National Conference with International Participation
ENGINEERING MECHANICS 2003
Svratka, Czech Republic, May 12 – 15, 2003

paper no.
112

CRACK GROWTH ASPECTS OF MIXED MODES

P. Brož[1], R. Novotný[2]

Summary: *To the interpretation of crack growth, it is substantial to estimate the plastic region geometry in front of a crack. Performed here is a study for predicting crack growth direction for mixed–mode I and II loading. It is suggested that under complex loading the crack will extend in the course where the radius of the plastic zone accomplishes a minimum value. There exists satisfactory correspondence between the predicted issues calculated on the basis of introduced criteria and testing data.*

In the paper presented, for initial mixed modes, the characteristics of propagation phenomena for fatigue cracks are given, in particular failure mechanisms and growth criteria by virtue of various factors.

1. Introduction

Criteria are desirable for the example of crack growth which evolves the formation of a branch crack, its angle with the initial crack, and for the propagation of the branch crack. Some of numerous criteria being based on linear elastic mechanics are alike to failure criteria for uncracked circumstances. To determine fatigue crack paths, the stipulation of the initial direction of crack growth is particularly important. In two dimensions this may be defined eg by the angle θ (Fig. 1).

Fatigue limits of technical materials are ordinarily stipulated under uniaxial loading. Fatigue failure criteria are required because a component can be subject to a complex stress state. Criteria are usually based either on stress constituents, or on principal stresses. Both alternating and mean stresses have to be taken into account. An additional complication is that applied loads are not allowed to be in phase. Fortunately, some simplification is feasible as conditions are usually fundamentally elastic.

In fatigue, failure of an uncracked specimen or component emerges on specific planes through the initiation and growth of cracks. Numerous criteria have been suggested, but none have become generally accepted. There are three main types of fatigue failure criteria. All contain no less than two material parameters that have to be determined experimentally. The

[1] Doc.Ing. Petr Brož, DrSc.: ČVUT v Praze, Kloknerův ústav, Šolínova 7, 166 08 Praha 6; tel.: +420 224 353 848, e-mail: broz@vc.cvut.cz
[2] Ing. Radimír Novotný, DrSc.: ČVUT v Praze, Stavební fakulta, Thákurova 7, 166 29 Praha 6; tel.: + 420 224 354 630, e-mail: radimir.novotny@fsv.cvut.cz

first group are named critical plane criteria. In these the mean and alternating membrane and shear stresses acting on various material planes are checked so as to determine the critical plane on which fatigue cracking is to be anticipated, and also to establish whether or not fatigue failure will develop. Different criteria apply various combinations of stresses. The maximum principal stress static failure criterion represents truly a critical plane condition.

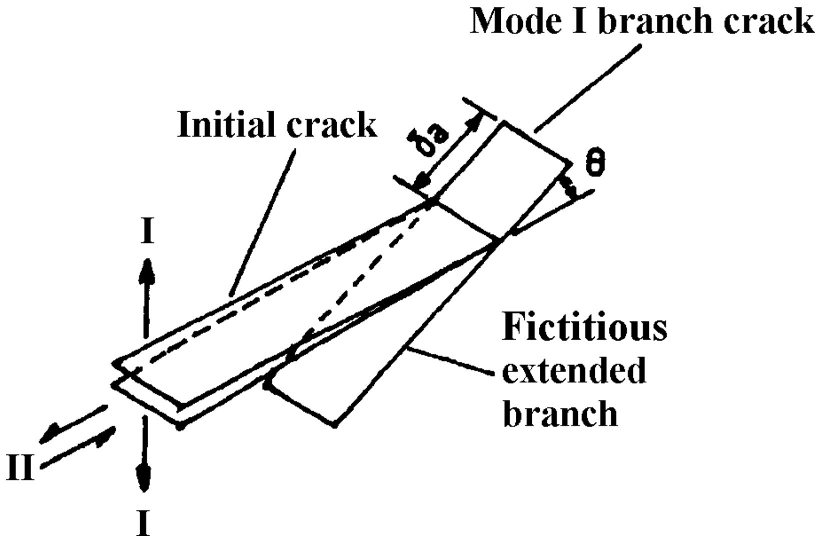

Fig. 1 Quasi two-dimensional mixed Modes I and II crack with fictitious extended branch crack.

