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EFFECT OF THE HEAT TRANSFER ON THE LAMINAR FLOW IN THE ENTRANCE REGION OF A CIRCULAR TUBE

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Summary: *An analytical-numerical model of the velocity and temperature field was created, whose parameters are expressed as functions of the thickness of the hydrodynamic and thermal boundary layers. This model converts the set of specific partial differential equations to a set of ordinary differential equations that are always valid in one of three regions into which the entrance region is divided by above mentioned boundary layers. Within the individual regions, these equations can be integrated and the algebraic equations obtained are then solved numerically. The model was tested for isothermal flow, where the calculated thickness of the velocity boundary layers agreed with the published values. In addition, the calculated tube wall temperatures exhibited excellent agreement with the values obtained experimentally. This model was employed to study the effect of the temperature dependent viscosity on changes of flow in the entrance region of a circular tube heated by constant heat flux.*

1. Introduction

In the description of pressure drops and heat transfer in the entrance region of a tube during the flow of very viscous liquids, the temperature and velocity profile in this area are of practical importance. In these liquids (characterized by high Prandtl numbers *Pr*), the development in the velocity profile occurs at a relatively large distance from the entrance into the tube and the viscosity changes substantially with the temperature. Entrance into a circular tube is almost a classical problem that has been described in a great many papers; nonetheless, however, a satisfactory solution has not yet been provided for the simultaneous development of a velocity and temperature profile, including temperature dependent viscosity. At the very beginning of the entrance section of the tube, which can include the entire region of hydrodynamic development of the flow, where there is singularity of the changes of the uniform velocity profile to a parabolic form, there are large velocity and temperature gradients and thus the numerical results are not very exact. Similarly, it is not possible to obtain detailed and reliable experimental data in this region. Thus, an analytical-numerical model of the velocity and temperature fields was created, whose parameters are

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expressed as a function of the thickness of the velocity and temperature boundary layers [1,8], which permits determination of the velocity and temperature profiles.

2. Model creation and validation

The model was created for a horizontal tube with a circular cross-section with a uniform wall heat flux. The flow of the liquid through the tube is laminar and steady state; the liquid is assumed to have constant thermal conductivity, specific heat and density and only the viscosity is temperature dependent. The terms of reference include knowledge of the initial temperature, pressure, and entrance velocity and wall heat flux. The task consists in determining the temperature of the wall of the tube, which determines the heat transfer. For this purpose, it is necessary to know the temperature and velocity fields in the studied part of the tube.

In creating the model, the starting point was the Navier-Stokes momentum equation in the axial direction and the continuity and energy equations, all in cylindrical coordinates. To simplify these equations, it is further assumed that the flow is axially symmetrical and the viscous dissipation and the action of body forces are neglected, because forced convection is involved. The momentum equation for the radial direction is not used, because the pressure distribution in the radial direction need not be studied in the solution in question. The momentum equation for the axial direction then has the form

$$\frac{\partial}{\partial r}\left(\mu\frac{\partial v_x}{\partial r}\right)+\frac{1}{r}\mu\frac{\partial v_x}{\partial r}=\rho\left(v_x\frac{\partial v_x}{\partial x}+v_r\frac{\partial v_x}{\partial r}\right)+\frac{\partial p}{\partial x}-\frac{\partial}{\partial r}\left(\mu\frac{\partial v_r}{\partial x}\right)-\frac{1}{r}\mu\frac{\partial v_r}{\partial x}-\frac{\partial}{\partial x}\left(2\mu\frac{\partial v_x}{\partial x}\right)$$

The last term in this equation can be omitted (the change in the quantity in the axial direction is negligible compared to the radial direction). It can also be stated that quantities v_r and $\partial v_x/\partial x$ are small compared to v_x and $\partial v_x/\partial r$ and thus can be replaced in the equation by the values centered across the radius of the tube. The same can be carried out with the quantity $\partial p/\partial x$ according to [2] (the Slezkin-Targ method). In addition to the above assumptions, the energy equation is simplified by neglecting the heat conduction in the axial direction against convection. In addition, instead of the convective terms, their values averaged across the thickness of the temperature boundary layer are used:

$$\frac{\lambda}{\rho c_p}\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}\right)=\frac{1}{k}\int_0^k\left(v_x\frac{\partial T}{\partial x}+v_r\frac{\partial T}{\partial r}\right)dr$$

