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SELF-EXCITED TWO-MASS SYSTEM WITH SOFT IMPACTS

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Summary: *To a basic one-mass self-excited system a tuned absorber is attached the motion of which is influenced by soft impacts. The self-excitation is considered as of van der Pol type one. The stops are symmetrically situated to the absorber mass equilibrium position. The results of numerical simulation are presented in diagrams showing the measure of extreme deflection of the basic system mass to the amplitude of the basic system vibration without absorber in dependence on the tuning coefficient of the absorber. The following effects are investigated: the ratio of the absorber mass to the basic system mass, the distance of the stops, absorber motion damping, and damping in the stops. Generally the stops do not represent a favourable effect.*

1. Introduction

Self-excited vibration induced due to different reasons, e.g. due to flow, relative friction etc., can represent a danger to some structures, machines and devices. Different means are used for suppressing these vibrations. One of the means is the tuned absorber that is attached to the basic self-excited system. For the basic analysis a two-mass model can be used to investigate different effects and to optimise the absorber action. The case of the tuned absorber consisting of an absorber mass, a spring, and a damper where the motion of the absorber mass is not influenced by stops has been analysed by several researchers (e.g. (Tondl, 1991) and the references mentioned therein) and its basic theory can be considered as closed.

This contribution deals also with a two-mass model but this one differs from the above mentioned in (Tondl, 1991) in that the motion of the absorber mass is influenced by soft stops situated symmetrically to the absorber mass equilibrium position (see Fig.1).

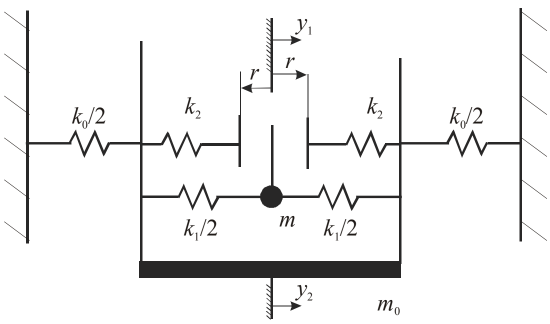

Fig. 1. Oscillator scheme

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2. Differential equations of motion

The basic self-excited system consists of mass m_0 on a spring having stiffness k_0, the deflection being denoted as y_2. The self-excitation of Van der Pol type is assumed. The absorber subsystem consists of absorber mass *m* on a spring having stiffness k_1, the absolute deflection is denoted as y_1. The identical stops are situated in distance *r* from the equilibrium position of the absorber mass, and, similarly as in (Tondl, Peterka 2002), are characterised by additional stiffness k_2. Linear viscous damping of the absorber mass motion is considered; the damping in the stops (as in (Tondl, Peterka 2002)) is proportional to the stops deformation, i.e. characterised by the term

$$(|y_1 - y_2| - r)[k_2 \operatorname{sgn}(y_1 - y_2) + k_3 \operatorname{sgn}(\dot{y}_1 - \dot{y}_2)].$$

Then the system is governed by the following differential equations:

$$m\ddot{y}_1 + k_1(y_1 - y_2) + b_1(\dot{y}_1 - \dot{y}_2) + f = 0,$$

$$m_0\ddot{y}_2 - b_2(1 - \delta\, y_2^2)\dot{y}_2 + k_0 y_2 - k_1(y_1 - y_2) - b_1(\dot{y}_1 - \dot{y}_2) - f = 0 \qquad (2.1)$$

where

$$f = \begin{cases} 0 & \text{for} \quad |y_1 - y_2| < r \\ (|y_1 - y_2| - r)[k_2 \operatorname{sgn}(y_1 - y_2) + k_3 \operatorname{sgn}(\dot{y}_1 - \dot{y}_2)] & \text{for} \quad |y_1 - y_2| > r, \end{cases}$$

and b_1, b_2 are positive parameters.

To transform these equations into dimensionless form new coordinates $x_s = y_s / Y_0$ (s = 1, 2) are introduced where

$$Y_0 = \frac{2}{\sqrt{\delta}} \qquad (2.2)$$

and the time transformation

$$\omega_0 t = \tau, \; (\omega_0 = \sqrt{\frac{k_0}{m_0}}) \qquad (2.3)$$

is used.

