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BIOMECHANICAL MODEL FOR OPTIMIZATION OF FOOD GRINDER DESIGN

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Summary: *A new proposition of a simulations methodology applicable to grain grinding is discussed in this article. This methodology represents a unified approach in which the essential features of previously developed analytic and numerical models are integrated. A systematic approach underlying the methodology facilitates separation of various features and phenomena characteristic of multiple disc grinder in the process of biomechanical model building. As an example, grain particles fields in milling are derived. These fields are used to estimate the actual tool-disc geometry and texture of the generated surface on the basis of measured cutting forces.*

1. Introduction

The grinding process was carried into effect using a cutting mill in the shape of a working assembly as in Fig. 1, Fig. 2 and Fig. 3. A particle in bulk was fed through the hole in the top fixed plate, and after grinding the product was released through the slot between the discs, and/or after vertical passing through the pack of discs with holes.

2. Energetic model of grinding

On introducing necessary reductions, there appears a new energetic model of grinding process, which is expressed by the following formula (Fig. 2 and Fig. 3):

$$E_c = \Delta E_{ST} + \Delta E_N + \Delta E_R + \Delta E_O + \Delta E_X$$

where:

- E_C – total energy supplied to system,
- ΔE_{ST} – energy assigned for process control,
- ΔE_N – energy lost in driving system,
- ΔE_R – energy lost in cutting mill,

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ΔE_O – energy assigned for process servicing,

ΔE_X – energy dissipated in system (vibrations, heat, sound, waves, etc).