In fatigue, failure of an uncracked specimen or component emerges on specific planes through the initiation and growth of cracks. Numerous criteria have been suggested, but none have become generally accepted. There are three main types of fatigue failure criteria. All contain no less than two material parameters that have to be determined experimentally. The first group are named critical plane criteria. In these the mean and alternating membrane and shear stresses acting on various material planes are checked so as to determine the critical plane on which fatigue cracking is to be anticipated, and also to establish whether or not fatigue failure will develop. Different criteria apply various combinations of stresses. The maximum principal stress static failure criterion represents truly a critical plane condition.

In the second group of criteria, stress invariants, usually the hydrostatic and deviatoric stresses or some combination, are tested to establish whether or not fatigue failure will take place. These criteria are symmetrical in the three principal stresses, and are therefore referred to static yield criteria. The bearings of a potential fatigue crack are not specified. In a stress average criterion some combination of membrane and normal shear stresses on a material plane within an elementary volume is examined. This combination is then averaged for all possible orientations of the plane. These criteria are also symmetrical in the three principal stresses, and no need for the orientation of a potential fatigue crack to be again determined.

There are two potential theoretical approaches. Whatever approach is used predicted crack growth courses are not strictly limited. Accordingly, minor deviations from isotropy due to microstructural irregularities can be awaited to possess a significant effect on the initial direction of crack growth. In the first method parameters associated with an infinitesimal straight branch crack (but not urgently Mode I) are examined to stipulate the course of growth of the branch crack, and also the requirements for it to progress to grow. This approach yields a lower bound as it provides a necessary condition for a branch crack to propagate, but not a sufficient qualification for branch crack forming. In the second approach it is supposed that the stress field connected with the initial (main) crack, by virtue of one parameter or another, is examining and the angular characteristics of that parameter are checked at or close to the crack tip. An appropriate maximum (or minimum) value then gives the direction in which crack growth proceeds, and also the condition for branch crack formation and continued crack growth to develop. Alternatively, a slender ellipse (or a narrow notch) may be tested.

In the treatise presented, deductions are notably pointed from the work [1] out.

2. Crack Propagation

Different criteria and interpretation are embraced in research work (eg [2], [4] and [5]) to investigate the crack growth direction problem.

The angled crack problem was tested by Erdogan and Sih. They suggested the criterion based on the stress field being presented just before the onset of fracture to predict the course of initial crack extension. In compliance with this criterion the crack enlarges in a radial direction pertaining to the maximum tangential stress and crack extension occurs when this maximum achieves a critical value. The study of the authors mentioned was based on the representation of the crack tip stress field via the stress function defined by Williams in the form:

$$\phi = r^{3/2} f_1(\theta) + r^2 f_2(\theta) + r^{5/2} f_3(\theta) + \ldots \tag{1}$$

where r is the radius and $f_1(\theta), f_2(\theta), f_3(\theta), \ldots$ stand for functions of θ. For the first term the stresses are expressed by the relations:

$$\sigma_\theta = \frac{1}{\sqrt{2\pi r}} \cos\frac{1}{2}\theta \left(K_I \cos^2\frac{\theta}{2} - \frac{3}{2} K_{II} \sin\theta \right) \tag{2}$$

$$\tau_{r\theta} = \frac{1}{2\sqrt{2\pi r}} \cos\frac{1}{2}\theta \left[K_I \sin\theta + K_{II}(3\cos\theta - 1) \right] \tag{3}$$

Erdogan and Sih proposed that the direction of crack growth is defined, as follows:

$$\frac{\mathrm{d}K_\theta}{\mathrm{d}\theta} = 0 \tag{4}$$

where

$$K_\theta = \cos^2\frac{\theta}{2} \left(K_I \cos\frac{\theta}{2} - 3K_{II} \sin\frac{\theta}{2} \right) \tag{5}$$

This is equivalent to

$$\cos\frac{\theta}{2}\left[K_I \sin\theta + K_{II}\left(3\cos\theta - 1\right)\right] = 0 \tag{6}$$

A better conformity between theoretical and experimental results has been obtained after using the first two terms of the eigenfunction expansion, Eq. (1), representing the stress field, and by employing a small but finite radius to put the maximum value of σ_θ.

