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ROTOR SUPPORTED ON MAGNETIC BEARINGS — NUMERICAL AND EXPERIMENTAL ANALYSIS

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Summary: *The dynamic properties of rotor supported on passive magnetic bearings is investigated by means of numerical solution and by measurement on the prototype of experimental set developed in Institute of Thermomechanics AS CR. Mathematical model was constructed both for linearized part of rotor with bearing and for strongly nonlinear system including impacts in retainer bearings. Comparison with results of measurement on prototype enables to ascertain real parameters of linearized part of magnetic bearings and the whole system.*

1. Introduction

New progressive elements in design of high speed rotor systems are magnetic bearings. Magnetic suspension has a lot of advantages over the conventional oil or ball bearings, particularly: reduction of bearing drag to zero, avoiding the presence of lubricant and wear, as well as applicability for high revolutions, low temperature, vacuum etc.

There are two main types of magnetic bearings: active and passive. In this contribution, we shall be engaged by an analysis of dynamic properties of stiff rotor supported on two passive magnetic bearings.

Magnetic bearings always have to contain so called retainer bearing in order to prevent from dangerous increase of oscillation and from destruction of bearings and rotor. The rotor/stator rub was investigated by many authors, but predominantly for rotor contact with rigid, not rotating inner surface of stator (e.g. Goldman, Muszynska, 1994). But retainer bearings, which are used together with magnetic suspension, are always rolling bearings. The inner ring of the retainer bearing can rotate after oblique impact with rotor pivot and adds one degree of freedom to the mathematical model of rotor bearing. Simple model introducing only radial Hertz stiffness, material contact damping, tangential dry friction and viscous damping (Půst, 2003, Půst & Kozánek, 2002), shows to be insufficient in some cases. This paper presents more exact dynamic model of rotor motion in passive magnetic bearings where the oscillations are restricted by a retainer bearing.

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The first step in dynamic analysis is the identification of parameters of linear model of rotor and bearings. Comparison of results gained by numerical solution with result of the measurement of experimental prototype developed in Institute of Thermomechanics AS CR enables to ascertain parameters of real machine. The main aim of this study is to gain the knowledge about the properties of such a system, and to constitute the base for more exact identification and diagnostic procedures of rotors supported on magnetic bearings.

2. Motion of rotor

Stiff rotor has 6 degrees of freedom. Displacement in z axis is neglected and velocity of rotation around this axis let be constant:

$$\omega_z = \omega\ .$$

Remaining 4 DOF are described by displacements x_1, x_2, y_1, y_2, in point 1,2 with distance l. Both bearings are identical and act on rotor in points 1,2 by forces F_{x1}, F_{x2}, F_{y1}, F_{y2} (Fig. 1).

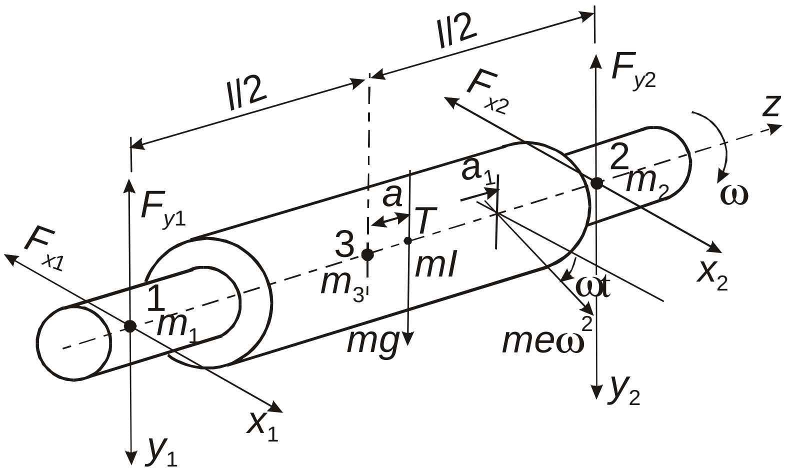


Fig. 1

Inertia properties of rotor are given by mass m [kg] and moment of inertia I [kg m^2] in point T which can be substituted by three masses m_1, m_2, m_3 in points 1,2,3. (Brepta et al., 1994).
Force of gravity mg acts in point T in distance a from geometric centre 3. Exciting force (unbalance) can act in arbitrary position on axis z. Let us suppose that it acts in the distance a_1 from geometric centre 3.

