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NONLINEAR BEHAVIOR ANALYSIS OF FILAMENT — WOUND COMPOSITE TUBES

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Summary: *Our recent experiments on filament-wound composite tube showed that unidirectional and cross filament-wound composite tubes had nonlinear behavior even under axial loading. This nonlinear behavior, in our opinion, is mainly caused by nonlinear shear behavior. In this paper, the nonlinear constitutive relation developed by Hahn and Tsai is adopted in our analysis where a piecewise linear approximation is used for unidirectional filament wound tubes. The nonlinear numerical and experimental results of unidirectional filament wound tubes have a good agreement. Also in these paper, some experimental results and analysis about cross filament wound tubes are given.*

1. Introduction

Filament-wound composite tubes are being increasingly used and are becoming an important class of engineering materials for a wide range of applications owing to their high strength-to-weight ratio and tailored design. Filament winding, one of the most common techniques for the manufacture of tubes, often produces interweaving of fibers inside the layer when cross filament winding is needed. Generally speaking, filament winding can be divided into two types, one is unidirectional filament winding (or called laminated filament winding) in which fiber tows are deposited on a mandrel side by side, such as axial winding, off-axis winding and hoop winding; the other is cross filament winding in which fiber tows are interlaced when they cross. Our recent experiments on the axial stiffness of filament-wound carbon-fiber-reinforced epoxy tubes showed that filament wound composite tubes had nonlinear behavior even under axial loading. The nonlinear behavior of unidirectional composite laminea has been studied by Hahn and Tsai[1]. Basing on the theory developed by Hahn and Tsai, Hahn[2] studied nonlinear behavior of laminated composite. Ishikawa and Chou[3] studied cross-plied woven fabric composites. In their paper they introduced Hahn and Tsai's nonlinear shear theory in their analysis and also developed fiber undulation and bridging models to explain the nonlinear behavior of woven fabric composites. However, no analysis is about filament wound composites.

In this paper, first, the nonlinear constitutive relation developed by Hahn and Tsai is introduced, then a piecewise linear approximation is used for unidirectional filament wound tubes.

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2. Nonlinear theory

One of the poorly understood characteristics of unidirectional fibrous composites is the appearance of severe nonlinearity in longitudinal shear stress-strain relation while in longitudinal tension or compression strain remains linearly related to stress. Deviation from linearity is also observed in transverse loadings; however, the degree of nonlinearity is not comparable to that in the longitudinal shear. Thus the problem may be effectively described within the framework of the nonlinear theory of elasticity.

Hahn and Tsai[1] developed the nonlinear constitutive relation which is adopted here. When a strain energy function per unit volume, $W(\varepsilon_{ij})$, exists, the stress-strain relations are derived from

$$\sigma_{ij} = \frac{\partial W}{\partial \varepsilon_{ij}}, \tag{1}$$

where σ_{ij} is the stress tensor and ε_{ij} the strain tensor. By introducing the complementary energy density W^*, strain can be determined in terms of stress.

$$\varepsilon_{ij} = \frac{\partial W^*}{\partial \sigma_{ij}} \tag{2}$$

For a state of plane-stress, a polynomial expansion for W^* of up to fourth-order will be

$$\begin{aligned} W^* &= \frac{1}{2}S_{11}\sigma_1^2 + \frac{1}{2}S_{22}\sigma_2^2 + S_{12}\sigma_1\sigma_2 + \frac{1}{2}S_{66}\sigma_6^2 + \frac{1}{3}S_{111}\sigma_1^3 + \frac{1}{3}S_{222}\sigma_2^3 + S_{112}\sigma_1^2\sigma_2 \\ &+ S_{122}\sigma_1\sigma_2^2 + S_{166}\sigma_1\sigma_6^2 + S_{266}\sigma_2\sigma_6^2 + \frac{1}{4}S_{1111}\sigma_1^4 + \frac{1}{4}S_{2222}\sigma_2^4 + \frac{1}{4}S_{6666}\sigma_6^4 + \mathrm{S}_{1112}\sigma_1^3\sigma_2 \\ &+ S_{1122}\sigma_1^2\sigma_2^2 + S_{1222}\sigma_1\sigma_2^3 + S_{1166}\sigma_1^2\sigma_6^2 + S_{2266}\sigma_2^2\sigma_6^2 + \mathrm{S}_{1266}\sigma_1\sigma_2\sigma_6^2. \end{aligned} \tag{3}$$