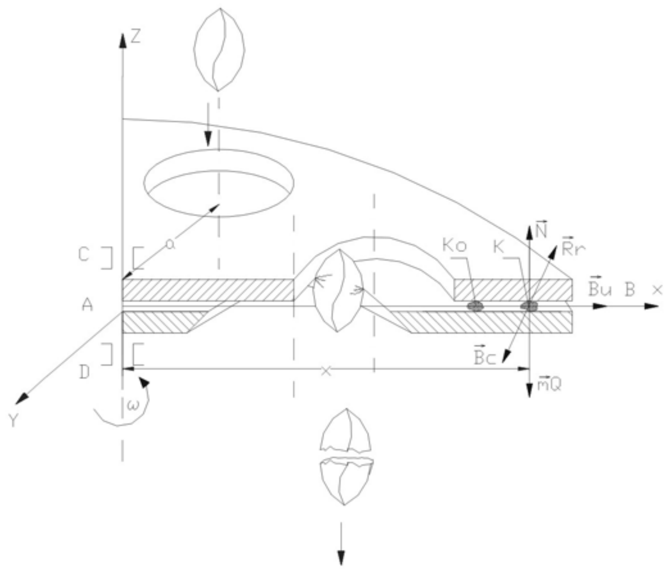

Fig. 1. A piece of grain in the inter-disc space of the grinder

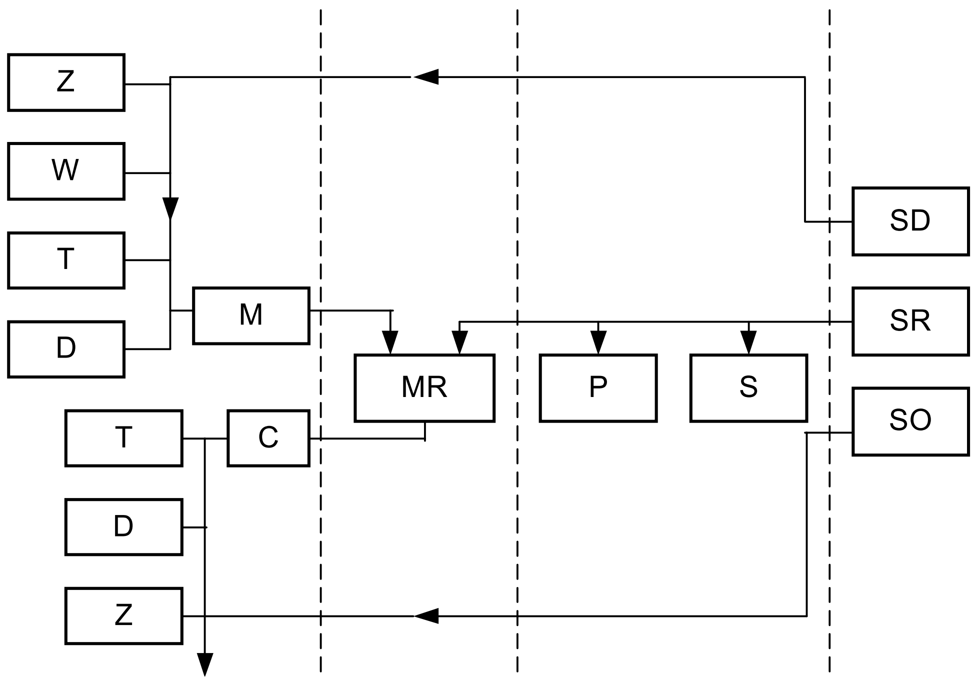

Fig. 2. A new unit, elements and relation in energetic model of grinding process

For design elements of multiple discs mill construction, the most important are phenomena occurring in the grinding and driving assembly. The design formulation of energetic problems simplifies the transmission and grinding relationship to the form:

$$E_c = \Delta E_N + \Delta E_R$$

what according to the relationship, with the assumption that the $E_C = E_E$ – electrical energy, allows one to determine an energy necessary for grinder of biomaterials as:

$$E_{R\lambda} = \int_0^T P_R(t) v(t) dt \qquad for \quad P_R \rangle 0, \quad v \rangle 0$$

where:

$P_R(t)$ – grinding force, N, $V(t)$ – grinding velocity, $m \cdot s^{-1}$, $E_{R\lambda}$ - energy necessary for mechanical deformation of grain sample during cycle to predetermined grinding degree, J, T - cycle duration, s.

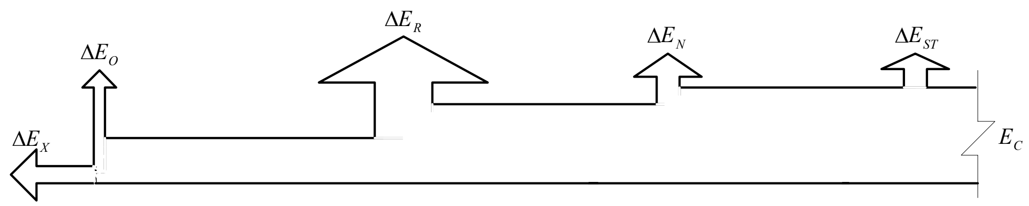

Fig. 3. A total and lost energy in grinding system

3. Mechanical model of system

At the same time there occurs the following relationship (Fig. 3 and Fig. 4):

$$E_{R\lambda} = E_E \cdot \eta_s \cdot \eta_p$$

where:

η_s – motor mechanical efficiency, – ,η_p - transmission mechanical efficiency, – , E_E – energy supplied at system input, J.

With stabilized motion, for full motion cycle or its multiple it may be assumed that:

$$E_{R\lambda} = E_{T\lambda} + \Delta E_M$$

where:

$E_{T\lambda}$ – energy needed for mechanical deformation of biomaterials - grain sample, in model conditions, to predetermined degree of grinding, J, ΔE_M – increase in energy losses for accomplishment of grinding in machine conditions, J.

Increase in energy for accomplishment of the grinding in machine conditions may be expressed in a coefficient form:

$$E_{T\lambda} = \alpha_R \cdot E_{R\lambda} \qquad \alpha_R = \frac{E_{T\lambda}}{E_{R\lambda}}$$

where:

α_R – factor of energetic relations as measure of model accomplishment in machine conditions, –.

Direct connection between transmission and functional (milling) system afford possibilities which define universal (general) efficiency (Fig.4):

$$\eta_o = \eta_s \cdot \eta_p \cdot \eta_r$$

where:

η_o – universal (global) efficiency of technical system to milling,

η_r – milling relative efficiency as a measure of model achievement in real conditions (α_R).