Equations of rotor motions are

$$\begin{aligned}
&m_1\ddot{x}_1 + m_3(\ddot{x}_1+\ddot{x}_2)/4 = -F_{x1} + (1/2 - a_1/l)me\omega^2\cos\omega t\\
&m_2\ddot{x}_2 + m_3(\ddot{x}_1+\ddot{x}_2)/4 = -F_{x2} + (1/2 + a_1/l)me\omega^2\cos\omega t\\
&m_1\ddot{y}_1 + m_3(\ddot{y}_1+\ddot{y}_2)/4 = -F_{y1} + (1/2 - a_1/l)me\omega^2\sin\omega t + (1/2 - a/l)mg\\
&m_2\ddot{y}_2 + m_3(\ddot{y}_1+\ddot{y}_2)/4 = -F_{y2} + (1/2 + a_1/l)me\omega^2\sin\omega t + (1/2 + a/l)mg\,,
\end{aligned} \tag{1}$$

where

$$m_1 = m\left(-\frac{a}{l}+\frac{2a^2}{l^2}\right)+\frac{2I}{l^2},\ m_2 = m\left(\frac{a}{l}+\frac{2a^2}{l^2}\right)+\frac{2I}{l^2},\ m_3 = m\left(1-\frac{4a^2}{l^2}\right)-\frac{4I}{l^2}\ , \tag{1a}$$

and F_{xi}, F_{yi}, i = 1,2 are reaction forces in bearings.

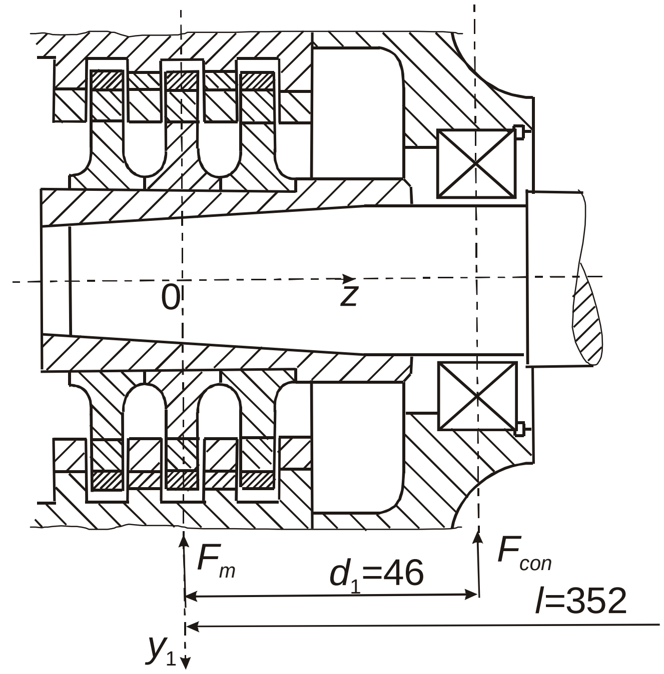


Fig. 2

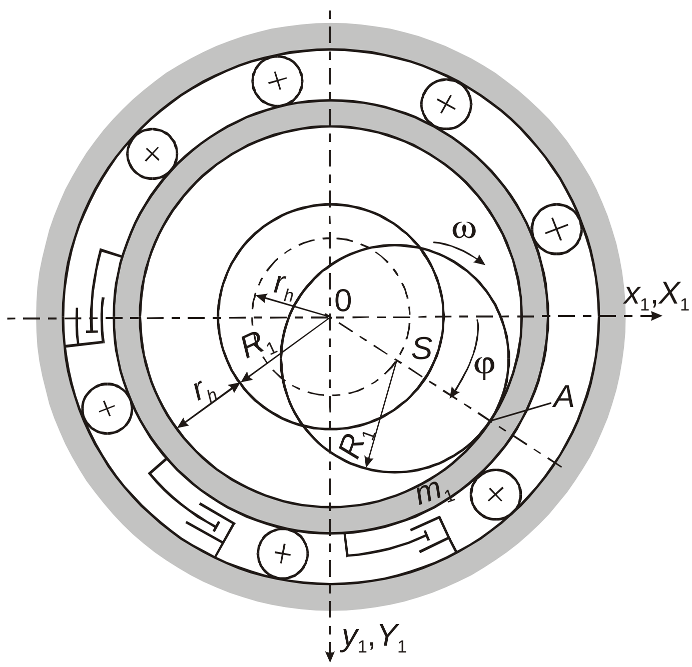


Fig. 3

The structure of passive magnetic bearing including retainer bearing is in Fig. 2. The resultant of magnetic forces F_m from support acts on the rotor in point 0.

