National Conference with International Participation
ENGINEERING MECHANICS 2003
Svratka, Czech Republic, May 12 – 15, 2003
paper no. 146

NON-NEWTONIAN LIQUIDS: DETERMINATION OF A VALUE OF VISCOSITY CHARACTERISING THE INITIAL NEWTONIAN PLATEAU

J. David*, **P. Filip***

Summary: *The present contribution indicates that precisely the same situation - as with the determination of the so-called yield stress of viscoplastic materials - concerns also the value of viscosity describing the initial Newtonian plateau. It is shown that the determination of this value from the experimental data is not unique, significantly subjects to the range of measured parameters, and the proper determination of the precise value is very questionable.*

1. Yield stress

The notion of the so-called yield stress is related with the materials denoted as viscoplastic. Their phenomenological description started in the 20s' in the last century when Bingham introduced the model still amply used for the description of the viscoplastic materials (see e.g. the overview paper on viscoplastic materials by Bird et al.(1983))

$$\tau = \left(\eta_p + \frac{\tau_0}{|\gamma|} \right) \gamma \tag{1}$$

where τ and γ represent shear stress and shear rate, respectively; η_p stands for viscosity, and τ_0 expresses the so-called yield stress. If an absolute value of the shear stress $|\tau|$ attains the value below this quantity there is no apparent external motion of the analysed liquid.

This model was further generalised by many authors all of them respecting the notion of a yield stress as e.g. Herschel-Bulkley model

$$\tau = \left(K|\gamma|^{n-1} + \frac{\tau_0}{|\gamma|} \right) \gamma \quad , \tag{2}$$

* Jiří David, Petr Filip, Institute of Hydrodynamics, Acad.Sci.Czech Rep., Pod Patankou 5, 166 12 Prague 6, Czech Republic, Tel.: +420.233 109 011, Fax: +420.233 324 361, e-mail: david@ih.cas.cz, filip@ih.cas.cz

Casson model (often used for description of biomaterials)

$$\tau = \left[\eta_p + \left(\frac{\tau_0}{|\gamma|} \right)^{1/2} \right]^2 \gamma \quad , \tag{3}$$

Vocadlo model (Vocadlo (1967), Parzonka and Vocadlo (1967, 1968), sometimes denoted as Robertson-Stiff model (Robertson and Stiff (1976))

$$\tau = \left[K|\gamma|^{\frac{n-1}{n}} + \left(\frac{\tau_0}{|\gamma|} \right)^{\frac{1}{n}} \right]^n \gamma \quad , \tag{4}$$

Shul'man (1968) model

$$\tau = \left[K^{\frac{1}{m}}|\gamma|^{\frac{1}{m}-\frac{1}{n}} + \left(\frac{\tau_0}{|\gamma|} \right)^{\frac{1}{n}} \right]^n \gamma \tag{5}$$

and a series of others (graphical representation of the above mentioned models is in Fig.1).

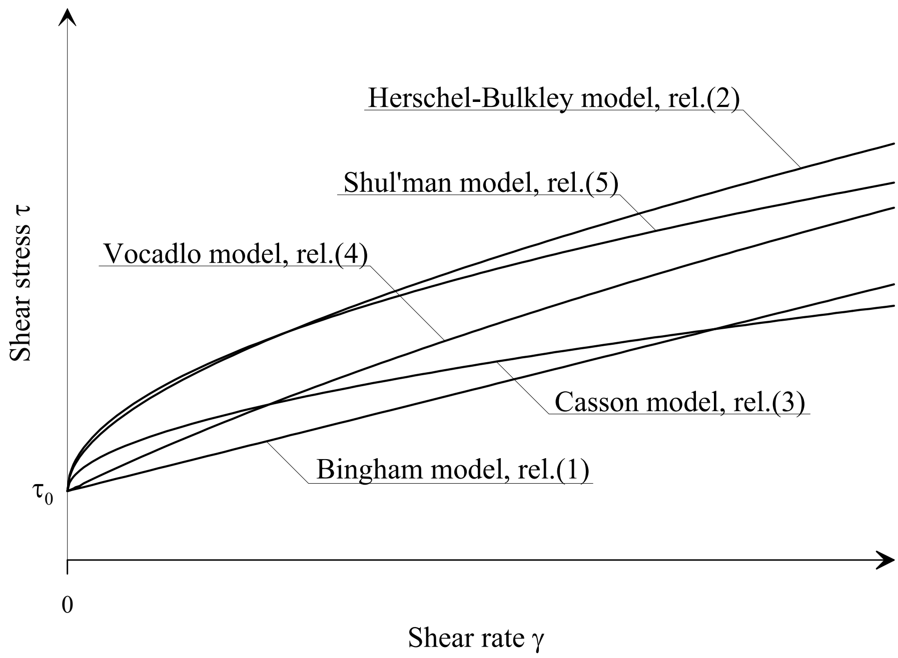

Fig.1 Behaviour of selected viscoplastic models.

However in the past approximately 20 years a wide debate has accompanied the notion of the yield stress starting with the fundamental paper by Barnes and Walters (1985) that questioned the introduction of this notion and emphasised the (non-) sensitivity of the measuring device for low values of shear rate. A series of contributions discussing this problem was summarised in Barnes (1999) concluding that at present there are two possibilities how to cope with this problem. Either the so-called ‘academic’ one considering the non-existence of this notion, or the so-called ‘engineering’ one using this notion whenever it is suitable e.g. in the calculation process. Nevertheless the question of existence or non-existence of the yield stress is still open.

2. Initial Newtonian plateau

The analogous situation also appears in the relation viscosity vs. shear stress (or viscosity vs. shear rate).