Comparing the previous relationships, it may be possible to determine an increase in energy for the grinding accomplishment in machine conditions:

$$\Delta E_M = E_E \cdot \eta_s \cdot \eta_p - E_{T\lambda}$$

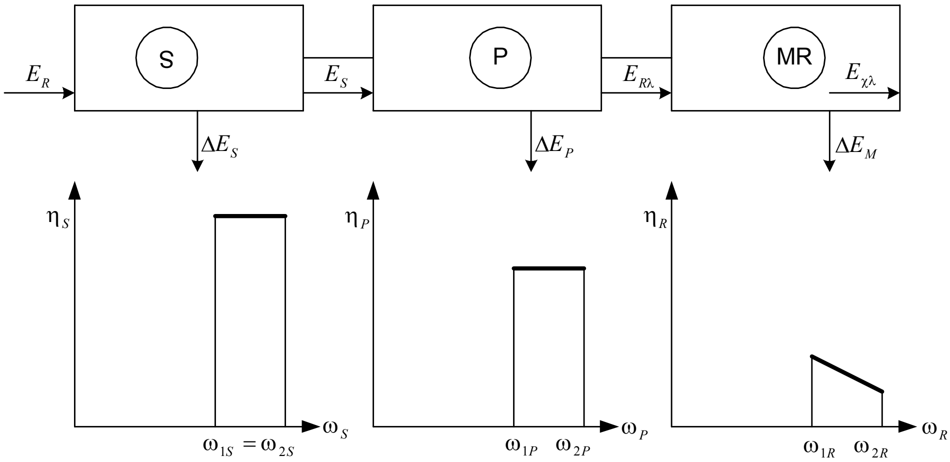

Fig. 4. Energy supplied at system input, motor mechanical efficiency, transmission mechanical efficiency, and grinder mechanical efficiency

The model described by the last relationship refers to the accomplishment of the process by the grinding assembly. It includes the energy dissipations, which do not relate directly to the grinded material deformation. It results from the energy conservation law that the kinetic energy of the grinding element before grinding is transformed into:

$$E_{T\lambda} = E_T + E_m + E_p$$

where: E_T – kinetic energy of working element after deformation of grain, J, E_m – kinetic energy of material particles after grinding, J, E_p – energy used for performing deformation work, J.

On the ground of grinding force equation:

$$P_R = k_j \cdot v_r + \sigma_{\max} \cdot F_r + \varepsilon \cdot F_r' \cdot v_r^2$$

where: k_j – coefficient resistances of running idle, $\mathrm{kg \cdot s^{-1}}$, v_r – grinding velocity, $\mathrm{m \cdot s^{-1}}$, $\sigma_{\max}$ – maximal stresses in grinding area between edges and grain particles, $\mathrm{N \cdot m^{-2}}$, F_r, F_r' – area of milling field section, $\mathrm{m^2}$, ε – coefficient of proportion, $N \cdot s^2 \cdot m^4$,

it may be possible to obtain simplified equation determining energy consumption during milling:

$$E_E = \frac{\left(k_j \cdot v_r \cdot \sigma_{\max} \cdot F_r + \varepsilon \cdot F_r' \cdot v_r^2\right) \cdot v_r \cdot t}{\eta_S \cdot \eta_P}$$

where:

t – milling time of relative field section, s.

This equation contains the main elements (parts) of efficient-energetic model of multiple disc shredder-grinder.

4. Biomechanical model of grinding

In consideration of milling specificity for fodder aims related to increase feed-efficiency, general optimization energetic model may be determined. There is a possibility to calculate a factor of biological value η_{bio} for definite section of each size groups of grinded material [1]:

$$\eta_{bio} = f_{<0,5}\eta_{bio<0,5} + f_{0,5-1,5}\eta_{bio0,5-1,5} + f_{>1,5}\eta_{bio>1,5}$$

where:

$\eta_{bio<0,5}$ – factor of biological value grinding products described by its dimension: $f_{<0,5}$,

$\eta_{bio0,5-1,5}$ – factor of biological value grinding products described by its dimension: $f_{0,5-1,5}$,

$\eta_{bio>0,5}$ – factor of biological value grinding products described by its dimension: $f_{>1,5}$ – fractions share of described dimension.