The expression for the temperature dependence of the viscosity was selected in a form that is suitable for the calculation and permits determining of the coefficients from the measured values.

$$\frac{1}{\mu}=a_0+a_1T+a_2T^2$$

The equations obtained with boundary conditions are converted to the dimensionless form in the subsequent solution. Integration of the equation was carried out separately in the individual regions, that are bounded by the temperature (k) and velocity (δ) limiting layers, as depicted in the figure 1, which is sketched for Prandtl number Pr>1 (this assumption is justified because the temperature dependence of the viscosity is important only at higher values and, thus, also higher *Pr*).

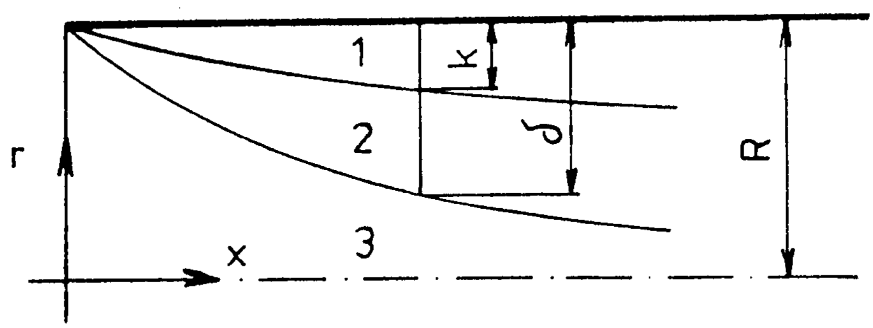

Fig. 1. Coordinate system and velocity and temperature boundary layer thickness in entrance region of the tube.

This separation has the advantage that affecting of the velocity profile of the temperature dependent viscosity occurs only for the thermal boundary layer region (1) and the velocity profile evolves isothermally in the remainder of the velocity boundary layer (2) and the velocity remains constant in the core region of the flow (3). After carrying out integration and rearrangement (the integral continuity equation is applied in three regions of the velocity profile), the temperature profile is expressed in terms of the parameter of the thickness of the temperature boundary layer, and the velocity profile is expressed differently in the three regions using the parameters of the thickness of the temperature and velocity boundary layers

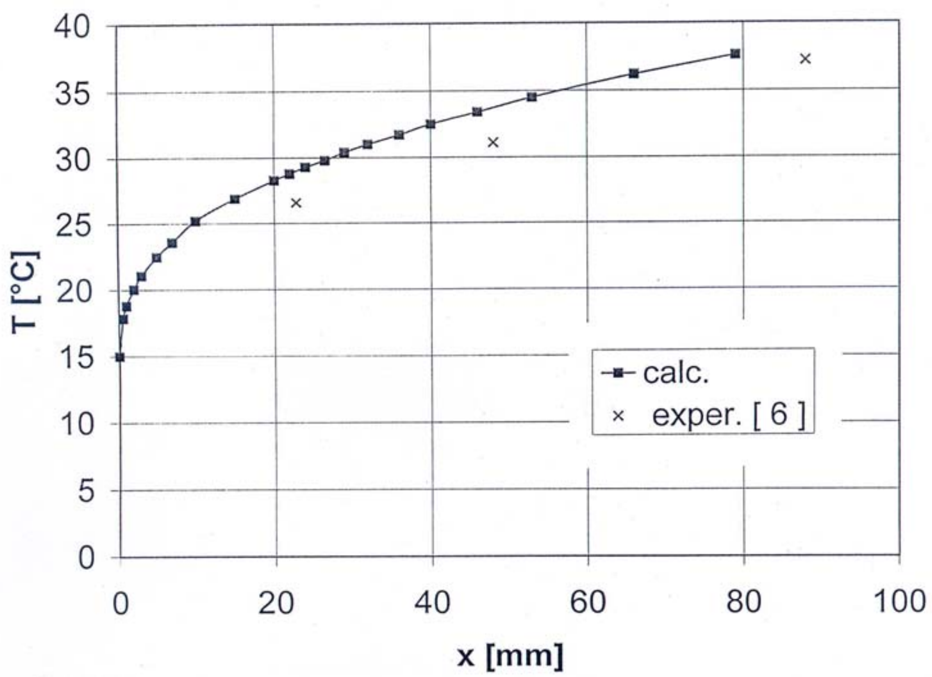

Fig.2. Tube wall temperatures in entrance region for sugar solution.

and the coefficients of the temperature dependent viscosity. In order to be able to utilize the relatively complex algebraic expressions obtained using this analytical approach, the literature relationships for the thickness of the velocity boundary layer [3] and temperature boundary layer [4] were employed and these relationships were used to carry out the relatively complex numerical calculations.

