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COMPARISON OF APPROACHES TO DETERMINATION OF HYDRAULIC FORCES OF LONG SQUEEZE-FILM DAMPERS

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Summary : *One possibility how to decrease lateral vibration of rotors excited by imbalance of the rotating parts is to couple the shaft with the stationary part through squeeze film dampers. An important factor that can significantly influence their dynamical properties is fluid inertia. For accommodation of damping forces in computational models several approaches have been developed : (i) a method based on calculation of the viscous forces by solving a Reynolds equation and determination of the inertia ones from the Lagrange equation and Reynolds transport theorem, (ii) a method based on averaging terms in the Navier-Stokes equation related to the direction of prevailing pressure gradient, and (iii) a method based on solving a set of Navier-Stokes equations. These methods have been implemented into the procedure for determination of the steady-state response of a rotor excited by imbalance of the rotating parts imploying a trigonometric collocation method. The particular approaches to calculation of the damping forces differ in the extent of their validity, in the number of arithmetical operations, and as application of the trigonometric collocation method results into solving a set of nonlinear algebraic equations they have different influence on convergence of the computational process.*

1. Introduction

One possibility how to decrease magnitude of lateral vibration of rotors consists in coupling the shaft with the stationary part through squeeze film dampers. The dampers are composed of two principal parts : of an outer and inner rings between which there is a layer of lubricant. The inner ring is coupled with the stationary part by a retainer spring that prevents its turning together with the shaft and makes possible its vibration relative to the machine frame.

In computational models the squeeze film dampers are usually incorporated by means of nonlinear force couplings. To determine components of the damping force it is necessary to know a pressure function that describes a pressure distribution on the damper gap.

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2. Calculation of damping forces using a classical lubrication theory (R)

The classical lubrication theory assumes that

- the inner and outer rings of the damper are absolutely rigid,
- surfaces of the inner and outer rings are absolutely smooth,
- cross section of the damper gap has an annular shape and its dimensions are not changed in the axial direction,
- the lubricant is incompressible Newtonian liquid,
- inertia properties of the lubricant are negligible,
- viscosity of the lubricant is constant,
- the oil adheres perfectly to surfaces of the inner and outer rings,
- the flow is laminar and isotermic,
- pressure in the radial direction is constant,
- velocity gradient in the radial direction is much greater than those in the axial and circumferential ones.

On these conditions the pressure distribution in the damper gap is described by a Reynolds equation

$$\frac{1}{R^2}\frac{\partial}{\partial\vartheta}\left(h^3\frac{\partial p_R}{\partial\vartheta}\right)+\frac{\partial}{\partial Z}\left(h^3\frac{\partial p_R}{\partial Z}\right)=\frac{6\eta}{R}\frac{\partial}{\partial\vartheta}\left[h\left(u_1+u_2\right)\right]+6\eta\frac{\partial}{\partial Z}\left[h\left(w_1+w_2\right)\right]+12\eta\frac{\partial h}{\partial t} \qquad (1)$$

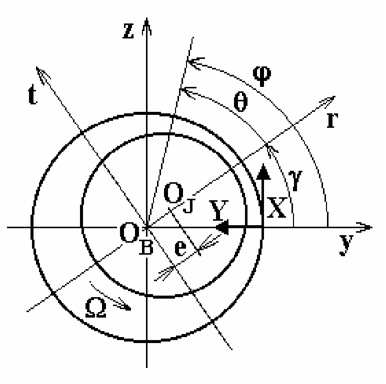

Fig.1 Scheme of the bearing

ϑ, Z - circumferential, axial coordinates (Fig.1),
e, γ - eccentricity of the rotor journal centre, position angle of the line of centres (Fig.1),
h_0, h - width of the gap at centric, eccentric position of the journal,
R, η - radius of the rotor journal, oil dynamical viscosity,
t - time,
p_R - pressure (pressure function),
u_1, w_1 - circumferential, axial velocity components of the points on the bearing shell surface,
u_2, w_2 - circumferential, axial velocity components of the points on the rotor journal surface.