The factor calculated as above is a numerical interval determined with a suitable probability or average number for investigational criteria of purpose. In consideration of mechanical grinding specificity for bio- fodder aims, related to increase feed-efficiency, general construction of grinder optimisation, efficiency energetic model may be determined:

$$e_R = \frac{\Delta E_{bio}}{E_E \cdot M_k} = \frac{\left(\eta_{bio} - \eta_z\right) \cdot E_{brutto}}{E_E \cdot M_k} = \frac{\left(\eta_{bio} - \eta_z\right) \cdot E_{brutto} \cdot \eta_S \cdot \eta_P}{\left(k_j \cdot v_r + \sigma_{\max} \cdot F_r + \varepsilon \cdot F_r' \cdot v_r^2\right) \cdot v_r \cdot t \cdot M_k}$$

where:

- η_{bio} – factor of biological value, described on the ground of riddled analysis and *in vitro* digestibility for grinding material, –,
- η_z – factor of bio-grain digestibility before grinding, –,
- E_{brutto} – average content of gross energy in 1 kg dry substance of grain, e.g.: rye – $15{,}7\,\mathrm{MJ \cdot kg^{-1}}$, wheat – $16{,}2\,\mathrm{MJ \cdot kg^{-1}}$, barley – $15{,}9\,\mathrm{MJ \cdot kg^{-1}}$, oat – $16{,}5\,\mathrm{MJ \cdot kg^{-1}}$,
- M_k – multiplier of mass attempt – if denominator components concern data different from 1 kg.

5. Estimation of grinder construction

The constructive features of the working set of the multiple discs grinder should be selected in such a way that the function achieves the maximal value (because of the e_R, indicator value) or minimal (because of the value of the unit energy consumption indicator E_R).

The point where the function value fulfils the required criterion is called problem solution: $x^* = \left(x_1^*, \ldots x_n^*\right)$. The solution is, of course, from the permissible area:

$$x^* \in \Phi$$

The principle of the optimization support in the direction of getting the extreme solution can be defined:

$$\left\{X^* \in \phi\right\} : \left\{ \bigwedge_{x \in \phi} Z(x) \geq Z\left(X^*\right) \right\},$$

in the case of minimization of energy consumption (Z=E$_R$)

$$\left\{X^* \in \phi\right\} : \left\{ \bigwedge_{x \in \phi} Z(x) \leq Z\left(X^*\right) \right\}$$

in the case of maximization of energetic milling indicator ($z = e_R$).

If the target point is known in the target space (e.g. $E_R < 10\,\mathrm{kJ/kg}$), it is possible to conduct the procedure aiming at approaching the given solution. The procedure means searching for such δ_F, δ_{ER} which are expressed by the following formula (Fig. 6 and Fig. 7):