The elastic strain energy dW stored in a parallel pipe of volume dV in the dominant zone of the deformed plate has been expressed as:

$$\frac{\mathrm{d}W}{\mathrm{d}V} = \frac{1+\nu}{4E}[\kappa_{1,2}\left(\sigma_x + \sigma_y\right)^2 + \left(\sigma_x - \sigma_y\right)^2 + 4\tau_{xy}^2] \tag{7}$$

where σ_x, σ_y, and τ_{xy} are the components of the stress tensor σ_{ij}, with $\kappa_{1,2}$ being:

$$\kappa_1 = \frac{1-\nu}{1+\nu}, \text{ for plane stress}$$
$$\kappa_2 = 1 - 2\nu, \text{ for plane strain} \tag{8}$$

Papadopoulos applied an elastic strain energy approach to suggest a criterion of fracture which takes into consideration the third stress invariant Det. (σ_{ij}),

$$\mathrm{Det.}(\sigma_{ij}) = \begin{vmatrix} \sigma_x & \sigma_{xy} \\ \sigma_{xy} & \sigma_y \end{vmatrix} \tag{9}$$

According to the Det.-criterion, the angle of crack extension for mixed-mode loading is determined from the premiss that the determination of the stress tensor must attains a maximum value. In a polar co-ordinate system this condition reads:

$$\left.\frac{\partial \mathrm{Det.}(\sigma_{ij})}{\partial\theta}\right|_{\theta=\theta^*} = 0, \quad \left.\frac{\partial \mathrm{Det.}(\sigma_{ij})}{\partial\theta}\right|_{\theta=\theta^*} < 0 \tag{10}$$

For the critical stress appurtenant to crack initiation it holds:

$$\mathrm{Det.}(\sigma_{ij}) = \mathrm{Det.}(\sigma_{ij})_{cr} \tag{11}$$

3. Uniaxial tension loading results

Let the W-theory be applied to the solution of the angled crack problem. Predictions for mixed–mode loading circumstances according to the W-criterion are compared with the commensurable predictions of the MTS and the Det.-criterion, with the correlation between testing data and calculations based on the W-theory for some special loading pattern being demonstrated, the configuration is indicated in Fig. 2.

