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EMPIRICAL MODE ANALYSIS OF THE NON-STATIONARY RESPONSE OF THE STRUCTURE VIBRATING IN THE WIND

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Summary: *The paper describes a new method of analysis of the non-stationary response of a bridge model subjected to the wind load. The analysis is based on the decomposition of the response signal into individual oscillating constituents and study of their frequency content and amplitudes. The decomposition for a given set of data is based on direct extraction of the energies associated with various time scales and can be viewed as an expansion of the data in terms of the corresponding modes called Intrinsic Mode Functions (IMF), which are derived from the data. The expansion can be linear or non-linear, as dictated by the data, and it is complete and almost orthogonal. Most important, because the IMFs are derived from the whole set of data, the expansion is auto-adaptive.*

1. Introduction

Signal processing is a fast growing area today and a desired effectiveness in utilization of bandwidth and energy makes the progress even faster. The deeper knowledge of the frequency and amplitude content of a signal is desirable also in wind engineering when dealing with complicated structural design often determined by the strong, non-stationary vibration in the wind.

There are several methods how to analyze the signal with changing properties in time, see e.g. (Cohen 1995). They are usually based on the Fourier series or Fourier transform methods. Most recently, the wavelet analysis is used for the study of various time series as described by Daubechies (1992), that cuts up data into different frequency components with resolution matched to its scale. There is a limitation however for the interpretation of the local event in the data that occurs in the low frequency range can be ambiguous, see (Huang 1998). Moreover, disadvantage of all these method is, however, that they have non-adaptive nature and they are not suitable for studying non-linear and non-stationary phenomena with abrupt changes.

2. Analytical signal

The frequency and amplitude modulation of a signal can be analyzed more deeply by means of so called strong analytic signal that is computed using the original signal $x(t)$

$$z(t) = x(t) + \mathrm{i}y(t) = E(t)\mathrm{e}^{i\psi(t)} \tag{1}$$

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where $E(t)$ is the amplitude of the analytical signal and $\psi(t)$ phase. Squaring the $E(t)$ we obtain a time-dependent expression for the "instantaneous power". The imaginary part $y(t)$ is computed via Hilbert transform of the original process $x(t)$. It can be written

$$\mathcal{H}x(t) = y(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{x(t)}{\tau - t} \mathrm{d}t, \tag{2}$$

if the integral exist. The P in the formula stands for the Cauchy principal value.

This equation (2) is a convolution of $x(t)$ and $1/(t - \tau)$ and so it emphasizes the local characteristics of the original signal. The formula (1) makes natural to introduce the notion of instantaneous angular frequency that can be calculated as derivative of the unwrapped phase with respect to time:

$$\omega(t) = -\frac{\mathrm{d}\psi_u(t)}{\mathrm{d}t} \tag{3}$$

The continuous, unwrapped phase is written

$$\psi_u(t) = \psi(t) + L(t) \tag{4}$$

where $L(t)$ is a integer multiple of π-valued function designed to insure a continuous phase function. For the rapid numerical computation ω can be rewritten in the form:

$$\omega(t) = \frac{x(t)y'(t) - y'(t)x(t)}{z(t)\overline{z}(t)} \tag{5}$$

where $\overline{z}(t)$ is the complex conjugate analytical signal. One can see here an important feature; though the local value can be obtained using this formula, it is actually a global characteristics of the process since it is using the whole signal.

3. Intrinsic mode functions and empirical mode decomposition

Application of the Hilbert transform to general time series with the broad frequency range alone would just lead to the analytic signal with highly fluctuating frequency and amplitude with no advantage to the analysis. The reason for this is that it is violating some specific restrictions on the analytical signal assuring the clear physical meaning.

The proper way how to deal with is first to resolve the original signal $x(t)$ into Intrinsic Mode Functions (IMF). This is done here by means of so called Empirical Mode Decomposition (EMD) first introduced by Huang et. al (1998). It allows the decomposition of the signal into components with special features and it is appropriate for example also in analysis of structural damage (Náprstek 1999).

As said before, in order that the signal $x(t)$ can be written as in the equation (1) it has to have physical meaning. This means primarily:

a) The process should comprise at the whole interval $t\epsilon(0 \div T)$ the same number of zero passages and local extreme values.

b) The process should be locally symmetrical with regard to the zero mathematical mean values; in other words, the mean value of the envelope connecting local maxima and envelope connecting local minima should equal zero in every point $t\epsilon(0 \div T)$.