The subsystem of magnetic and retainer bearings in one bearing unit is presented in the plane x_1, y_1 in Fig. 3.

3. Motion without impacts

If the displacements of rotor pivots are smaller than clearance r_h

$$r_i = \sqrt{x_i^2 + y_i^2} \le r_h\,, \qquad i=1,2 \tag{2}$$

the only shearing magnetic force acts on rotor in support. This force is centrally symmetrical and shows the softening characteristic with weak cubic nonlinearity

$$F = kr - K_3 r^3\,. \tag{3}$$

Components of this force are

$$\begin{aligned} F_{xi} &= \left(k - k_3 r_i^2\right) x_i \\ F_{yi} &= \left(k - k_3 r_i^2\right) y_i\,. \end{aligned} \qquad i = 1,2 \tag{4}$$

Passive magnetic bearings have very low damping, almost zero. But there are external damping forces from air drag, eddy currents, etc. Therefore small linear dampings $b\dot{x}_1$, $b\dot{x}_2$, $b\dot{y}_1$, $b\dot{y}_2$ are added to the bearing forces F_{xi}, F_{yi} in mathematical model of bearings.
Differential equations of rotor motion (4 DOF) are

$$\begin{aligned}
&(m_1 + m_3/4)\ddot{x}_1 + m_3/4\ddot{x}_2 + b\dot{x}_1 + \left(k + k_3\left(x_1^2 + y_1^2\right)\right)x_1 = (1/2 - a_1/l)me\omega^2\cos\omega t \\
&(m_2 + m_3/4)\ddot{x}_2 + m_3/4\ddot{x}_1 + b\dot{x}_2 + \left(k + k_3\left(x_2^2 + y_2^2\right)\right)x_2 = (1/2 + a_1/l)me\omega^2\cos\omega t \\
&(m_2 + m_3/4)\ddot{y}_1 + m_3/4\ddot{y}_2 + b\dot{y}_1 + \left(k + k_3\left(x_1^2 + y_1^2\right)\right)y_1 = (1/2 - a_1/l)me\omega^2\sin\omega t + (1/2 - a/l)mg \\
&(m_2 + m_3/4)\ddot{y}_2 + m_3/4\ddot{y}_1 + b\dot{y}_2 + \left(k + k_3\left(x_2^2 + y_2^2\right)\right)y_2 = (1/2 + a_1/l)me\omega^2\sin\omega t + (1/2 + a/l)mg\,.
\end{aligned} \tag{5}$$

Introducing non-dimensional variables and parameters

$$\alpha = \frac{2a}{l},\quad \alpha_1 = \frac{2a_1}{l},\quad \delta = \frac{mg}{2kr_h},\quad X_i = \frac{x_i}{r_h},\quad Y_i = \frac{y_i}{r_h},\quad E_c = e/(2r_h),\quad \tau = t\sqrt{k/m}\,,$$

$$\eta = \omega\sqrt{m/k}\,,\qquad B = b/\sqrt{km}\,,\qquad \kappa = r_h^2 k_3/k\,,\qquad \mu_1 = m_1/m = \left(-\alpha + \alpha^2 + \rho^2\right)/2,$$

$$\mu_2 = m_2/m = \left(\alpha + \alpha^2 + \rho^2\right)/2,\qquad \mu_{34} = m_3/4m = \left(1 - \alpha^2 - \rho^2\right)/4\,,\qquad \rho^2 = \frac{4I}{ml^2}\,, \tag{6}$$