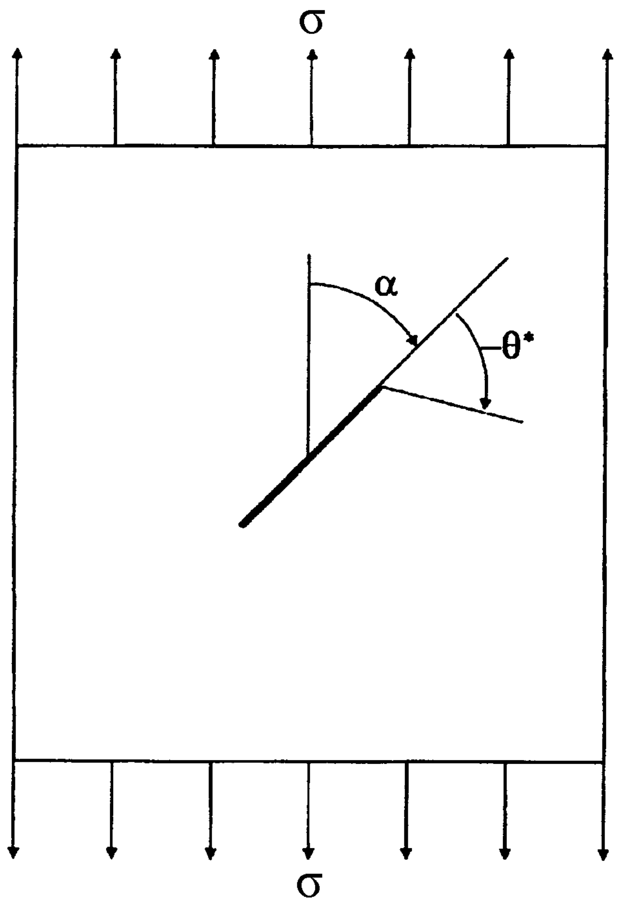

Fig. 2 The uniaxial tension problem

Predictions of the course of initial crack extension θ^* in compliance with the different criteria are stated in Table 1. Four various cracks bearings have been checked. For $\alpha \geq 45°$, the magnitudes of θ^* based on the Det.-criterion are more than those given by the MTS and the W-criterion. The data that have been calculated on the basis of the MTS and *W*-criteria differ by as much as 7,5° (Fig. 3). Values of the crack growth angle under uniaxial loading, which are determined from the *W*-criterion have been compared with testing data for LY12 – CZ [3] and also to other criteria. The material stated possesses (analogous, in the Czech denomination, to the alloy EN AW – 421401 (AlCu4Mg1)) the following mechanical characteristics: 0.2 % proof stress = 332 MPa; ultimate tensile strength = 456 MPa; Young´s modulus = 70.6 GPa; Poisson´s ratio = 0.33; and elongate to fracture = 11.7 %.

Table 1 Predicted values of the crack initiation angle

α (°)	MTS $-\theta^*$ (°)	Det. criterion	Proposed W-criterion
15	66.0	97.5	73.51
30	61.0	82.75	63.49
45	53.6	63.75	53.13
60	43.0	33.30	41.49

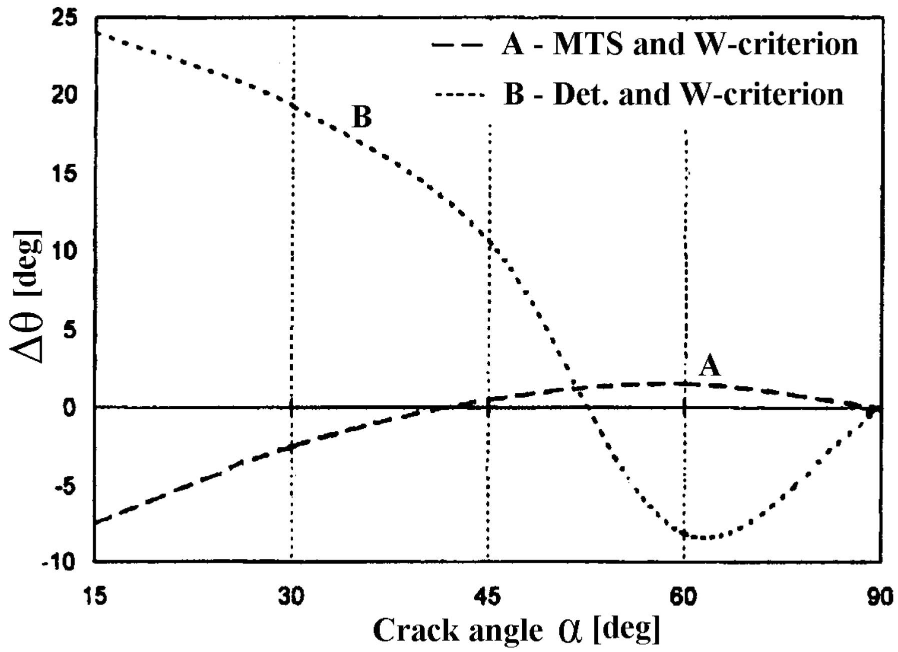

Fig. 3 Predictions of crack propagation angles based on the suggested *W*-criterion and other fracture criteria

The specifics of the crack geometry jointly with the specimen dimensions are demonstrated in Fig. 4. The length of the crack was given as the constant value while the angle α altered from $\alpha = 15°$ to $\alpha = 90°$. Fig. 5 presents the modification of the W factor being a function of θ^* for magnitudes of α = 15°,30° 45°, 60°, and 75°. Fig 6 performs the predicted values of the crack growth angle as a function of α for the Det. and *W*-criteria.