If geometry and design parametres of the damper make possible to consider it as long (length to diameter ratio greater than 0.25, sufficient sealing at the damper faces), and taking into account usually accepted boundary conditions for velocities

$$u_1 = 0 \quad , \quad u_2 = 0 \quad , \quad v_1 = 0 \quad , \quad w_1 = 0 \quad , \quad w_2 = 0 \qquad (2)$$

the Reynolds equation (1) is transformed into a simplier form

$$\frac{1}{R^2}\frac{\partial}{\partial\vartheta}\left(h^3\frac{\partial p_R}{\partial\vartheta}\right)=-12\eta\left(\dot{e}\cos\vartheta+e\dot{\gamma}\sin\vartheta\right) \qquad (3)$$

To perform its solution two additional conditions must be added

$$p(\vartheta_0)=p_0 \qquad (4)$$

$$p(\vartheta)=p(\vartheta+2\pi) \qquad (5)$$

p_0, ϑ_0 - pressure magnitude, specified circumferential coordinate.

Condition (4) defines magnitude of the pressure at a specified location of the damper gap. (5) represents a condition of periodicity.

The circumferential velocity component depends on the pressure gradient and the radial one is determined from satisfying the equation of continuity

$$u = \frac{1}{2\eta R}\frac{\partial p_R}{\partial \vartheta} Y^2 + C_1 Y + C_2 \tag{6}$$

$$w = 0 \tag{7}$$

$$v = -\int_0^Y \left(\frac{1}{R}\frac{\partial u}{\partial \vartheta} \right) dY \tag{8}$$

u, v, w - circumferential, radial, axial velocity component.

The integration constants C_1, C_2 are calculated from the relationship defining the velocity boundary conditions given by (2).

If magnitude of the pressure at some location in the damper gap drops under a certain limit p_{cav}, a cavitation takes place and the Reynolds equation stops to hold. The experiments carried out by Zeidan and Vance [7] at the early 90-ties proved that pressure of the medium in cavitated regions remains approximately constant. Then for the pressure distribution around the circumference of the damper it can be assumed

$$p_d = p_R \quad \text{for} \quad p_R \geq p_{cav} \quad , \quad p_d = p_{cav} \quad \text{for} \quad p_R < p_{cav} \tag{9}$$

Radial and tangential components of the damping force are then given by its integration around the damper circumference

$$F_r = -RL\int_0^{2\pi} p_d \cos\vartheta \, d\vartheta \quad , \quad F_t = -RL\int_0^{2\pi} p_d \sin\vartheta \, d\vartheta \tag{10}$$

3. The method based on calculation of the viscous forces by solving a Reynolds equation and inertia ones by using a Lagrange equation and Reynolds transport theorem (S)

Substance of the damping forces (10) consists only in fluid viscosity. In the 80-ties some researcher proved that another factor that can significantly influence dynamical properties of squeeze film dampers is fluid inertia. On the other hand they reported that it does not greatly afflict the velocity field predicted by the classical lubrication theory.

In the early 90-ies El-Shafei [5] developed an approach to determination of damping forces of long squeeze film dampers taking into account the fluid viscosity and inertia. He assumed that the radial and tangential inertia forces consisted of two components

$$F_{ir} = F_{ir1} + F_{ir2} \tag{11}$$

$$F_{it} = F_{it1} + F_{it2} \tag{12}$$

F_{ir}, F_{it} - radial, tangential component of the fluid inertia force.