The decomposition is made by the iterative sifting process, that filters out the components with energies associated with various time scales (frequencies) until there remains the so-called trend of the original process. The original signal is written in the sum of its constituents that are usually narrow-banded:

$$x(t) = \sum_{i=1}^{n} p(t) + r_n(t) \tag{6}$$

The second condition modifies the requirement of otherwise global character to the one having a local property character. It means that the instantaneous frequency defined by equation (3) must not contain any sudden fluctuations brought about by the asymmetry of the wave. In an ideal case an application of this requirement should result in the mathematical mean value equal zero. In case of a non-stationary process it is necessary to work in local time scale.

Defining IMF in this way, every cycle contains only one oscillation mode which, however, is not limited to the narrow-band signal. Its amplitude and frequency may still being non-stationary.

Referring to the previous paragraph, the conditions imposed on IMF, however, are relatively very severe. The processes obtained by measurements do not comply with them in most cases. Again, the process usually contains several oscillation modes in every moment and, consequently, the Hilbert transform itself cannot describe fully the frequency content of the process. In contrast to other methods (see Huang 1996) the EMD method is intuitive and based purely on the particular data set within a time interval $t\epsilon(0 \div T)$. The method is capable to adjust itself respecting the internal structure of the process investigated and its advantage consequently, consists in its high adaptability.

4. Numerical algorithm

The original signal is decomposed into individual IMF from the shortest to the longest scales and the possible residue which is not of oscillation character and can be considered as a certain trend. Should the record of the process be longer, the residue could become another IMF. The iterative algorithm of the decomposition can be outlined as follows:

1. Initialize $r_o(t) = x(t)$ and the counter i.
2. Extract the $i-$th IMF
   (a) Initialize $d_0 =$ and inner counter j.
   (b) Extract the local minima and maxima of d_{j-1}
   (c) Interpolate the local maxima and the local minima by cubic spline. This forms the envelopes of the signal.
   (d) Subtract the mean of envelopes m_{j-1} from the d_{j-1}.
   (e) The results $d_j(t) = d_{j-1}(t) - m_{j-1}(t)$ is the approximation of the IMF $p_i(t)$.
   (f) If the first approximation of the IMF does not satisfy the conditions given above, the iteration should start again at point (b) with increase counter.
3. Create the new initial process $r_i(t) = r_{i-1}(t) - p_i(t)$
4. If r_i still has at least two extrema then increase the counter i and repeat the iteration starting at point 2. Otherwise, the analysis is finished and r_i is the residue r_n.

Writing any function in the form of a decomposition, it is always necessary to assess the completeness of this decomposition with reference to the base functions applied. In this particular case the base is not determined a priori, but is generated dynamically in the course of the decomposition process. This process will terminate, when the norm of the residue is sufficiently small, so that the completeness of decomposition results from its very algorithm, if the iterative process is convergent.

Another problem consists in the possible orthogonality of the functions within the interval $t\epsilon(0 \div T)$. So far no exact proof has been provided, but in individual cases in the meaning of numerical integration have been always orthogonal with acceptable accuracy.

The expansion can be linear or non-linear, as dictated by the data, and it is complete and almost orthogonal. The most important is the fact, that it is adaptive, as the IMFs are derived from the original signal. Therefore the complex non-stationary data originating for example from a real wind measurement can be analyzed relatively easily.

The interpretation of the physical meaning of each IMF is rather knotty, because there are anyway a certain scale of a phenomenon intermittent and each component could contain more scales. Therefore, the decomposition only gives a physical meaning in totality by means of Hilbert amplitude spectrum. Since both the amplitude and frequency are functions of time, the Hilbert spectrum, $H(\omega, t)$ which is amplitude contoured on the frequency-time plane in a three dimensional plot, is more appropriate. The data as in the equation (6) can be represented in the three dimensional plots and are given by the triplet $\{t, \omega(t), E(t)\}$. The example of such spectra is shown below on Figure 3.

5. Practical example

The previous explanation is applied and documented on the signal obtained from the measurements of the vibration of a bridge model. It had been measured in a aeronautical wind tunnel in VZLU Prague. Since the response were in some cases highly non-stationary it is difficult and in some sense also meaningless to analyze its frequency and amplitude content using conventional tools. Using Hilbert transform at every intrinsic mode we are able to do it in more detail.

This signal has been obtained from the measurement of the self excited response of sectional bridge model with 2DOF immerged in the wind at the onset of instability onset, called flutter. The response is the combination of heave and rotation as a function of time and wind speed. The onset is characterized mainly by the amplitude modulation, i.e. it has non-stationary characteristics.