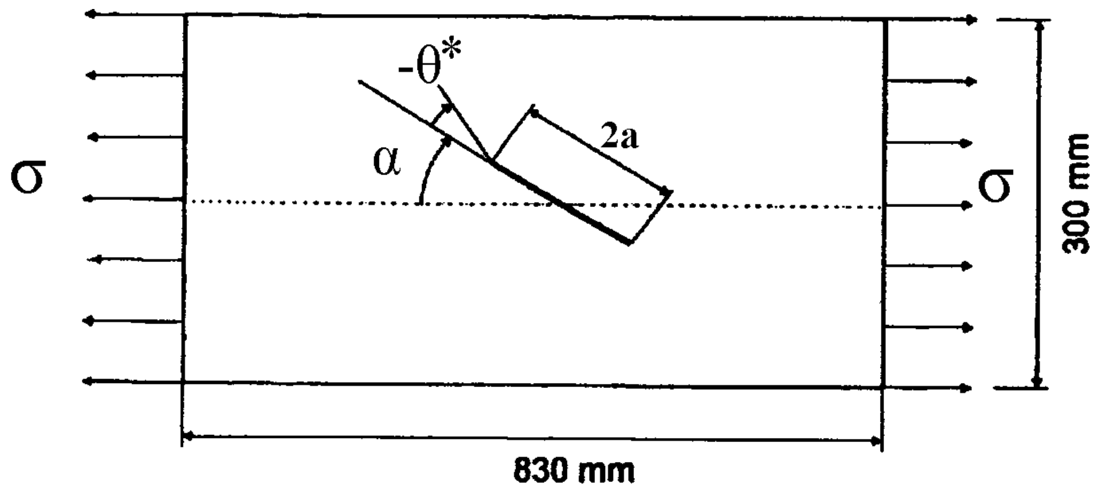

Fig. 4 Specimen geometry

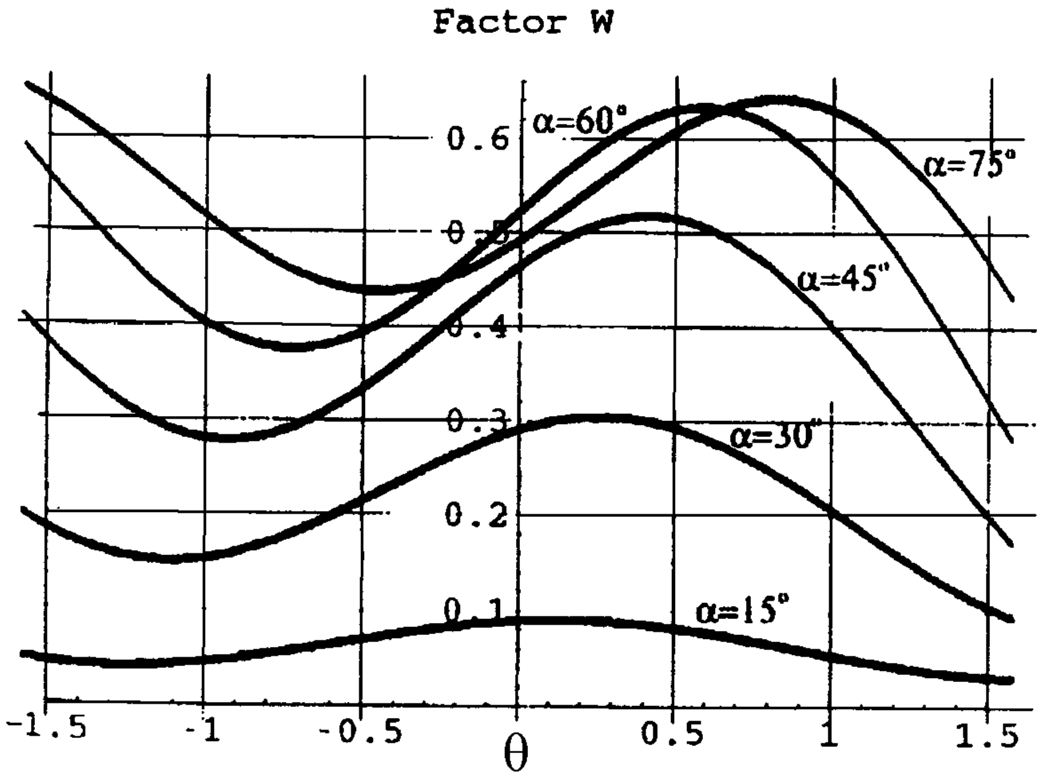

Fig. 5 Stationary values of the *W* factor

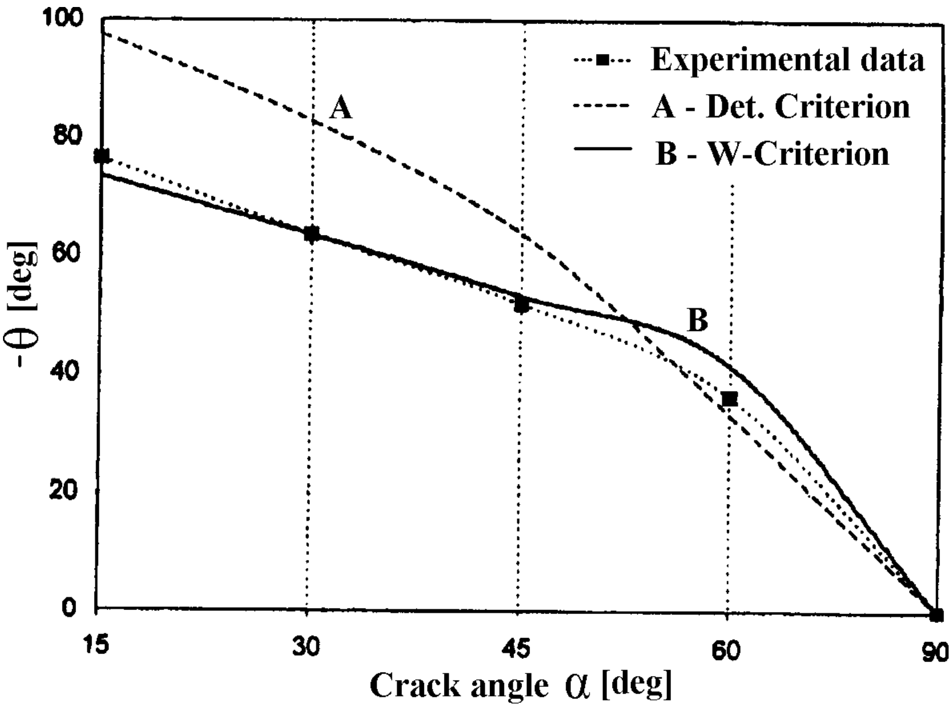

Fig. 6 Comparison of both the *W*-criterion and the Det.-criterion with experimental data for LY12-CZ

4. Conclusion

To involve the effect of the plastic region near the crack tip on the fracture process, a different qualification has been suggested, named the *W*-criterion. It is based on the analysis of the plastic zone shape when predicting the crack propagation course under mixed-mode loading. The values of *W* may presage the direction of crack growth. The conformity between angles of crack propagation determined from the *W*-criterion and testing data is fair. The criterion is straightforward to use employing the discrete numerical methods when predicting crack growth course in the case of various material features and boundary conditions. It may be resumed that the derived factor W is the competent means to give an account of crack growth charecteristics of real materials. For ductile materials, the studies will continue to test the influences of different material qualities, geometry, and loading modes, in terms of the suggested *W*-criterion.

5. Acknowledgements

The author gratefully acknowledges the financial support of the presented research by the Grant Agency of the Academy of Sciences of the Czech Republic (project IAA 2811201).

6. References

[1] Wasiluk, B. & Golos, K. (2000) Prediction of crack growth direction under plane stress for mixed-mode I and II loading. *Fatigue Fract. Engng Mater. Struct.*, 23, pp. 381-386.

[2] Pook, L. P. (1971) The effect of crack angle on fracture toughness. *Engn Fracture Mech.,* 3, pp. 205-218.

[3] Ui, Y. X. & Fan, W. X. (1987) Combined mode fracture of a ductile material under plane stress conditions. *Fatigue Fract. Engng Mater. Struct.*, 10, pp. 51-57.

[4] Golos, K., Osinski, Z. & Wasiluk, B. (1996) Mixed mode I + II model of fatigue crack growth. In: *Mechanism and Mechanics of Fatigue* (ECF 11) ESIS I, EMAS, UK, pp. 1107-1112.

[5] Miller, K. J., Brown, M. W. & Yates, J. R. (1999) Some observations on mixed-mode fatigue behaviour of polycrystaline metals. In: *Standard Technical Publication* 1359, American Society for Testing and Materials, pp. 229-257.