The first part arises from the change of the flow in uncavitated area and can be determined from the Lagrange equation of the second order

$$F_{ir1} = -\frac{d}{dt}\left(\frac{\partial W_{KL}}{\partial \dot{e}}\right) + \frac{\partial W_{KL}}{\partial e} \tag{13}$$

$$F_{it1} = -\frac{1}{e}\frac{d}{dt}\left(\frac{\partial W_{KL}}{\partial \dot{\gamma}}\right) + \frac{1}{e}\frac{\partial W_{KL}}{\partial \gamma} \tag{14}$$

W_{KL} - kinetic energy of the fluid in uncavitated region.

The second component has its origin in the flux through the boundary between the cavitated and uncavitated regions. Its magnitude is given by the Reynolds transport theorem

$$F_{ir2} = -\int_S \frac{\partial w_{KL}}{\partial \dot{e}} \mathbf{Vn}\, dS + \int_S w_{KL} \frac{\partial (\mathbf{Vn})}{\partial \dot{e}} dS \tag{15}$$

$$F_{it2} = -\frac{1}{e}\int_S \frac{\partial w_{KL}}{\partial \dot{\gamma}} \mathbf{Vn}\, dS + \frac{1}{e}\int_S w_{KL} \frac{\partial (\mathbf{Vn})}{\partial \dot{\gamma}} dS \tag{16}$$

S - border surface between cavitated and uncavitated areas,
V - velocity vector with respect to the surface S,
n - outward normal vector on the surface S,
w_{KL} - specific kinetic energy (kinetic energy per unit volume).

If the damper is uncavitated, the force components F_{ir2}, F_{it2} are zero.

Kinetic energy of the fluid in uncavitated region (in the case of long bearings it is assumed that the fluid flow is significant only in the circumferential direction) is defined by the following relationship

$$W_{KL} = \frac{1}{2}\int_{\vartheta_1}^{\vartheta_2}\int_0^h\int_{-\frac{1}{2}L}^{\frac{1}{2}L} \rho\, u^2 R\, d\vartheta\, dY\, dZ \tag{17}$$

To perform appropriate manipulations El-Shafei applied the velocity profiles obtained from solving the Reynolds equation. Consequently he derived relations (in a closed form) for fluid inertia forces that hold in a special case : (i) extension of the oil film around circumference of the damper is π or 2π and (ii) the rotor journal centre exhibits a central circular orbit.

The radial and tangential components of the damping force are then composed of the inertia and viscous parts that result from solving the Reynolds equation

$$F_r = m_r e\dot{\gamma}^2 - C_{rt} e\dot{\gamma} \tag{18}$$

$$F_t = -m_t e\dot{\gamma}^2 - C_{tt} e\dot{\gamma} \tag{19}$$

F_r, F_t - radial, tangential components of the damping force,
m_r, m_t - inertia coefficients,
C_{rt}, C_{tt} - viscous coefficients.

Both the viscous and inertia coefficients depend only on the relative eccentricity [5].

4. The method based on averaging terms in the Navier-Stokes equation related to the circumferential direction (ANS)

Another approach to taking into accout fluid inertia in computational models of squeeze film dampers is based on averaging terms in the Navier-Stokes equation related to the circumferential direction (to the direction of prevailing pressure gradient) across the film thickness

$$\frac{\partial p}{\partial \vartheta} = -R\,\frac{\rho}{h}\left(\int_0^h \frac{\partial u}{\partial t}\,dY + \int_0^h \frac{u}{R}\frac{\partial u}{\partial \vartheta}\,dY + \int_0^h v\,\frac{\partial u}{\partial Y}\,dY + \int_0^h w\,\frac{\partial u}{\partial Z}\,dY\right) + R\,\frac{\eta}{h}\int_0^h \frac{\partial^2 u}{\partial Y^2}\,dY \qquad (20)$$

To carry out appropriate manipulations the velocity profiles (6) - (8) predicted by the classical lubrication theory are applied.