Y_0 is the approximate vibration amplitude of the basic system without absorber (see e.g. Tondl, 1991). In this way the following equations are obtained:

$$x_1'' + \Phi = 0,$$

$$x_2'' - \beta(1 - 4x_2^2)x_2' + x_2 - \mu\Phi = 0 \qquad (2.4)$$

where

$$\Phi = Q^2(x_1 - x_2) + \kappa(x_1' - x_2') + Q^2 F, \; Q^2 = \frac{k_1 / m}{\omega_0^2}; \; \kappa = \frac{b_1}{m\omega_0^2}, \; \mu = \frac{m}{m_0},$$

$$F = \begin{cases} 0 & \text{for } |x_1 - x_2| < \rho,\ \rho = \dfrac{r}{Y_s} \\ \left(|x_1 - x_2| - \rho\right)\left(\dfrac{k_2}{k_1}\operatorname{sgn}(x_1 - x_2) + \dfrac{k_3}{k_1}\operatorname{sgn}(x_1' - x_2')\right) & \text{for } |x_1 - x_2| > \rho,\ \rho = \dfrac{r}{Y_s}. \end{cases}$$

Parameter Q represents the tuning coefficient of the absorber, ratio k_3 / k_2 is the coefficient of the damping in the stops. The damping in the stops corresponds, e.g. to the damping in rubber stops or in stops consisting of a set of leaf springs.

3. Results of numerical simulation

The main aim of applying an absorber is to reduce the vibration of the basic system. As a measure of the absorber efficiency the extreme deflection of the coordinate x_2 can be taken, denoting it as X_2.

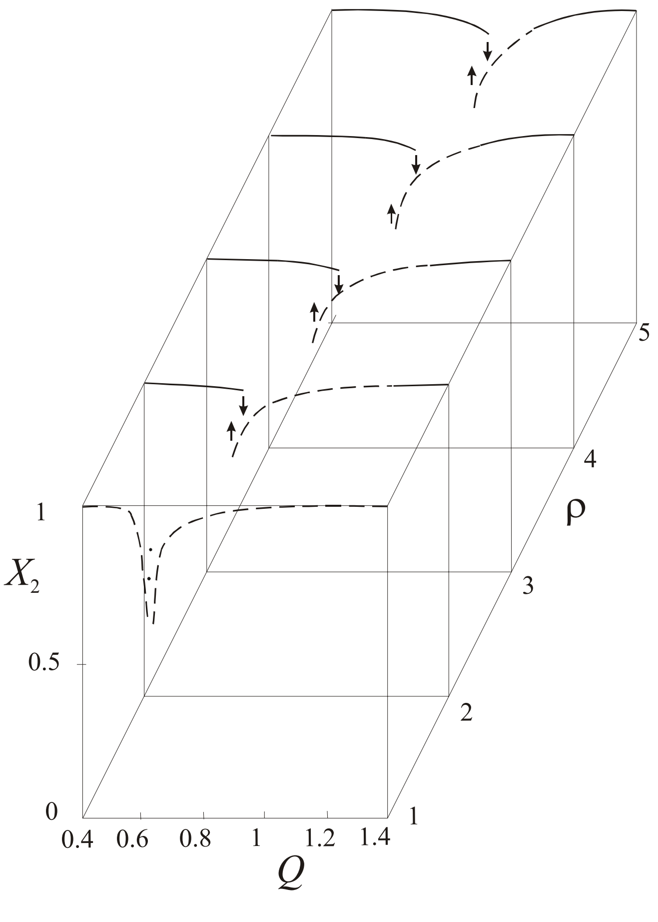

Fig. 2. Relative extreme deflections X_2 in dependence on tuning coefficient Q for different values of ρ ($\rho = \frac{r}{Y_s}$ - relative stops distance) ($\mu = 0.01,\ \kappa = 0.05,\ k_2 / k_1 = 2,\ k_3 / k_2 = 0.2$)

The diagrams of X_2 in dependence on the tuning coefficient Q show the efficiency of the absorber. For $X_2 < 1$ the absorber has a positive effect in comparison to the system without an absorber. For further illustration the time histories of x_1, x_2 and the trajectories in the phase plane ($x_1 - x_2, x_1' - x_2'$) are presented.