we get simpler form of differential equation of rotor motion

$$\begin{aligned}
&(\mu_1 + \mu_{34})X_1'' + \mu_{34}X_2'' + BX_1' + X_1\left(1 + \kappa\left(X_1^2 + Y_1^2\right)\right) = (1 - \alpha_1)E_c\eta^2\cos\eta\tau \\
&(\mu_1 + \mu_{34})X_2'' + \mu_{34}X_1'' + BX_2' + X_2\left(1 + \kappa\left(X_2^2 + Y_2^2\right)\right) = (1 + \alpha_1)E_c\eta^2\cos\eta\tau \\
&(\mu_1 + \mu_{34})Y_1'' + \mu_{34}Y_2'' + BY_1' + Y_1\left(1 + \kappa\left(X_1^2 + Y_1^2\right)\right) = (1 - \alpha_1)E_c\eta^2\sin\eta\tau + (1 - \alpha)\delta \\
&(\mu_1 + \mu_{34})Y_2'' + \mu_{34}Y_1'' + BY_2' + Y_2\left(1 + \kappa\left(X_2^2 + Y_2^2\right)\right) = (1 + \alpha_1)E_c\eta^2\sin\eta\tau + (1 + \alpha)\delta\,.
\end{aligned} \tag{7}$$

The inertia properties of rotor (m, I, a) can be ascertained very easily from real structure or from drawing. The prototype realized in Institute of Thermomechanics has rotor mass

m=7.379 kg, moment of inertia to the axis x I=945.7 kg cm^2 and shift of centre of gravity along axis z a=0.5mm. Distance between magnetic bearings is l=352 mm.
For this values we get

$$\rho^2 = 0.4137,\ \mu_1 = 0.2040,\ \mu_2 = 0.2097,\ \mu_3 = 0{,}3863,\ \mu_{34} = 0.1466,\ \alpha = 0.00575. \tag{8}$$

The other values (k, b, κ, …) must be ascertained from measurement.

4. Measurement and identification

The prototype of the approx. 7 kg rotor supported in radial direction on two passive magnetic bearings (shown in Fig. 2 - permanent magnetic rings magnetized in axial direction with shear radial forces) was designed and realized in the Institute of Thermomechanics AS CR in the solution of project "Stability and control of the rotor on magnetic bearings". This rotor was stabilized in the axial direction by the active control system with one electromagnet. Rotor was situated in a very stiff stator enabling the possibility of complicated adjustment of the clearances not only in radial but also in axial directions.

Very important were problems: the design, the mathematical simulation of control subsystem and putting into operation the control back-loop system. In (Pavelka 2001) was proposed the control of system on the PID analogue principle using the special electromagnet. The main difficulty was caused by the necessary great axial force of the electromagnet and by the very small (0.4 mm) axial working clearance. The negative influence of the strong electromagnetic field on the steel balls of the retainer rolling bearings was eliminated by replacing the above mentioned bearings by the special type with ceramic balls.

But at using these ceramic balls, it was unfortunately impossible to verify by means of simple electric conductance whether the rotor with working axial control feed-back loop has no contact with stator. For determination of this very important property, the dynamical measurement and identification of the rotor in several different axial positions was proposed. The reproducibility of dynamical response proves that there are no contacts with stator and the rotor moves freely in the space.

During this measurement, the rotor was excited by the random signal gained from the Modal Analysis program PULSE 6.1 in the Dynamic laboratory IT AS CR which was led into the electromagnetic exciter B&K Mini Shaker 4810. The applied force was recorded by the Force-Transducer 8203. The response was measured by the contactless laser vibrometr system Polytec OFV 3000, with Sensor Head OFV 302 (Software Noise Vibration Analysis 7700). The point and direction (radial or axial) of the force and dynamic response was fixed on the free end of rotor because the second end of the rotor was occupied by the electromagnet of axial control system.

Some experimental results are on Fig. 4-10. Amplitude of the velocity response as function of forced frequency of the radial resp. axial vibration in the case of switched off control circuit is depicted in Fig 4 (resp. Fig. 5). It is shown that the undesirable and undetermined contact between the rotor and left retainer bearing caused a lot of frequencies in response spectrum.