The right-hand side of (20) is a complicated function of the circumferential coordinate ϑ and that's why the pressure function p cannot be expressed in a closed form. But as it is 2π periodic, it can be approximated by a finite number of terms (N_H) of a Fourier series

$$p = a_0 + \sum_{j=1}^{N_H} a_j\cos(j\vartheta) + b_j\sin(j\vartheta) \qquad (21)$$

and the Fourier coefficients a_j, b_j can be calculated by means of a trigonometric collocation method. This approach requires to specify N_P collocation points (angles)

$$\vartheta_k = \frac{2\pi}{N_P}.(k-1) \qquad \text{for} \qquad k = 1, 2, \ldots N_P \qquad (22)$$

and then substitution of its first derivative into (20) yields a set of linear algebraic equations

$$\sum_{j=1}^{N_H} -j\,a_j\sin(j\vartheta_k) + j\,b_j\cos(j\vartheta_k) = f_P(\vartheta_k) \qquad \text{for} \qquad k = 1, 2, \ldots N_P, \qquad (23)$$

f_p - right-hand side of (20).

To avoid singularity of the coefficient matrix the number of collocation angles must be greater than is the number of the unknown Fourier coefficients. But in this case solving the set of linear algebraic equations (24) must be performed utilizing a matrix pseudoinversion.

The absolute coefficient a_0 is determined from satisfying the boundary condition (4)

$$a_0 = p_0 - \sum_{j=1}^{N_H} a_j\cos(j\vartheta_0) + b_j\sin(j\vartheta_0) \qquad (24)$$

Taking into account a cavitation the pressure distribution in the damper gap is described by the following relations

$$p_d = p \quad \text{for} \quad p \geq p_{cav} \qquad , \qquad p_d = p_{cav} \quad \text{for} \quad p < p_{cav} \qquad (25)$$

Radial and tangential components of the damping force F_r and F_t are then obtained by integration of p_d around the damper circumference

$$F_r = -RL\int_0^{2\pi} p_d\cos\vartheta\,d\vartheta \qquad , \qquad F_t = -RL\int_0^{2\pi} p_d\sin\vartheta\,d\vartheta \qquad (26)$$

More details on this procedure are provided by [1] and [2].

5. The method based on solving a set of Navier-Stokes equations (NS)

This approach assumes that the flow in the damper gap as 3D. Calculation of the pressure distribution starts from solving a set of Navier-Stokes equations completed with the equation of continuity

$$\rho \frac{\partial \mathbf{c}}{\partial t} + \eta \text{ rot rot } \mathbf{c} + \text{grad } p = 0 \tag{27}$$

$$\text{div } \mathbf{c} = 0 \tag{28}$$

c - vector of velocity components,
p - pressure (pressure function),

To carry out the computation the boundary conditions for velocities must be imposed. There are several possibilities :

- zero velocity of the flow at Γ,
- zero velocity of the flow at P, K, Γ,
- zero velocity of the flow at Γ, zero pressure at P, K.

S, Γ - surfaces of the inner, outer rings,
P, K - surfaces perpendicular to the axis of the damper at its faces.

It is assumed that the lubricant adheres perfectly to surfaces of the damper rings and faces. If the damper can be considered as long, then the first or second type of the boundary conditions should be applied.

Solution of the set of Navier-Stokes equations is performed by means of a control volumes method using curvilinear coordinates. This procedure approximates the pressure distribution and the velocity components by a Bézier body which matches the boundary conditions and geometry of the damper. A detailed derivation of the resulting relations is given in [3], [4].

The next manipulations arrive at elimination of velocity components of the centre of the inner damper ring. The transformation relationships have the form of convolutory integrals

$$c_i = \int_0^t \alpha_{ij}(t-\tau)\, v_j(\tau)\, d\tau \quad , \quad p = \int_0^t \beta_i(t-\tau)\, v_i(\tau)\, d\tau \tag{29}$$

c_i - i-th velocity component,
α_{ij} ,β_i - velocity, pressure functions,
v_j , v_i - i-th, j-th velocity component of the inner damper ring.