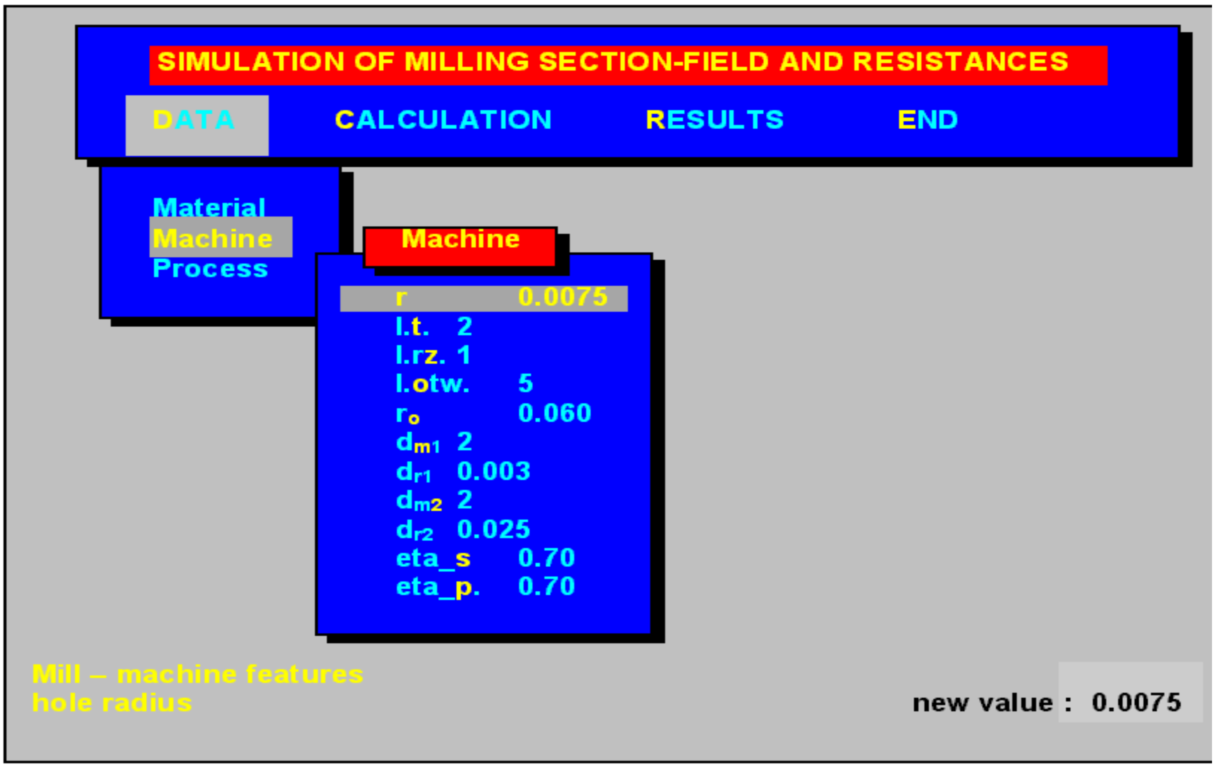

Fig. 6. The simulation data of milling field and forces

$$\delta F = \frac{F_{max} - F_{min}}{F_{sr}} \Rightarrow 0,$$

$$\delta E_R = \frac{E_{R\,max} - E_{R\,min}}{E_{sr}} \Rightarrow 0$$

This way a new purpose function is obtained. It is in the form of the distance between the target condition and the countess condition in the target space:

$$Z_d(x) = \left\| Z_{min} - Z(x) \right\|$$

where:

Z_d – the distance between the solution quality vector Z(x) and the target solution Z_{min}.

In the case of Euclidean norm, the distance is expressed by the following formula:

$$Z_{dl}(x) = \left\{ \sum_{i=1} \left[Z_{i\,min} - Z_i(x) \right]^2 \right\}^{\frac{1}{2}}$$

where:

Z_{min} – the value of unitary energy consumption for the target solution,

$Z_i(x)$ – the value of the unitary energy consumption for the designed solution

Conception	I	II	III
Number of disks	3	5	7
Number of rows in discs	2	2	2
Number of holes in the first disc	5	9	9

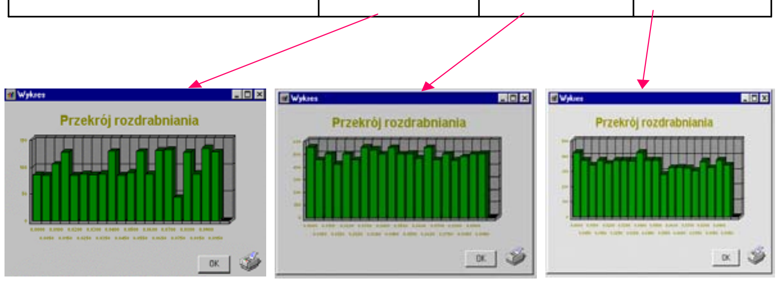

Fig. 7. Results of the section area simulation

6. Summary

Based on this study, the following specific conclusions can be drawn.

There exists discrepancy between the calculated construction indicators and energetic efficiency indicators – determined for the machine built on the basis of the carried out support procedures. The discrepancy achieves the value of even several percent (the obtained result is the most advantageous when the discrepancy between the calculated construction and the constructed mill with the energy – consumption $E_R = 17{,}1 kJ \cdot kg^{-1}$ – with the criterion $E_R < 15 kJ \cdot kg^{-1}$ is 12%).

In the process of searching for processing machines properties, it is necessary to include the following procedures:

- to use the scientific basis of machine construction and exploitation,
- to create new solutions on the basis of individual ideas taking into consideration the nature of needs and the up-to date condition of possibilities, motivations, know-how, capital, outlet,
- to take into consideration the complexity of technical systems to implement the stated processing function – steering, drive, service, repairing, power supply, damages, scrapping and others.

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