Three basic types of motion have been found:

(a) Periodic motion without impacts (for higher values of ρ) – in the diagrams marked by full lines.
(b) Periodic motion with impacts – in the diagrams marked by hatched lines.
(c) Non-periodic chaotic motion – in the diagrams marked by points.

In most cases the possible magnitude of the absorber mass is limited and thus the relative mass ratio μ is small. Two alternatives have been analysed: μ = 0.01 and 0.05. For all alternatives the value β = 0.1 is common. Fig. 2 shows the diagrams of X_2 in dependence on Q for several values of ρ and for μ = 0.01, κ = 0.05, k_2/k_1 = 2 and k_3/k_2 = 0.2. We can see that with the exception of a relatively narrow interval of Q the value of X_2 is very close to 1, i.e. the efficiency of the absorber is small. Also at optimal tuning the efficiency is not high. The optimal value of the tuning coefficient decreases with decreasing ρ.

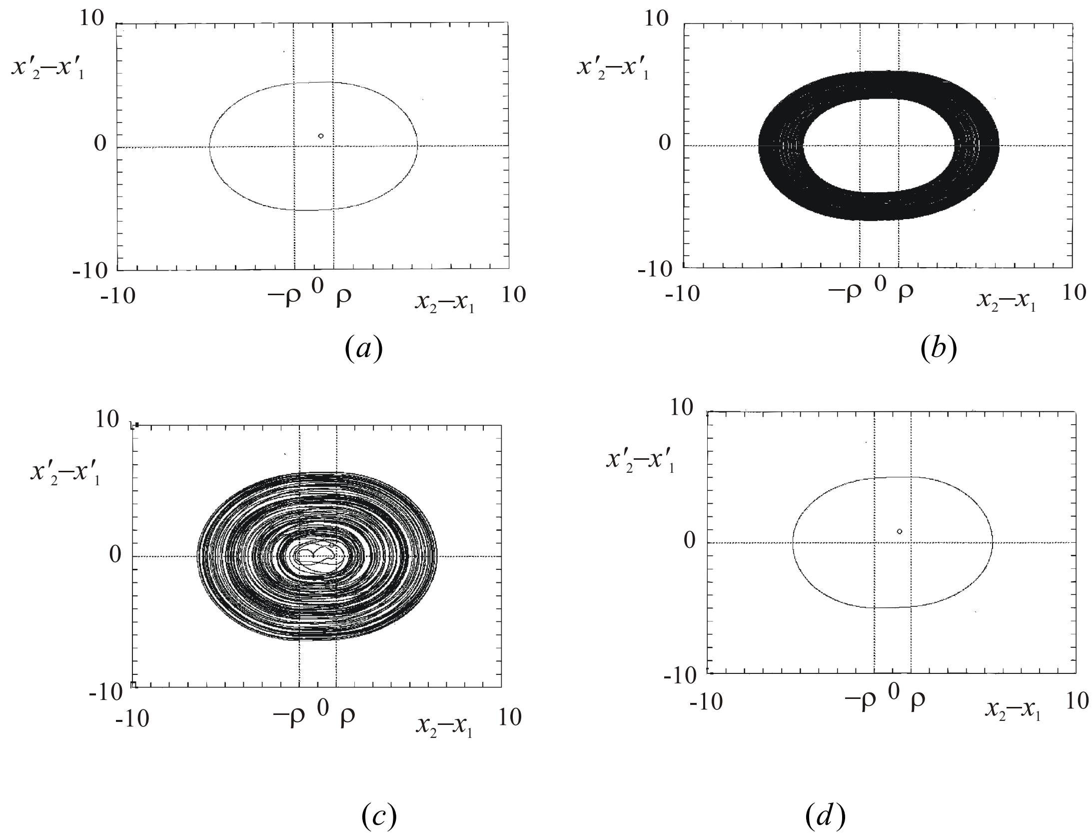

Fig. 3. Trajectories in the phase plane ($x_1 - x_2, x_1' - x_2'$)
((a) for Q = 0.6, (b) for Q = 0.62, (c) for Q = 0.63, (d) for Q = 0.64)