This approach separates movement of the inner damper ring from the flow of lubricant in the damper gap which brings the advantage : coefficients of additional mass and damping depend only on the journal position and not on its velocity and therefore their magnitudes can be pre-calculated. Radial and tangential damping forces (F_r, F_t) are then expressed

$$\begin{bmatrix} F_r \\ F_t \end{bmatrix} = -\mathbf{A}_M \begin{bmatrix} a_r \\ a_t \end{bmatrix} - \mathbf{A}_B \begin{bmatrix} v_r \\ v_t \end{bmatrix} \tag{30}$$

$\mathbf{A}_M$, $\mathbf{A}_B$ - square matrices of additional mass, damping coefficients,
v_r, v_t, - radial, tangential components of the velocity of the inner damper ring centre,
a_r, a_t, - radial, tangential components of the acceleration of the inner damper ring centre.

6. Response of a rotor supported by squeeze film dampers on imbalance excitation

The assumed model rotor systems are assigned the following properties

- the shaft is represented by a beam-like body that is discretized into finite elements,
- the stationary part is absolutely rigid and motionless,
- the disks are axisymmetric absolutely rigid bodies,
- inertia and gyroscopic effects of the rotating parts are taken into account,
- the rotor is supported by rolling-element bearings and squeeze film dampers,
- the dampers are accommodated in the computational model by means of force couplings,
- material damping of the shaft is viscous, other kinds of damping (except the dampers) are linear,
- the rotor rotates at constant angular speed,
- the rotor is loaded by its weight and by centrifugal forces due to its imbalance.

Lateral vibration of the assumed rotors is governed by the equation of motion and by the relationship for boundary conditions

$$\mathbf{M}.\ddot{\mathbf{x}} + (\mathbf{B} + \eta_V.\mathbf{K}_{SH} + \Omega.\mathbf{G}).\dot{\mathbf{x}} + (\mathbf{K} + \Omega.\mathbf{K}_C).\mathbf{x} = \mathbf{f}_A + \mathbf{f}_V + \mathbf{f}_H(\mathbf{x}, \dot{\mathbf{x}}) \tag{31}$$

$$\mathbf{x}_{BC} = \mathbf{x}_{BC}(t) \tag{32}$$

$\mathbf{M}$, $\mathbf{G}$, $\mathbf{K}$ - mass, gyroscopic, stiffness matrices of the rotor system,
$\mathbf{B}$, $\mathbf{K}_C$ - (external) damping, circulation matrices of the rotor system,
$\mathbf{K}_{SH}$ - stiffness matrix of the shaft,
$\mathbf{f}_A$, $\mathbf{f}_V$, $\mathbf{f}_H$ - vectors of applied, constraint, hydraulical forces acting on the rotor system,
$\mathbf{x}, \dot{\mathbf{x}}, \ddot{\mathbf{x}}$ - vectors of generalized displacements, velocities, accelerations of the rotor system,
$\mathbf{x}_{BC}$ - vector of boundary conditions,
Ω, η_V - angular speed of the rotor rotation, coefficient of viscous damping (material of the shaft).

The steady-state response of such rotors on excitation caused by imbalance of the rotating parts can be determined for a certain class of problems by application of a trigonometric collocation method. This approach assumes that

- the response is a periodic function of time,
- its period can be derived from the period of excitation,
- it can be approximated by a finite number of terms of a Fourier series.