The model was first tested for isothermal flow, where the calculated thickness of the velocity boundary layer agreed with the published data. In addition, the calculated tube wall temperatures were compared with both the experimental data [5,6] obtained for a sugar solution, see Fig.2. (R = 6 mm, entrance velocity and temperature $v_V = 0.2$ m/s and $T_V = 15$°C, heating with a constant heat flux $q = 20.28$ kW m^{-2}, $\mu = 31$ mPa s, $\rho = 1263$ kg m^{-3}, $\lambda = 0.419$ W m^{-2} K^{-1}, $c_P = 2\,858$ J kg^{-1} K^{-1}, $Re = 98$, $Pr = 211$) and the experimental data [7] obtained for a heavy oil, see Fig.3. ($R = 25$ mm, $v_v = 2$ m/s, $q = 6$ kW m^{-2}, $T_v = 40$°C, other properties at 25°C: $\mu = 0.219$ Pa s, $\rho = 879$ kg m^{-3}, $\lambda = 0.124$ W m^{-2} K^{-1}, $c_p = 1555$ J kg^{-1} K^{-1}, $Re = 401$, $Pr = 2\,746$).

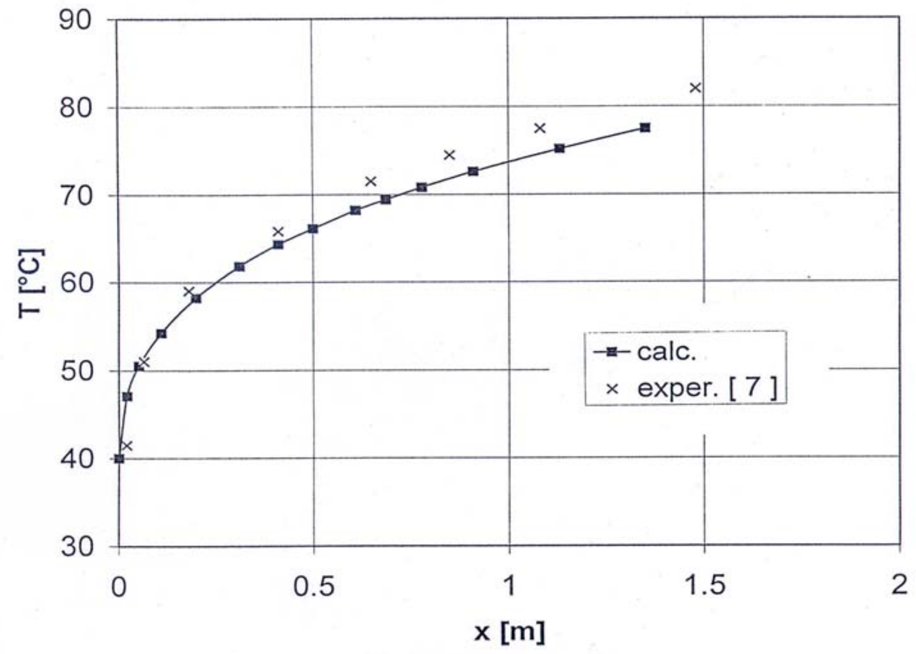

Fig.3. Tube wall temperatures in entrance region for heavy oil.

Calculated wall temperatures shown on figures 2. and 3. turn aside symmetrically from the measured values only in larger distance from the entrance. The model can be used in a broad spectrum of causes because there were used widely different liquids and different heating. The influence of mechanical energy dissipation on temperature of the liquid was verified in a more viscous liquid. According to the calculated value of Brinkman number *Br* there is no influence of the dissipation on the liquid temperature.