If the axial control circuit is switched-on, the rotor moves to the right and releases the contact with stator. In order to make sure that no contact exists, three consecutive measurements for three different axial positions of rotor were repeated. It is evident, that the radial response functions on the Fig. 6 (middle position 0), Fig. 7 (shift +0,1 mm), Fig. 8 (shift –0.1 mm) are practically the same. It follows from this fact that rotor was free in the space. The same situation was observed at the axial excitation. All three responses for all three positions (-0.1 mm, 0, 0.1 mm) were effectively the same. As an example, the axial response for –0.1 mm position is presented in Fig. 9.

The above radial measurements can be used also for the identification of unknown stiffness and damping parameter of the permanent magnetic radial support after its assembly.

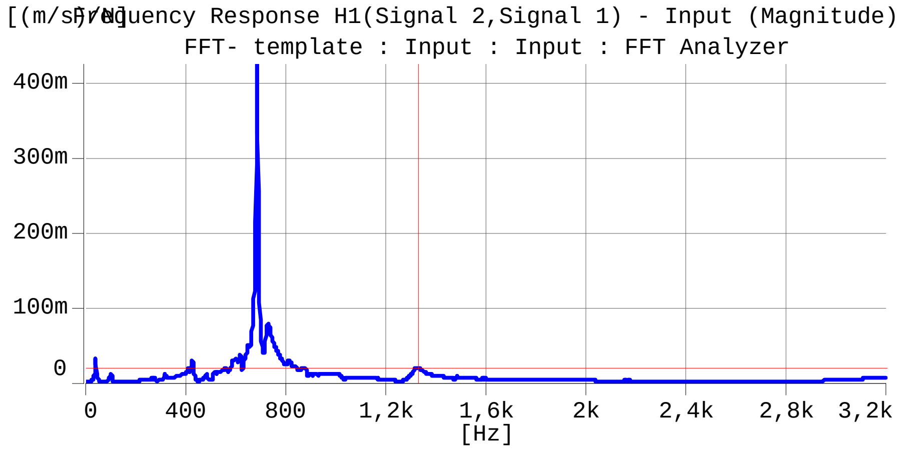


Fig. 4 Amplitude of the response function for radial rotor vibration. Axial control is switched-off

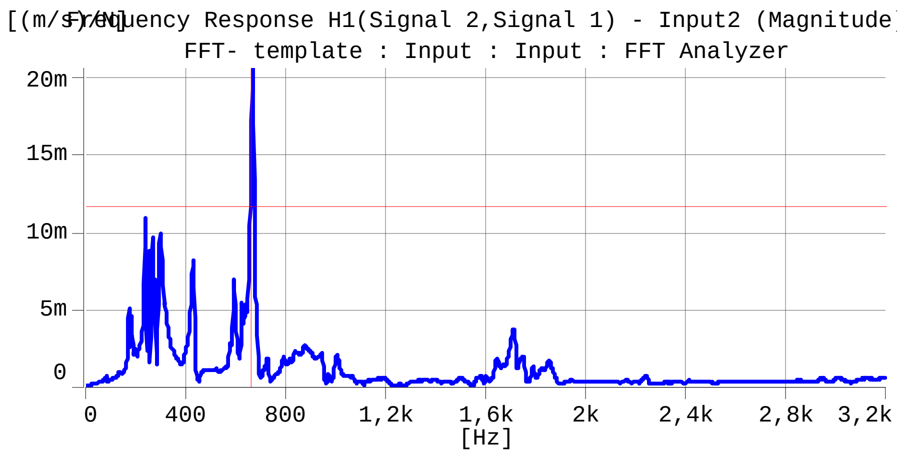


Fig. 5 Amplitude of the response function for axial rotor vibration. Axial control is switched-off