To be satisfied the boundary conditions (32) at any moment of time the equation of motion (31) is transformed into this form

$$\mathbf{A}_2.\ddot{\mathbf{y}} + \mathbf{A}_1.\dot{\mathbf{y}} + \mathbf{A}_0.\mathbf{y} = \mathbf{b} \tag{33}$$

where $\mathbf{A}_2$, $\mathbf{A}_1$, $\mathbf{A}_0$, $\mathbf{y}$, $\dot{\mathbf{y}}$, $\ddot{\mathbf{y}}$ and $\mathbf{b}$ are obtained from matrices $\mathbf{A}_2^*, \mathbf{A}_1^*, \mathbf{A}_0^*$ and vectors $\mathbf{x}$, $\dot{\mathbf{x}}$, $\ddot{\mathbf{X}}$, $\mathbf{b}^*$ by omitting their rows and columns that correspond to the degrees of freedom to which the boundary conditions are assigned

$$\mathbf{A}_2^* = \mathbf{M} \tag{34}$$

$$\mathbf{A}_1^* = \mathbf{B} + \eta_V.\mathbf{K}_{SH} + \Omega.\mathbf{G} \tag{35}$$

$$\mathbf{A}_0^* = \mathbf{K} + \Omega.\mathbf{K}_C \tag{36}$$

$$\mathbf{b}^* = \mathbf{f}_A + \mathbf{f}_H - \mathbf{A}_2^*.\ddot{\mathbf{x}}_{BC} - \mathbf{A}_1^*.\dot{\mathbf{x}}_{BC} - \mathbf{A}_0^*.\mathbf{x}_{BC} \tag{37}$$

The steady-state solution of (33) is approximated by a finite number of terms of a Fourier series

$$\mathbf{y} = \mathbf{y}_0 + \sum_{j=1}^{N} \mathbf{y}_{Cj} . \cos\left(j \frac{2\pi}{T} t \right) + \mathbf{y}_{Sj} . \sin\left(j \frac{2\pi}{T} t \right) \qquad (38)$$

$\mathbf{y}_0$, $\mathbf{y}_{cj}$, $\mathbf{y}_{sj}$ - vectors of Fourier coefficients (j = 1, 2, ... , N).

A trigonometric collocation method requires to specify N_C collocation points of time. Then substitution of the assumed solution and its derivatives into (33) for all collocation points results into a set of nonlinear algebraic equations. The unknowns are Fourier coefficients of all displacements of the rotor system to which no boundary conditions are assigned.

7. Example - response of a rotor on imbalance excitation

The investigated rotor (Fig.2) consists of a shaft (SH) and of two disks (D1, D2) attached to its overhanging end. The shaft is coupled with a rigid foundation plate (FP) through two rolling-element bearings and squeeze film dampers (SD1, SD2).

The rotor rotates at constant angular speed and is loaded by its weight (and by forces of constant magnitude acting on the shaft at the disk D1 location in the horizontal direction). In addition the system is excited by centrifugal forces caused by the disks imbalances.

The task was to analyze the steady-state component of the induced vibration.

In the computational model the shaft was represented by a beam-like body that has been discretized into 5 finite elements. Both dampers were considered as long.

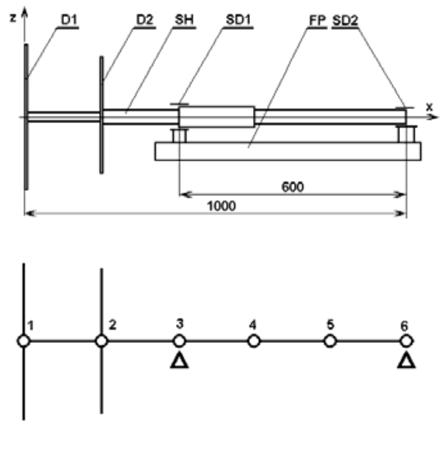

Fig.2 Scheme of the rotor system

Dependence of the diagonal elements of the matrix of additional mass corresponding to the radial and tangential directions on the relative eccentricity of the rotor journal in damper SD1 is given in Fig.3.

In Fig.4 there are compared forms of orbit of the rotor journal centre in damper SD1 determined by different approaches to calculation of the damping forces. It is evident that all arrive approximately at the same results. The procedure based on solution of the Navier-Stokes equations shows slightly greater damping.