3. Results

The model was employed to study the effect of the temperature dependent viscosity on the flow conditions in the entrance part of the tube heated by constant heat flux. Liquids with

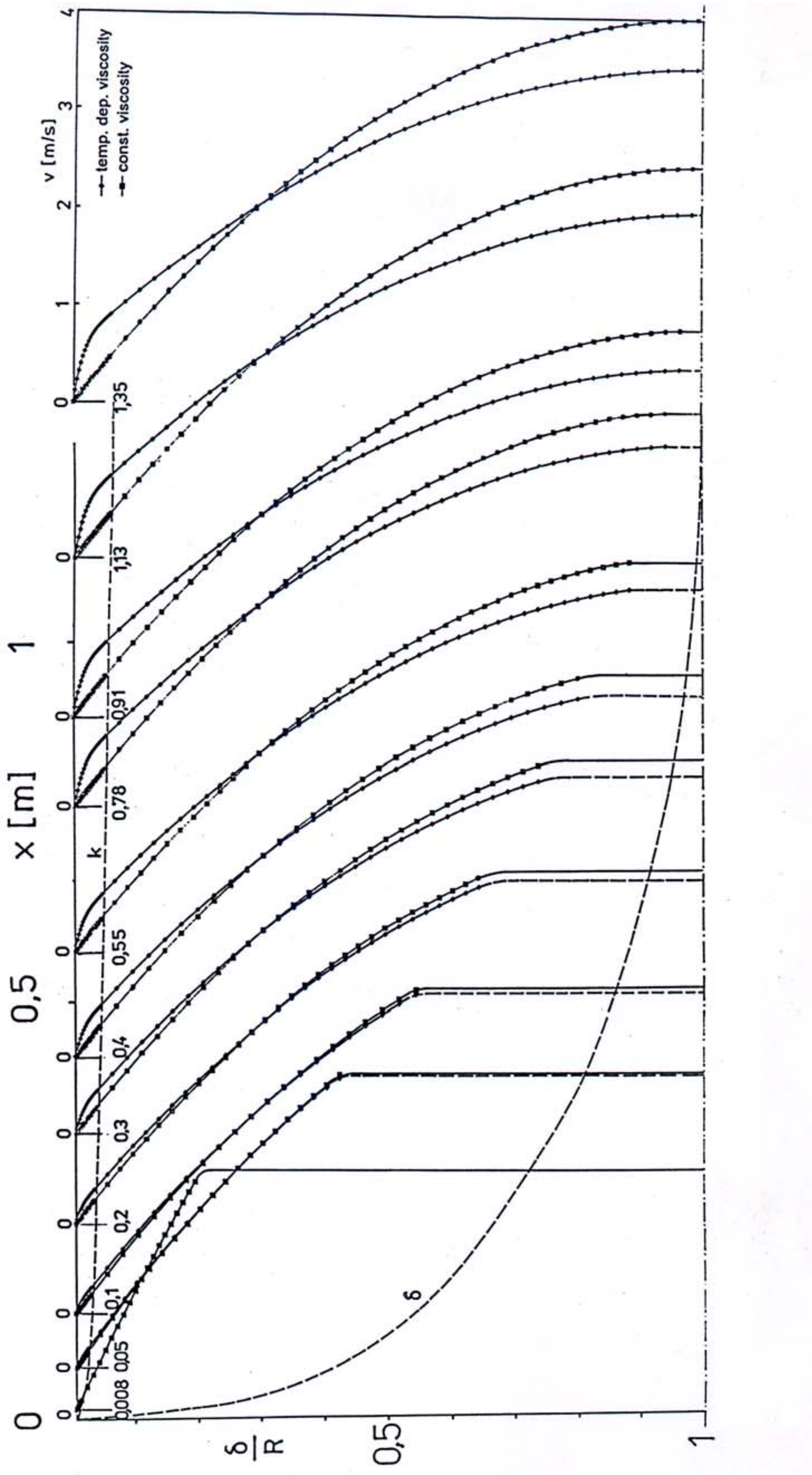

Fig.4. Development of the velocity profile of heavy oil [7] in hydrodynamic entrance region of the tube.

various viscosity were used and the differences were evaluated between isothermal flow and flow during heating of the wall by a constant heat fluxes taking into consideration the temperature dependent viscosity. The shapes of the isothermal and non-isothermal velocity profiles, the temperature profiles, the changes in the thickness of the hydrodynamic and temperature boundary layers and the changes in the velocity gradient at the wall for isothermal and non-isothermal flow were obtained in the region of development of the velocity profile. An example of obtained results is in the Fig.4. There is shown the velocity profile development in both isothermal and non-isothermal flow of heavy oil [7] in hydrodynamic entrance region of the tube.