Experiments in [1] show that the coupling between σ_1 or σ_2 and σ_6 is negligible, i.e., coefficients like S_{166} in W^* are negligible. Experiments in [1] also show the absence of nonlinear coupling between σ_1 and σ_2. Therefore, Equation (3) can be reduced to

$$W^* = \frac{1}{2}S_{11}\sigma_1^2 + \frac{1}{2}S_{22}\sigma_2^2 + S_{12}\sigma_1\sigma_2 + \frac{1}{2}S_{66}\sigma_6^2 + \frac{1}{4}S_{6666}\sigma_6^4. \tag{4}$$

Thus the stress-strain relation deduced from Equations (2) and (4) is given by

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix} + S_{6666}\sigma_6^2 \begin{Bmatrix} 0 \\ 0 \\ \sigma_6 \end{Bmatrix}, \tag{5}$$

or

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} + S_{6666}\tau_{12}^2 \begin{Bmatrix} 0 \\ 0 \\ \tau_{12} \end{Bmatrix}, \tag{6}$$

where γ_{12} and τ_{12} are used instead of ε_6 and σ_6. This equation explicitly shows that according to the fourth-order theory only one fourth-order constant S_{6666} is needed to account for the nonlinear shear behavior of the composite lamina.

3. Analysis

Basing on the Equation (6), we analyze the nonlinear behavior of filament wound tubes. We use a small increment of tensile stress for our analysis, in which a piecewise linear approximation is used. This kind of incremental analysis is applicable only to quasi-linear materials, which have no coupling among various stress components in the nonlinear range so that superposition of stresses is permissible.

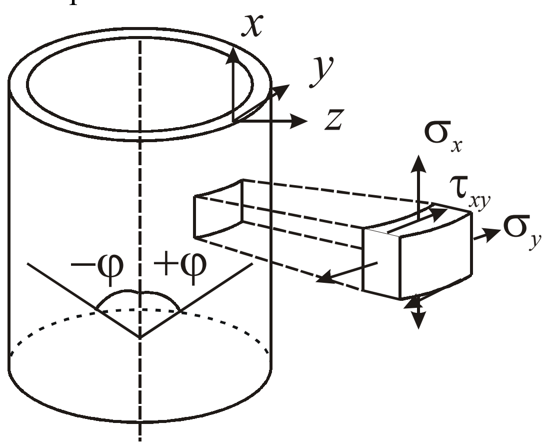

Figure 1. An element from a tube

A thin element laminate consisting of N laminae of unidirectional fiber composites (Figure 1) is considered. The coordinates (x, y, z) are fixed in the laminate with x direction concordant with the tube axial direction, while the coordinates (1, 2, 3) are taken parallel to the material principal axes of each individual lamina with the 1 axis parallel to the fiber direction in each lamina.

The following assumptions, similar to the classic laminate theory, have been made for the present analysis:

(1) The interlamina bonding is perfect.

(2) A plane stress state exists. Thus, the stress components vanish in the thickness (z) direction, i.e. $\sigma_z=\tau_{zx}=\tau_{yz}=0$.

(3) All the laminae and, hence, the laminate experience the same strain in the x-y plane.