In Fig.5 and 6 there are drawn time histories of the oil pressure during one period obtained by the classical lubrication theory (R) and by the approach based on averaging inertia terms in the Navier-Stokes equation (ANS). The results confirm that influence of the fluid inertia properties rises with increasing magnitude of the Reynolds number calculated according to [5].

Comparison of orbits of the rotor journal centre in damper SD1 calculated by means of a Reynolds and averaged Navier-Stokes equations is carried out in Fig.7 and 8. Its evident that contribution of the fluid inertia to the damping effects depends on magnitude of the Reynold's number.

Fig.9 and 10 show trajectory of the rotor journal centre in damper SD1 in dependence on the force magnitude acting on the shaft at disk D1 location in the vertical direction. For larger eccentricities difference between the results obtained by NS and ANS methods is greater.

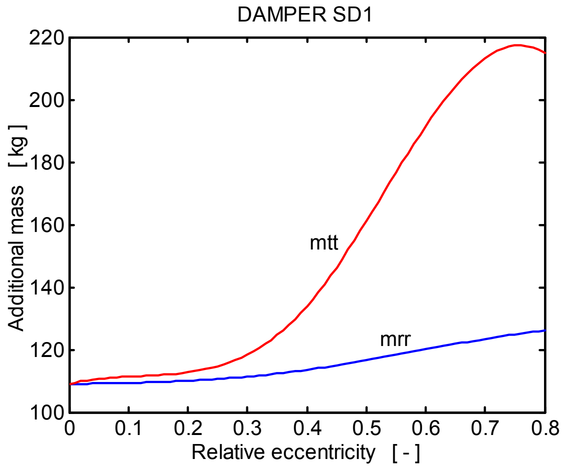

Fig.3 Additional mass coefficients

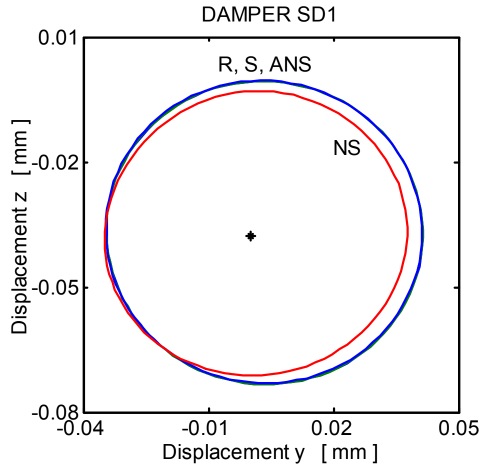

Fig.4 Orbit of the rotor journal centre

