then these results are important for judgement of the rotor system from the point of view of a limit state of deformation. Trajectories of the rotor journal centres in bearings B1 and B2 are drawn in Fig.4 and time histories of their eccentricities in Fig.6. Its evident that the journals move inside the holes of the bearing shells and that no impacts between the rotor and the stationary part take place.

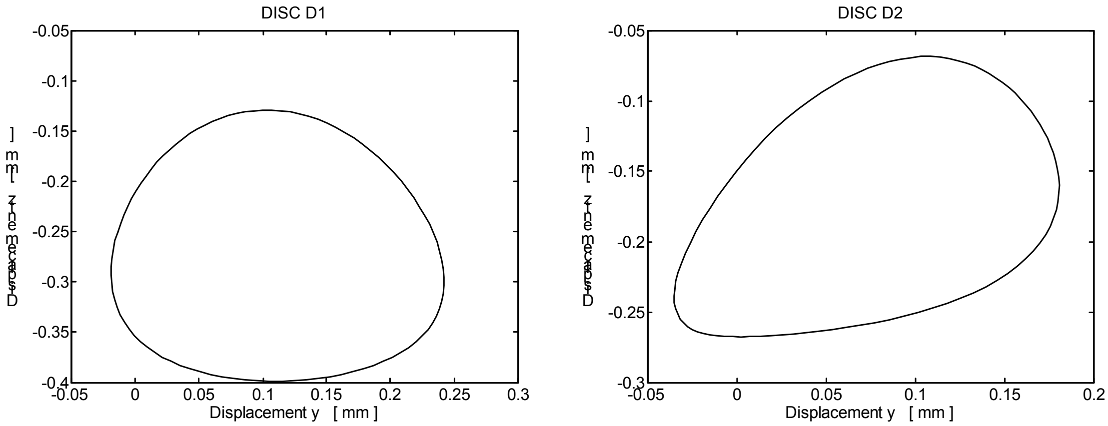

Fig.3 Trajectories of the discs D1 and D2 centres

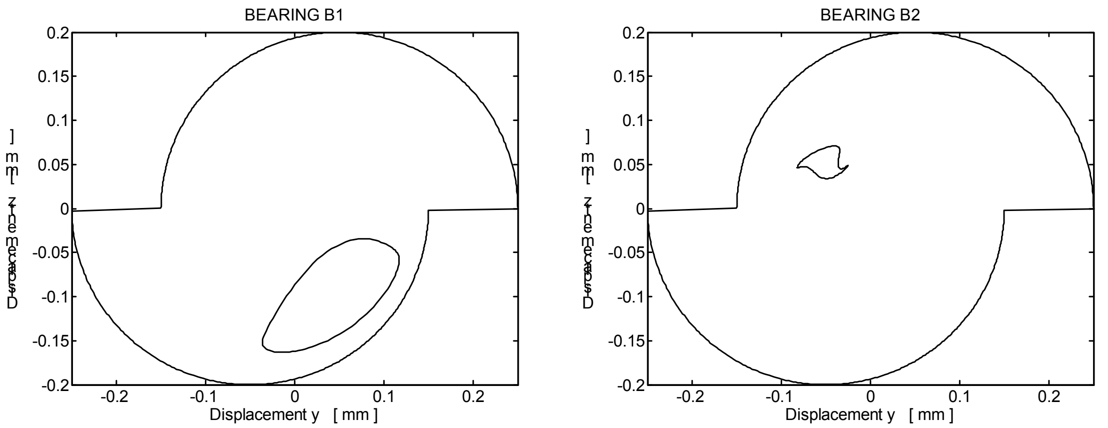

Fig.4 Trajectories of the rotor journal centres in bearings B1 and B2

Fig.5 shows images of the Fourier transformation of z-displacements of the rotor journal centres in bearings B1 and B2.

Distribution of eigenvalues of the transition matrix set up over the span of time of one period in a Gauss plane are drawn in Fig.7. All of them are situated inside a unit circle and it means that the investigated vibration is periodic.