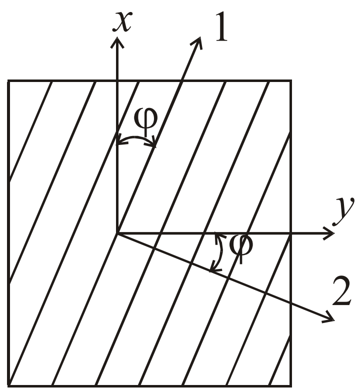

Figure 2. A lamina

The linear stress-strain relations of the unidirectional lamina are valid for infinitesimal deformations. The initial elastic modules of unidirectional fiber composite can be calculated using the elastic modules of the constituents (matrix and fibers) and fiber volume fraction. In the following, the stress-strain relations for the k-th lamina are denoted by $[S_{ij}]_k$ in the (1, 2, 3) coordinate system.

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & 0 \\ S_{12} & S_{22} & 0 \\ 0 & 0 & S_{66} \end{bmatrix}_k \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} \tag{7}$$

where

$$S_{11} = \frac{1}{E_1}, \quad S_{12} = -\frac{\nu_{21}}{E_2} = -\frac{\nu_{12}}{E_1}$$
$$S_{22} = \frac{1}{E_2}, \quad S_{66} = \frac{1}{G_{12}}. \tag{8}$$

Note: at the first step loading, S_{66} will have the form in Equation (8), after that when shearing stress is obtained, S_{66} will be nonlinear function of shearing stress.

Inverting Equation (7), we obtain

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}, \tag{9}$$

where the stiffness is the inverse of the compliance, i.e. $Q_{ij}=S_{ij}^{-1}$.

In order to analyze the finite deformation of the lamina, the linear elastic relation is assumed between the small increments of stress and strain. So, Equation (9) is rewritten as:

$$\begin{Bmatrix} \Delta\sigma_1 \\ \Delta\sigma_2 \\ \Delta\tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}_k \begin{Bmatrix} \Delta\varepsilon_1 \\ \Delta\varepsilon_2 \\ \Delta\gamma_{12} \end{Bmatrix}. \tag{10}$$

The linear elastic relation between the small stress and strain increments of the k-th lamina can be transformed to the x, y, z coordinates (Figure 2).

$$\begin{Bmatrix} \Delta\sigma_x \\ \Delta\sigma_y \\ \Delta\tau_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \Delta\varepsilon_x \\ \Delta\varepsilon_y \\ \Delta\gamma_{xy} \end{Bmatrix}, \tag{11}$$

where $\overline{Q}_{ij}$ is off-axis stiffness. By integrating the stress through the whole thickness of the laminate, we obtain

$$\begin{Bmatrix} \Delta N_x \\ \Delta N_y \\ \Delta N_{xy} \\ \Delta M_x \\ \Delta M_y \\ \Delta M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon_x \\ \Delta\varepsilon_y \\ \Delta\varepsilon_{xy} \\ \Delta k_x \\ \Delta k_y \\ \Delta k_{xy} \end{Bmatrix}, \tag{12}$$

or in the condensed form

$$\begin{Bmatrix} \Delta N \\ \Delta M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \Delta\varepsilon \\ \Delta k \end{Bmatrix} \tag{13}$$

where the A matrix represents the in-plane stiffness, the B matrix defines the bending-stretching coupling and the D matrix represents the bending stiffness in the classical laminate theory.

By inverting Equation (13), we obtain

$$\begin{Bmatrix} \Delta\varepsilon \\ \Delta k \end{Bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{Bmatrix} \Delta N \\ \Delta M \end{Bmatrix}, \tag{14}$$

where A', B', C' and D' are from the classical laminate theory. If the tube is bending free, which means Δk=0, it holds

$$\Delta k = 0 = C'\Delta N + D'\Delta M, \tag{15}$$

and then

$$\Delta M = -(D')^{-1} C'\Delta N. \tag{16}$$

Therefore, we have

$$\Delta\varepsilon = [K]\Delta N, \tag{17}$$

where

$$[K] = A' - B'(D')^{-1} C' \tag{18}$$

is the compliance matrix of the laminate.

