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DYNAMICS OF GEAR MESH SOLUTION USING MATLAB

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Summary: *This paper presents a solution of high-frequency dynamic effects occurring in the mesh of pair of gears that was formulated and calculated using the MATLAB development environment. The mathematical model used to describe the system takes into account the mesh unevenness as a result of both the alternation of the number of teeth in the mesh as well as variable mesh stiffness depending on the mesh position.*

1. Introduction

The adopted mathematical model of the high-frequency effects in the gear mesh is an ordinary linear differential equation of the second order that describes generally damped stimulated oscillations where the reduced mass and damping parameters are considered to be constant while the only variable excitation is represented by the zero order term that includes the mesh stiffness and the mesh transmission unevenness functions. The periodic solution of this differential equation is found by one of standard numerical integration methods of MATLAB. Properties of this periodic solution are then discussed as to their implications regarding resonance and related phenomena.

2. Model formulation and its solution

One of the simplest forms of mathematical model describing periodic high-frequency effects in the gear mesh transmission that take into account the elastic properties of the gear material can be expressed as the following differential equation (Doležal, 1995):

$$m.q''(t) + k.q'(t) + b.c(t).(q(t) - \nu(t)) = 0 \qquad (1)$$

where m is the reduced mass of the gear pair defined by the equation

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$$1/m = r_1^2/I_1 + r_2^2/I_2 \qquad (2)$$

k is the viscous damping constant coefficient, b is the gear pair width, c(t) is the gear mesh stiffness function and ν(t) is the gear mesh transmission unevenness function. The quantities of r_i and I_i are base circle radii and angular momentums respectively of the gear i = {1, 2} of the gear pair. The quantity of q(t) is the relative displacement of the driving gear relative to the driven gear of the gear pair as measured in units of distance on the mesh line.

The periodic solution of the equation (1) is found by means of a standard numerical integration method of MATLAB namely the ordinary differential equation solver termed ode15s. The convergence criteria of the iteration process of finding the periodic solution were set at quite a stringent level of relative change of the quantity q(t) and its first temporal derivative q'(t) having to be less than 0.1 ‰ between the tooth mesh period limit values, i.e. values at the beginning and end of the period. As to the very method employed to calculate the solution, several numerical techniques have been utilized to speed up the iteration process, namely using the results already calculated for preceding frequencies (rotations per minute) to predict the initial values of quantities for the iteration calculation at the next frequency. For this purpose up to quadratic extrapolation method was used.

The calculated results were plotted in three-dimensional graphs where one of the horizontal coordinates is the mesh position on the mesh line. The other horizontal coordinate is the driving gear rotations per minute. The vertical coordinate being either total mesh force or mesh tooth pair force.

3. Values of model parameters

The above outlined model has been encoded in MATLAB development environment code taking advantage of the conciseness of the tool and utilizing the wide spectrum of MATLAB code available from its libraries. The constructed MATLAB code has been tested on a particular set of input parameters.

The input parameters used were as follows:

Reduced mass of the gear pair	m = 3.226 [kg]
Viscous damping constant coefficient	k = 1.5954e+04 [kg/s]
Gear pair width	b = 75 [mm]
Nominal load force loading the mesh	Fn = 17035 [N]
Number of the teeth of the driving gear	z1 = 21 [1]
Number of the teeth of the driven gear	z2 = 91 [1]

Although the gear mesh stiffness function c(t) and the gear mesh transmission unevenness function ν(t) have been used in a pre-calculated form of a Fourier series it is more advantageous to calculate them from their constituent parts namely from the tooth pair stiffness functions that are determined by dimensionless tooth pair flexibilities expressed as a fourth order regression polynomial in gear mesh position.

4. Discussion of the calculated results

The three-dimensional plots of the quantities calculated in this model reveal some of the properties of the modeled physical system, namely the resonance properties that are determined on one hand by the elastic properties of the material of the gears, their rigidity or stiffness coupled by their reduced mass and on the other hand by the mesh internal excitations arising from both the alternation of the number of teeth in the mesh and the resulting variations of dynamic loads and from the variable mesh stiffness depending on the mesh position all impacted by the viscous damping.

One should also note the limitations of this model as it yields results in certain regions in the frequency (rotations per minute) domain that are hard to interpret physically.

5. Conclusions

The calculated results clearly show the region of strong resonance at the driving gear rotations per minute of around 11000 [RPM] with several secondary resonances at lower frequencies for the particular set of input parameters used. Furthermore the high-frequency end of the spectrum shows gradual diminution of the resonance response of the system.

6. Acknowledgements

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7. References

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