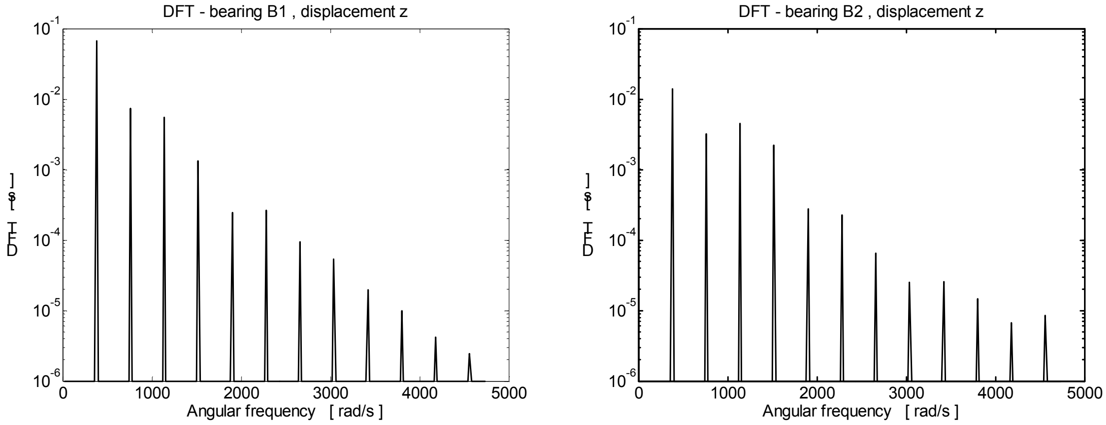

Fig. 5 Fourier transformation (z-displacements of the rotor journal centres in B1 and B2)

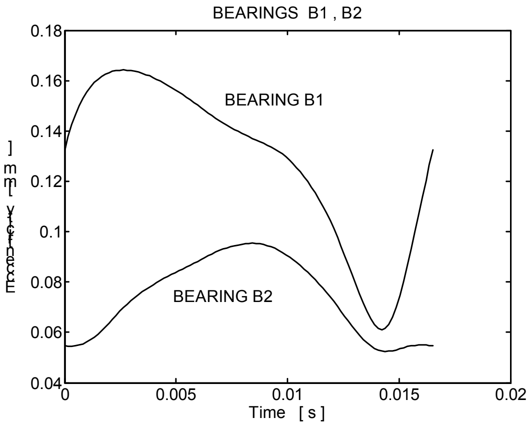

Fig.6 Eccentricities of the rotor journal centres

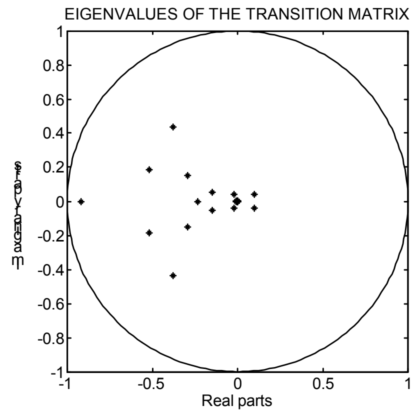

Fig.7 Eigenvalues of the transition matrix

7. CONCLUSIONS

The described numerical approach represents a comperhensive method for stability investigation of rotors supported by short fluid film bearings of non-circular cross section. The attention is paid especially to determination of the pressure distribution in the bearing gap. The proposed procedure makes possible to accommodate influence of the rotor journal and bearing shell vibration on the boundary conditions related to the flow velocity components and to include rupture of the oil film into the computational model. The pressure function can be expressed in a closed form which contributes to speeding up the analysis.

The carried out computer simulations brought a some practical experience that can be briefly summarized.

- A trigonometric collocation method makes possible to find a steady-state solution only if those is a periodic function of time.

- The most difficult step of its application consists in solving a set of nonlinear algebraic equations. For this purpose a Newton-Raphson and modified Newton-Raphson methods were applied.
- Iteration proces of these numerical procedures can stop to converge if (i) the response after dying out the initial transient component of the vibration is not a periodic function of time (e.g. it is chaotic), (ii) its period is not estimated correctly, (iii) the load increment is too large, (iv) during the computational process a bifurcation takes place or if (v) the pressure function is not calculated with enough accuracy.
- Advantage of the trigonometric collocation method is that it also enables to find an unstable solution at which one cannot arrive by application of a direct integration of the equation of motion.
- Setting up a transition matrix by means of the described procedure requires to divide period of the response into a large number of short time subintervals.

As a conclusion it can be said that investigation of rotors supported by hydrodynamical bearings of non-circular cross section is an important but also rather complicated technical problem. To its solution a computer modelling method can be applied. The computer simulations are valuable especially if they are performed for a series of running conditions or design parameters of the rotor system.

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