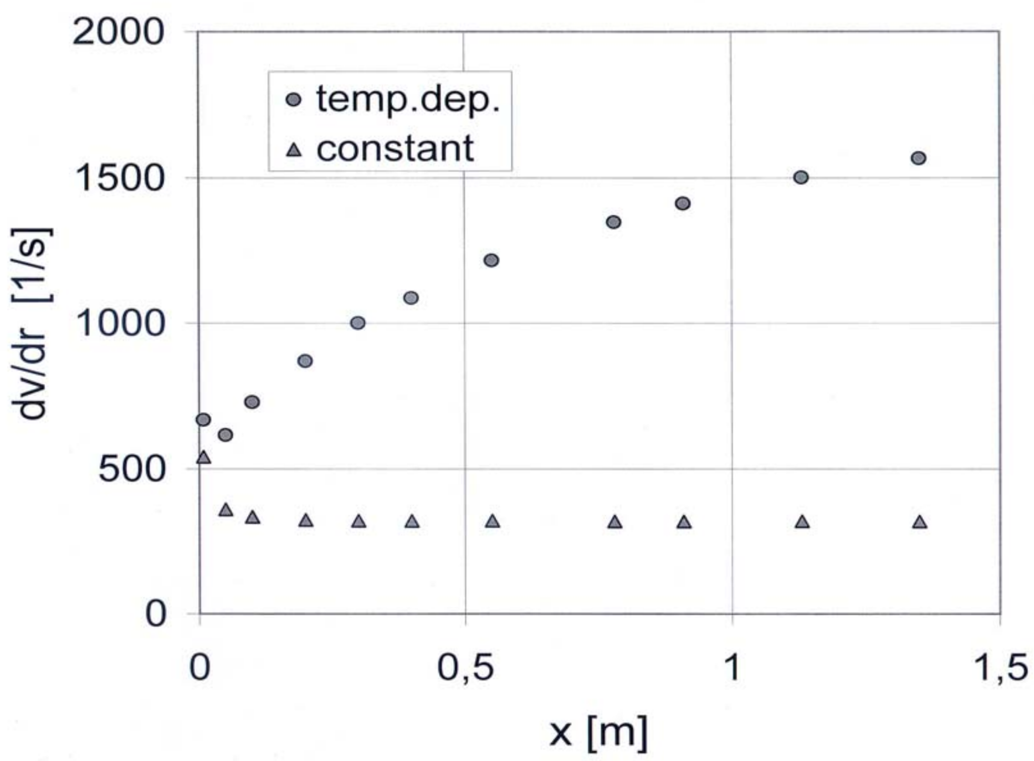

Fig.5. Wall velocity gradient in entrance region for heavy oil [7].

The course of the wall velocity gradient for constant and temperature dependent viscosity is drawn in Fig.5. for the same liquid in the region of hydrodynamic entrance length.

All obtained data could be used to quantitatively evaluate the effect of the temperature-dependent viscosity on the flow and heat transfer in the entrance region of the tube.

4. Acknowledgment

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5. List of symbols

a_0, a_1, a_2 coefficients

c_p	specific heat [J kg^{-1} K^{-1}]
k	thickness of the temperature boundary layer [m]
q	heat flux [W m^{-2}]
R	radius of tube [m]
r	radial coordinate [m]
T	temperature [^{o}C]
v	velocity [m s^{-1}]
x	axial coordinate [m]
δ	thickness of boundary layer [m]
λ	thermal conductivity [W m^{-2} K^{-1}]
μ	viscosity [Pa s]
ρ	density [kg m^{-3}]
Br	Brinkman number $\mu\,(v_k)^2/\lambda(T_w - T_k)$ [-]
Pr	Prandtl number $c_p\,\mu/\lambda$ [-]
Re	Reynolds number $2\,R\,v\,\rho/\mu$ [-]

Subscripts

k	temperature boundary edge
o	axis of the tube
r	radial
v	entrance
w	wall
x	axial

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