Based on the Equations (6)--(18), the incremental analysis can proceed. Only S_{66} deserves special attention, which will be changed at each step of loading to account for the effect of shearing stress.

Considering the laminate under uniaxial tension in the x direction (see Figure 1) by an increment $\Delta N_x \neq 0$ ($\Delta N_y = \Delta N_{xy} = 0$) at l-th step loading, the corresponding increments of strain of the laminate are

$$\begin{Bmatrix} \Delta\varepsilon_x \\ \Delta\varepsilon_y \\ \Delta\gamma_{xy} \end{Bmatrix}^{(l)} = [K]^{(l)} \begin{Bmatrix} \Delta N_x \\ 0 \\ 0 \end{Bmatrix}. \tag{19}$$

Using Equation (11), $(\Delta\sigma_x)_k^{(l)}$, $(\Delta\sigma_y)_k^{(l)}$, and $(\Delta\tau_{xy})_k^{(l)}$ of k-th lamina can be obtained. Transforming them to the material principal coordinates (1, 2, 3), we obtain

$$\begin{Bmatrix} \Delta\sigma_1 \\ \Delta\sigma_2 \\ \Delta\tau_{12} \end{Bmatrix}_k^{(l)} = [T]_k^{\{l\}} \begin{Bmatrix} \Delta\sigma_x \\ \Delta\sigma_x \\ \Delta\tau_{xy} \end{Bmatrix}_k^{(l)}, \tag{20}$$

where $[T]_k^{(l)}$ is the transform matrix of stress. Then the stresses of k-th lamina at l-th step are

$$\begin{aligned} (\sigma_1)_k^{(l)} &= (\sigma_1)_k^{(l-1)} + (\Delta\sigma_1)_k^{(l)} \\ (\sigma_2)_k^{(l)} &= (\sigma_2)_k^{(l-1)} + (\Delta\sigma_2)_k^{(l)} \\ (\tau_{12})_k^{(l)} &= (\tau_{12})_k^{(l-1)} + (\Delta\tau_{12})_k^{(l)} \end{aligned}. \tag{21}$$

The average axial stress and strain of tube are

$$\begin{aligned} (\sigma_x)^{(l)} &= (\sigma_x)^{(l-1)} + \Delta N_x^{(l)}/t \\ (\varepsilon_x)^{(l)} &= (\varepsilon_x)^{(l-1)} + (\Delta\varepsilon_x)^{(l)} \end{aligned}. \tag{22}$$

And

$$(S_{66})_k^l = \frac{1}{G_{12}} + S_{6666}\left(\tau_{12}^2\right)_k^{(l)}. \tag{23}$$

This S_{66} is then taken back to the Equation (7), it will make some modification to the $[S_{ij}]_k$, therefore, to $[Q_{ij}]_k$ and $[\overline{Q}_{ij}]_k$. All these modified compliance and stiffness are then prepared for next step increment loading.

4. Comparison of Numerical Results and Experimental Results

Using method mentioned in previous section, we calculated several filament wound tubes. Their winding angles are 26°, 31°, 36°and 46°. The details of these tubes are listed in Tables 1 and 2.

Table 1. Detail of tubes

Mark	Winding Angle [deg]	Mandrel Diameter [mm]	Wall Thickness [mm]	Fiber	Matrix	Fiber volume Fraction[%]
X26	25.8	26	1.11	T700	P418	55
X31	31.2	26	1.17	T700	P418	55
X36	36.0	26	1.23	T700	P418	55
X46	45.8	26	1.42	T700	P418	55

Table 2. Properties of fiber and matrix

	El [Gpa]	Et [Gpa]	Glt [Gpa]	vlt
T700	235	15	5	0.35
P418	5	5	1.6	0.4

Before doing numerical calculation, we need S_{6666}. Considering the main stress in 46° winding angle is shearing stress, we use data from that tube to obtain S_{6666}. S_{6666} we obtained is 40Gpa^{-3}, then this S_{6666} is also used for the calculation of the other tubes' calculation. Numerical results and experimental results of tube stress and strain are given in Figures 3, 4,5 and 6. In each figure, curve 1 represents the experimental results of unidirectional filament wound tubes, curve 2 represents the numerical results and curve 3 represents the experimental results of cross filament wound tubes, respectively.