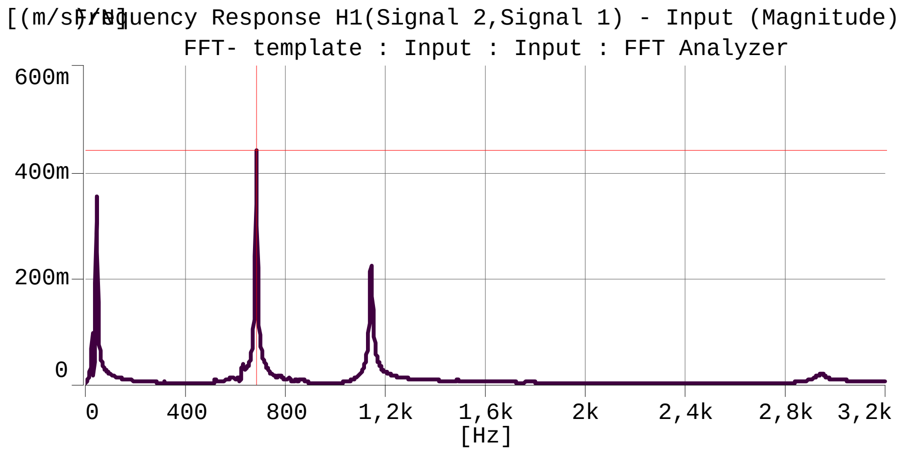


Fig. 6 Amplitude of the response function for radial rotor vibration. Axial control is switched-on in the 0 axial position (identified eigenvalues (Hz): -2.5+46i, -3.1+684i, -4.6+1142i.

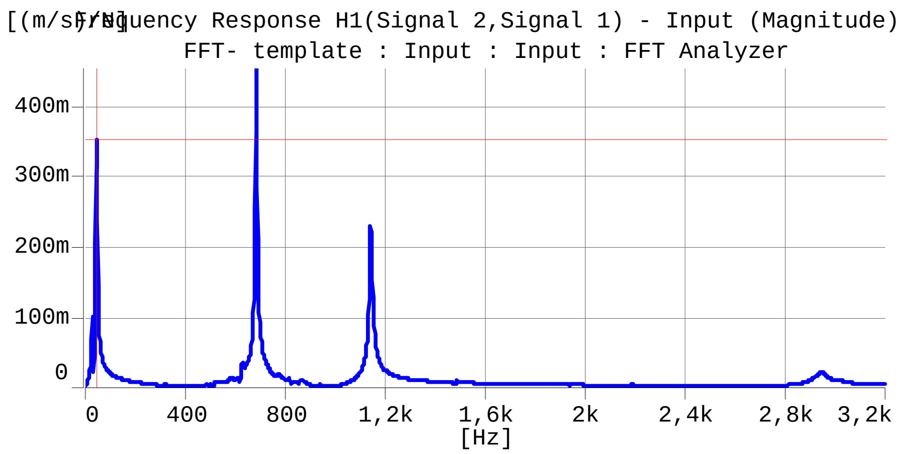


Fig. 7 Amplitude of the response function for radial rotor vibration. Axial control is switched-on in the +0.1 mm axial position

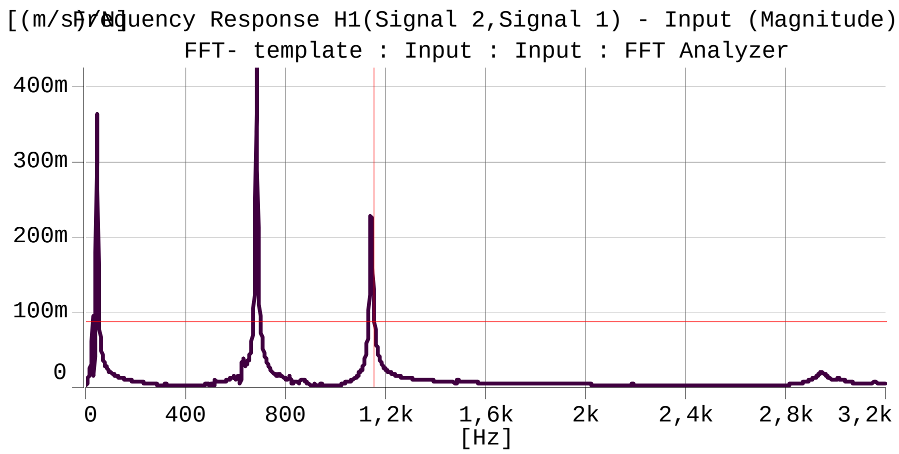


Fig. 8 Amplitude of the response function for radial rotor vibration. Axial control is switched-on in the -0.1 mm axial position

On the Fig. 10a is the detailed response function corresponding to the vibration of the rotor as the very stiff solid state body. Axial control is switched-on in the 0 axial position. Two complex eigenvalues were identified from this response

$$-2.4 + 28.5\ i\ (\text{Hz}) \ \text{ and } \ -2.5 + 46\ i\ (\text{Hz}) \tag{8}$$

By means of mathematical model (7) together with values (7a) and by comparison of its response on excitation force acting on the left end (a_1=-235,5 mm, α_1=-1.338) we get the simulated response curve shown in Fig. 10b.