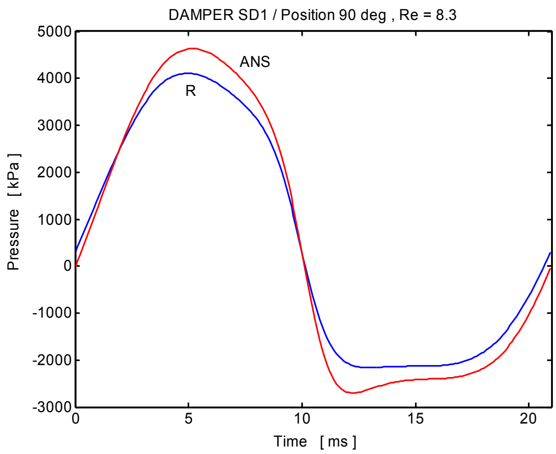

Fig.5 Time history of the pressure

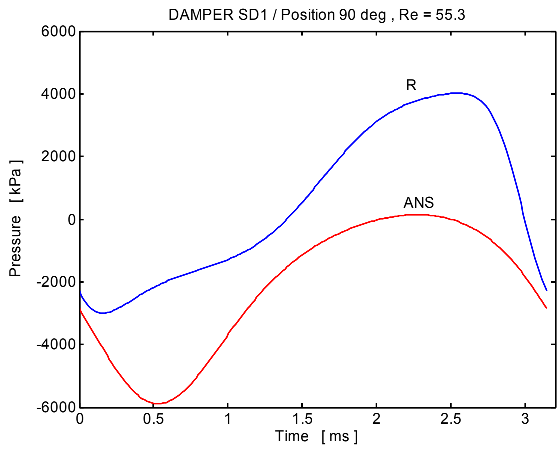

Fig.6 Time history of the pressure

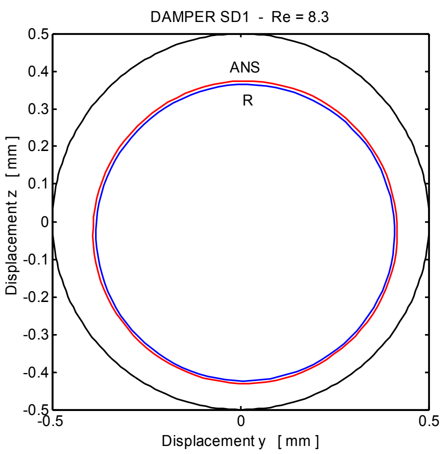

Fig.7 Orbit of the rotor journal centre

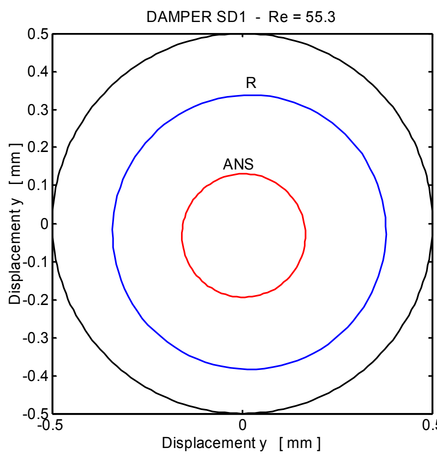

Fig.8 Orbit of the rotor journal centre

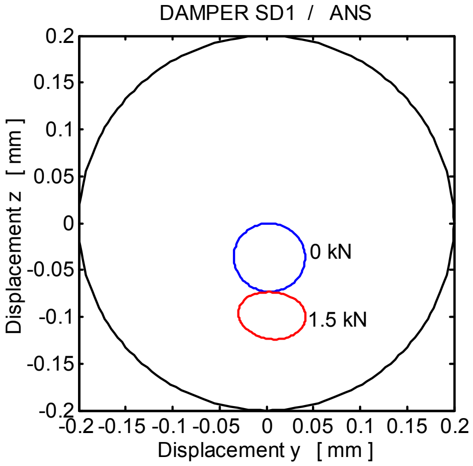

Fig.9 Orbit of the rotor journal centre

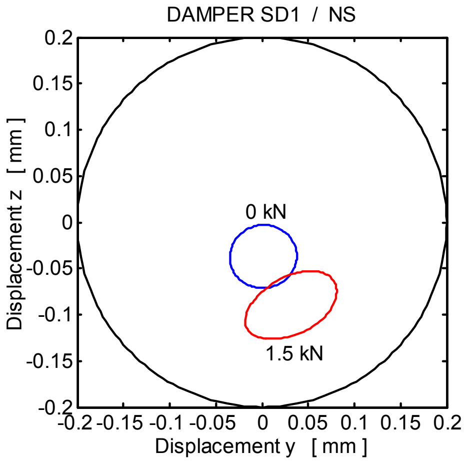

Fig.10 Orbit of the rotor journal centre

8. Conclusions

In computational models the squeeze film dampers are usually incorporated by means of nonlinear force couplings. Four approaches to determination of the damping forces have been implemented into the procedure for calculation of the steady-state response of a rotor on imbalance excitation and have been compared. Results of the computer simulations show that contribution of the fluid inertia to dynamical properties of the dampers becomes significant if magnitude of the Reynolds' number is large.

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