For ρ = 1 in the narrow interval around Q = 0.63 a chaotic motion occurs. The value of X_2 at Q = 0.63 is higher than for values of Q in a small interval around Q = 0.63. This can be seen when comparing the results for different values of Q, presented in trajectories in the phase plane ($x_1 - x_2, x_1' - x_2'$) and in the time histories. Fig. 3 shows the trajectories in the mentioned phase plane [(a) for Q = 0.6, (b) for Q = 0.62, (c) for Q = 63, (d) for Q = 0.64]. Fig. 4 shows the time histories [(a) for Q = 0.62, (b) for Q = 0.63]. The motion is periodic for higher values of ρ: either with or without impacts. For $\rho \geq 2$ there exists an interval of Q where both types mentioned can exist.

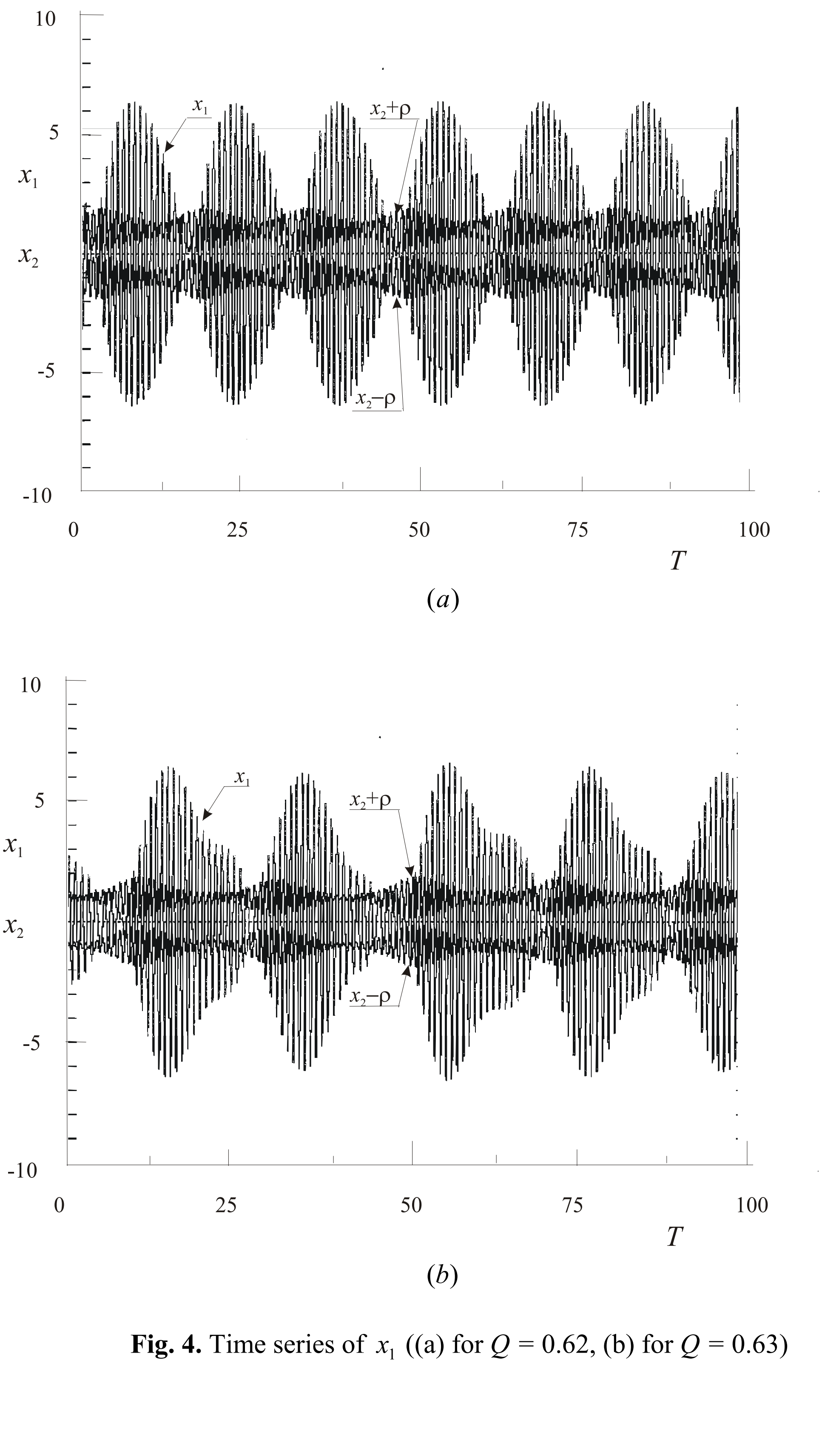

Fig. 4. Time series of x_1 ((a) for Q = 0.62, (b) for Q = 0.63)

For a higher value of μ (μ = 0.05) the results (for κ = 0.05, k_2 / k_1 = 2, k_3 / k_2 = 0.2) are presented in Fig. 5. We can see that more favourable results have been obtained, in comparison to the previous case, thanks to the higher value of μ. With the exception of ρ = 1 the motion is periodic (with or without impacts).