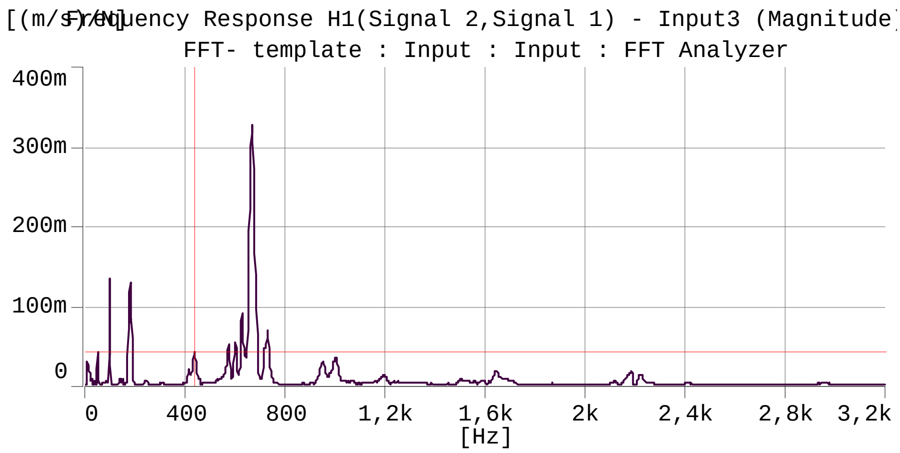
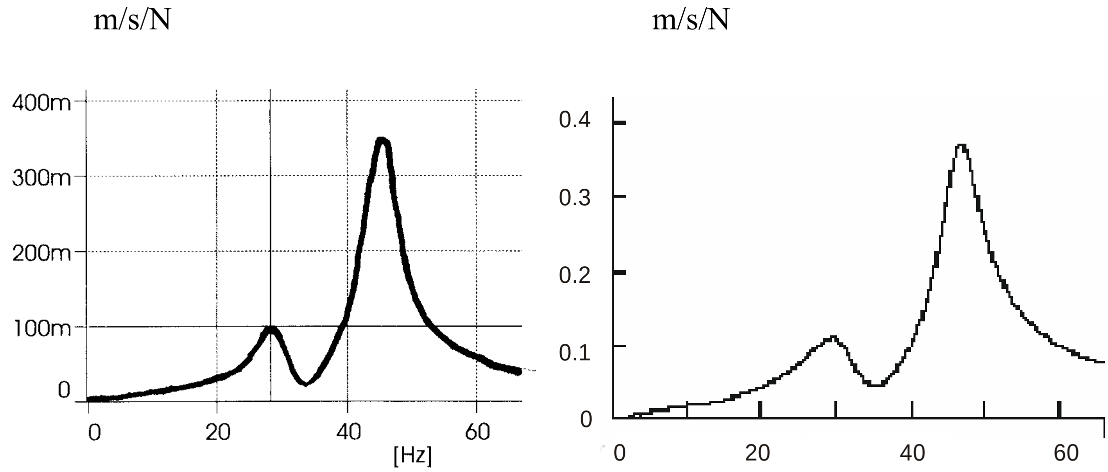


Fig. 9 Amplitude of the response function for radial rotor vibration.
Axial control is switched-on in the -0.1 mm axial position



a) b)

Fig. 10 Velocity amplitudes of response function for radial rotor vibration – axial control in 0 position. a) measurement, b) simulation

The coincidence of response curves in both Figures 10a and 10b proves that mathematical model with parameters (8) and with identified values $k = 1.25.10^5$ kgs^{-2}, $\kappa = 0$, describes very well the dynamic properties of linear part of prototype of PMB rotor realized in IT AV ČR. Damping was found to be strongly dependent on frequency. The law $b/\sqrt{km} = 0.38/\left(1-2.6\eta+2.4\eta^2\right)$ is convenient for the frequency interval 0-60 Hz.