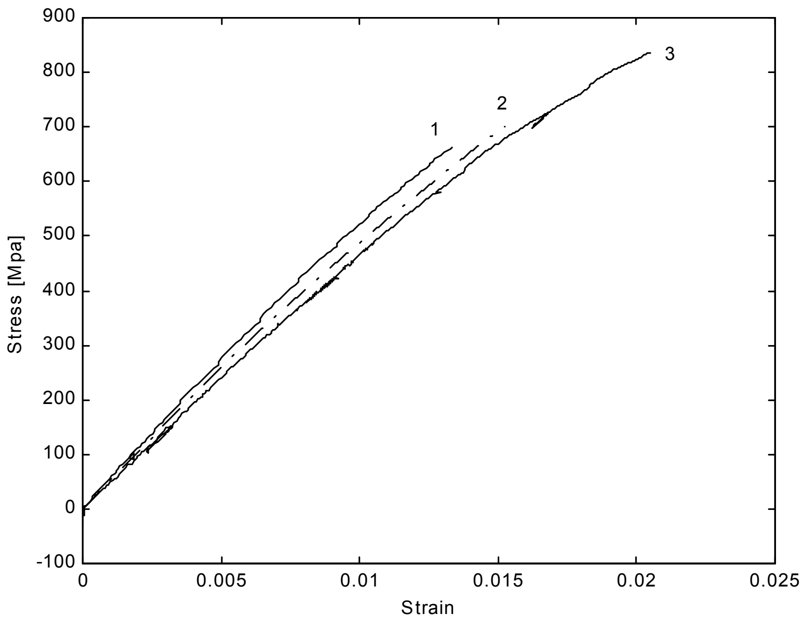

Figure 3. X26 results

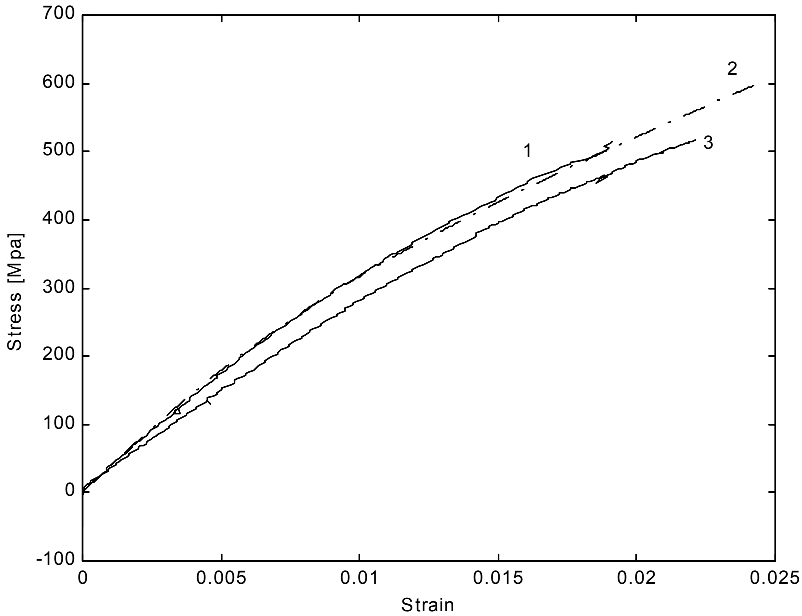

Figure 4. X31 results

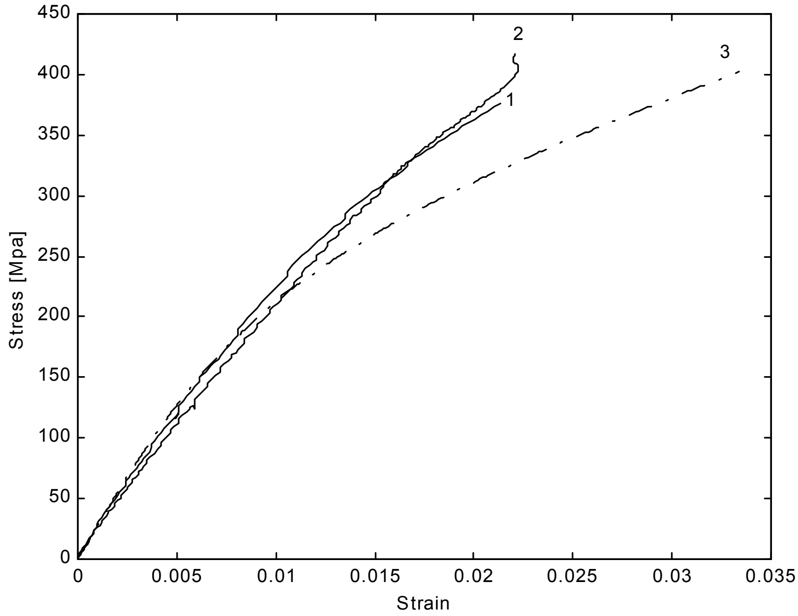

Figure 5. X36 results

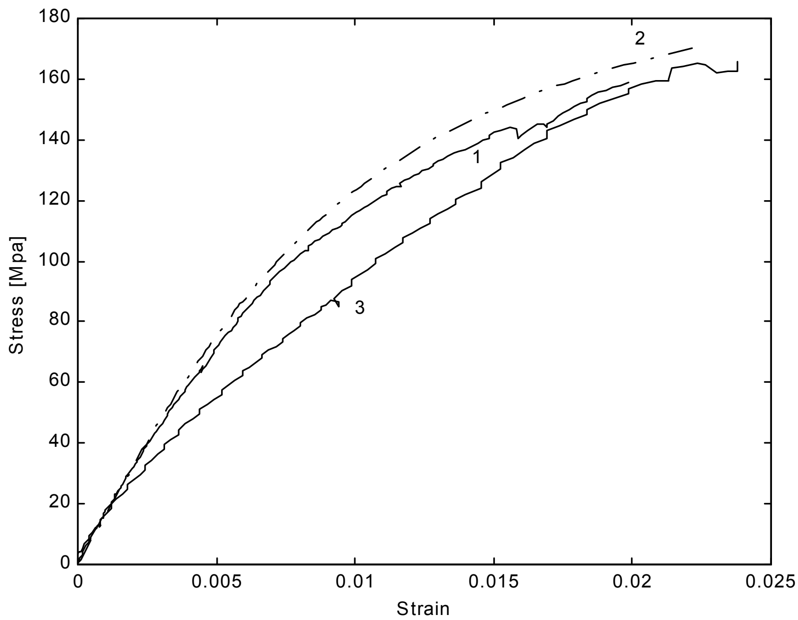

Figure 6. X46 results

5.Conclusions

From Figure 3 to 6, we can see

(1) All numerical results except 36° have a good correlation with experimental results of unidirectional filament wound tubes.

(2) The stiffness of unidirectional filament wound tubes is little greater than that of cross filament wound tubes in elastic range (the linear part). The reason for that is the in-plane stiffness reduction due to the fiber interlace which causes fiber direction change in lamina thickness direction. Also, this fiber interlace causes nonlinear shear behavior in lamina thickness direction, which makes nonlinear behavior of cross filament wound tubes more complicated, or in other words, the nonlinear behavior of cross filament wound tubes is more serious than that of unidirectional filament wound tubes.

(3) Nonlinear behavior of filament wound tubes is mainly caused by nonlinear shear behavior of composite.

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Reference

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