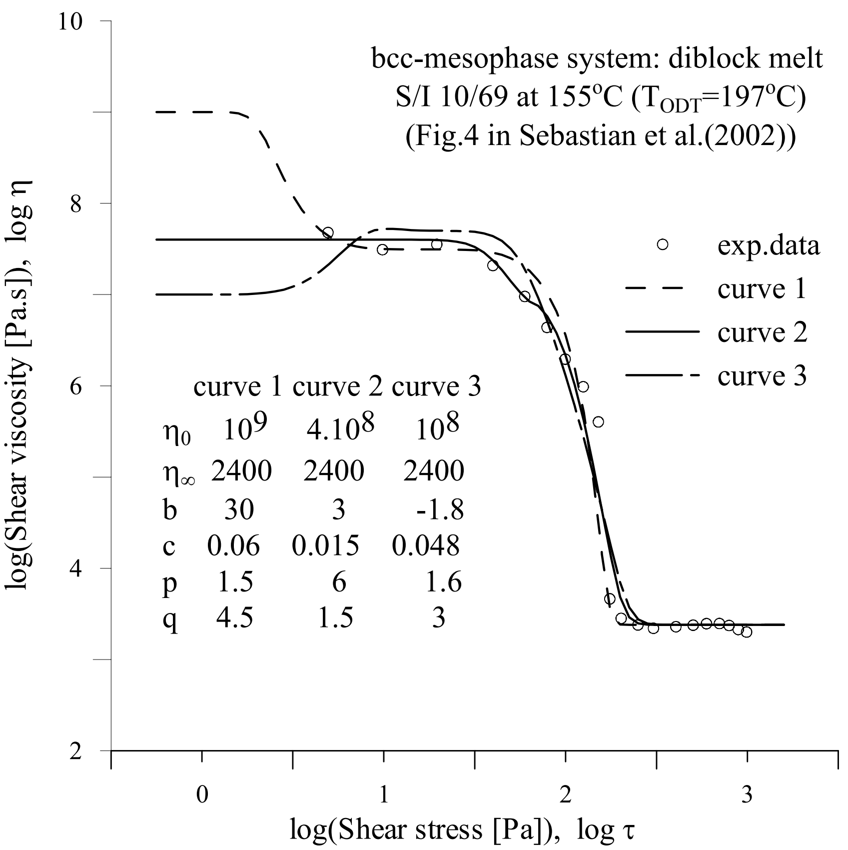

Fig.2 Ambiguity of the initial Newtonian plateau.

Let us consider the following 6-parameter model

$$\eta = \frac{\eta_\infty \exp\left(f(\gamma; c, p, q)\right) + \eta_0 \exp\left(-f(\gamma; c, p, q)\right)}{b + \exp\left(f(\gamma; c, p, q)\right) + \exp\left(-f(\gamma; c, p, q)\right)} \qquad (6a)$$

where

$$f(\gamma; c, p, q) = sign\left(\log(c\gamma)^p\right) \cdot \left|\log(c\gamma)^p\right|^q \quad . \tag{6b}$$

The non-negative parameters η_0 and η_∞ determine the initial and terminal Newtonian plateau, respectively. The parameters c, p, q are supposed to be positive, the denominator in rel.(6a) requires b>-2.

If this model is applied to the experimental data by Sebastian et al. (2002) we can see (Fig.2) that there exists for this model an infinite number of possibilities how to predict the possible behaviour of the material studied for very low values of shear rates (the so-called initial Newtonian plateau). The problem of this ambiguity is very similar to that of the introduction of the yield stress and its solution will be closely related with the development of the new sophisticated experimental devices.

3. Acknowledgement

The authors wish to acknowledge GA AS CR for the financial support of Grant No. A2060004.

4. References

Barnes,H.A. (1999) The yield stress – a review or 'παντα ρει' – everything flows? *J.Non-Newt.Fluid Mech.*, 44, pp.133-178.

Barnes,H.A. & Walters,K. (1985) The yield stress myth. *Rheol.Acta*, 24, pp.323-326.

Bird,R.B., Dai,G.C. & Yarusso,B.J. (1983) The rheology and flow of viscoplastic materials. *Rev.Chem.Eng.*, 1, pp.1-70.

Gorislavec,V.M., Smol'skii,B.M. & Shul'man,Z.P. (1968) *Rheophysics and rheodynamics of fluid systems* (in Russian). Nauka i technika, Minsk – U.S.S.R.

Parzonka,W. & Vocadlo,J. (1967) Modèle à trois paramètres pour les corps viscoplastique. Solution pour le viscosimètre rotatif type Couette. *C.R.Acad.Sc.Paris*, 264, Série A, Apr., pp.745-748.

Parzonka,W. & Vocadlo,J. (1968) Méthode de la caractéristique du comportement rhéologique des substances viscoplastique d´après les mesures au viscosimètre de Couette (modèle nouveau à trois paramètres). *Rheol.Acta*, 7, pp.260-265.

Robertson,R.E. & Stiff,H.A. (1976) An Improved Mathematical Model for Relating Shear Stress to Shear Rate in Drilling Fluids and Cement Slurries. *Soc.Pet.Eng.J.*, Feb., pp.31-36.

Sebastian, J.M., Graessley, W.W. & Register, R.A. (2002) Steady-shear rheology of block copolymer melts and concentrated solutions: Defect-mediated flow at low stresses in body-centered-cubic systems, *J.Rheol.*, 46, pp.863-879.

Vocadlo,J. (1967) *Ph.D.Thesis*, Institute of Hydrodynamics, Czechoslovak Academy Sciences, Prague, Czechoslovakia.