5. Impacts in retainer bearings

Linear model of rotor expresses the dynamic properties in the case of small displacements. During the operation eg. in resonance, transient motion etc., impacts in retainer bearing can occur and therefore the rotor system must be described by strongly nonlinear mathematical model. Situation at the beginning of contact is shown in Fig. 3, where the rotor (radius R_1 touches in point A the inner ring of retainer bearing.

In contradiction to the case of rotor impact on not rotating inner surface of stiff stator, the inner ring of retainer bearing rotates after oblique impact and the mathematical model must respect both the radial deformation of bearing and tangential forces arising at the contact between rotor rotating by angular velocity ω and inner ring having another velocity. The mathematical models of supports must therefore contain differential equations of inner rings.

Dynamic Hertz's contact force $F_r = K_p (r - r_h)^{3/2} (1 + b_h (\dot{x}x + \dot{y}y)/r)$ is supposed to express the radial component of force. Dry friction with coefficient f in contact describes the tangential force component force

$$F_\tau = F_r . f \,\mathrm{sgn}((\dot{y}x - \dot{x}y)/r + R_1\omega - v), \qquad r = \sqrt{x^2 + y^2}\ ,$$

where k_n, b_n are Hertz's coefficients, $(\dot{x}x + \dot{y}y)/r$ and $(\dot{y}x - \dot{x}y)/r$ are radial and tangential velocities of rotor axes, v is circumferential velocity of inner retainer bearing ring.

Support forces F_{xi}, F_{yi}, (i=1,2) in equations (1) contain except linear and weakly nonlinear terms $b\dot{x}_i + (k + k_3(\dot{x}_i^2 + \dot{y}_i^2))x_i$, … (see eqs- (5)) also strong nonlinear forces

$$H(r_i - r_h)\{k_h (r_i - r_h)^{3/2} (1 + b_h (\dot{x}_i x_i + \dot{y}_i y_i)/r_i)[x_i / r_i - y_i / r_i\, f \,\mathrm{sgn}((\dot{y}_i x_i - \dot{x}_i y_i)/r_i + R_1\omega - v_i)]\},$$

and $\qquad (i=1,2) \qquad (9)$

$$H(r_i - r_h)\{k_h (r_i - r_h)^{3/2} (1 + b_h (\dot{x}_i x_i + \dot{y}_i y_i)/r_i)[y_i / r_i + x_i / r_i\, f \,\mathrm{sgn}((\dot{y}_i x_i - \dot{x}_i y_i)/r_i + R_1\omega - v_i)]\}.$$

Velocities v_1, v_2 are given by additional two equations describing the motion of inner rings of retainer bearings:

$$m_4\dot{v}_i + b_4 v_i - H(r_i - r_h) f \left[k_h (r_i - r_h)^{3/2} (1 + b_h (\dot{x}_i x_i + \dot{y}_i y_i)/r_i)\mathrm{sgn}((\dot{y}_i x_i - \dot{x}_i y_i)/r_i + R_1\omega - v_i)\right]$$

$$(i=1,2) \qquad (10)$$

Heaviside functions $H(r_i - r_h)$ switch-on or switch-off the multiplied expressions into equations in question, if $r_i > r_h$ (H=1) or $r_i \le r_h$ (H=0) during the solution.

The numerical solution of the set of 6 equations, (i.e. 4 equations (5)) with additional strongly nonlinear forces (9) and 2 equations (10)) enables to analyse the rotor space motion in the case, when contact of stiff rotor with one or both retainer bearings occur.

Some examples of periodic, quasiperiodic and chaotic motion of rotor will be shown at presentation.

6. Conclusion

- The mathematical model of space oscillations of stiff rotor supported in two passive radial magnetic bearings is derived for linear (or weakly nonlinear) system.
- The special PID analogue control device for axial rotor position adjustment was developed and realized.
- By means of simple identification measurement on experimental set developed in Institute of Thermomechanics AS CR and by comparison with result of numerical simulation of vibration with small amplitudes (without impact in retainer bearing), the main parameters of rotor and bearings properties were ascertained.
- New mathematical model of rotor motion with impacts in retainer bearings is derived in the form of 6 differential equations containing strong nonlinear terms - Hertz's dynamic contacts, dry friction etc.

7. Acknowledgement

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