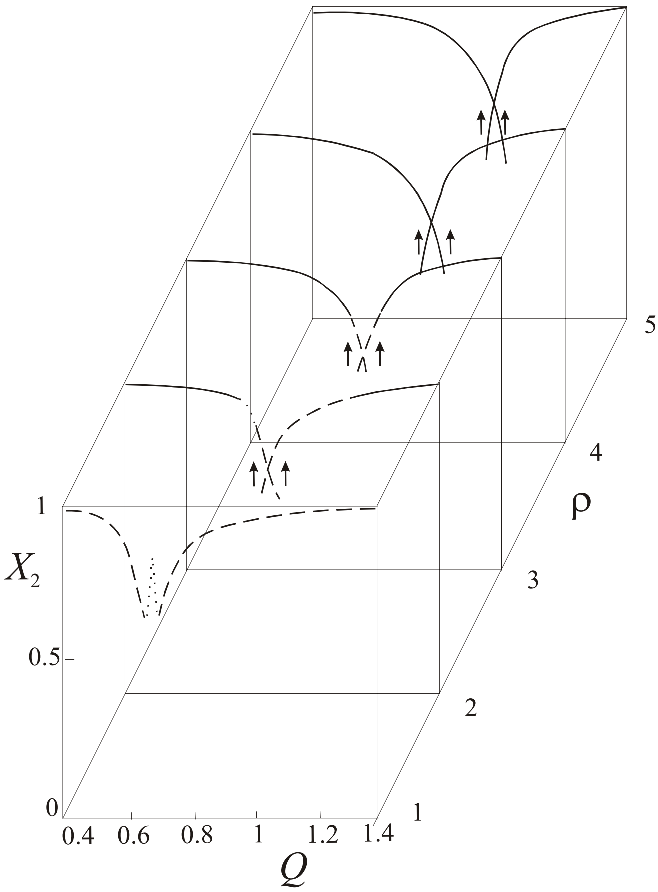

Fig. 5. Relative extreme deflections X_2 in dependence on tuning coefficient Q for different values of ρ ($\mu = 0.05$, $\kappa = 0.05$, $k_2 / k_1 = 2$, $k_3 / k_2 = 0.2$)

Similarly, as in case for μ = 0.01, the optimal value of Q increases with increasing value of ρ. For $\rho \geq 4$ no impact motion occurs.

4. Conclusion

The optimal value of the tuning coefficient Q of the absorber decreases with decreasing the relative stop distance ρ. The efficiency of the absorber increases with increasing the ratio of the absorber mass to the basic system mass. Three types of motion have been found: periodic motion without impacts, periodic motion with impacts, and chaotic motion with impacts. The latter one has an unfavourable effect on the absorber efficiency. The effect of damping in stops on the absorber efficiency is not important. Generally it can be stated that the impact motion in the considered absorber system has a rather unfavourable effect on the efficiency of the absorber.

Reference

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