Here for the illustrative purposes, the rotational response is shown at the first rows if Figures 1 and 2. The Figure 1 shows the start of the instable vibration which is characterized mainly by the amplitude modulation. The representative signal from the section model in flutter oscillation is shown on the top part of the figure 2. These processes are decomposed into generally n intrinsic mode function depending on the convergency criteria, each of them is characterized with the rather narrow band frequency content and certain energy contribution to the whole process. Instead of analyzing them by the Fourier transform the Hilbert transform is used and the local attributes of the signal are obtained. In the right columns of the figure instantaneous frequencies are depicted showing the prevalent frequency of the corresponding IMF.

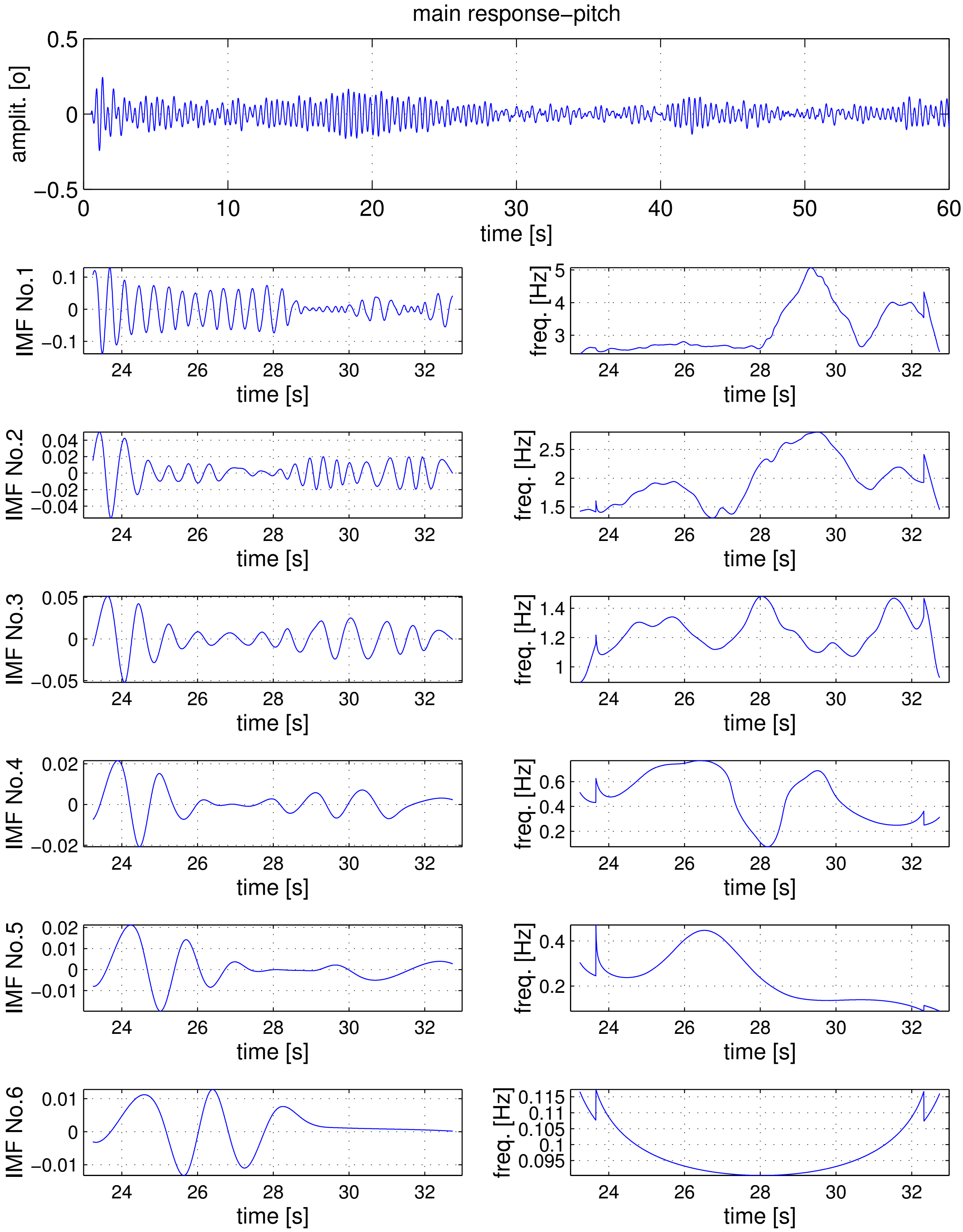

Figure 1. Empirical mode decomposition of a rotational motion of a bridge. On the top graph, the main process is depicted. Graphs bellow show individual mode functions (left column) and corresponding instantaneous frequency (right column)

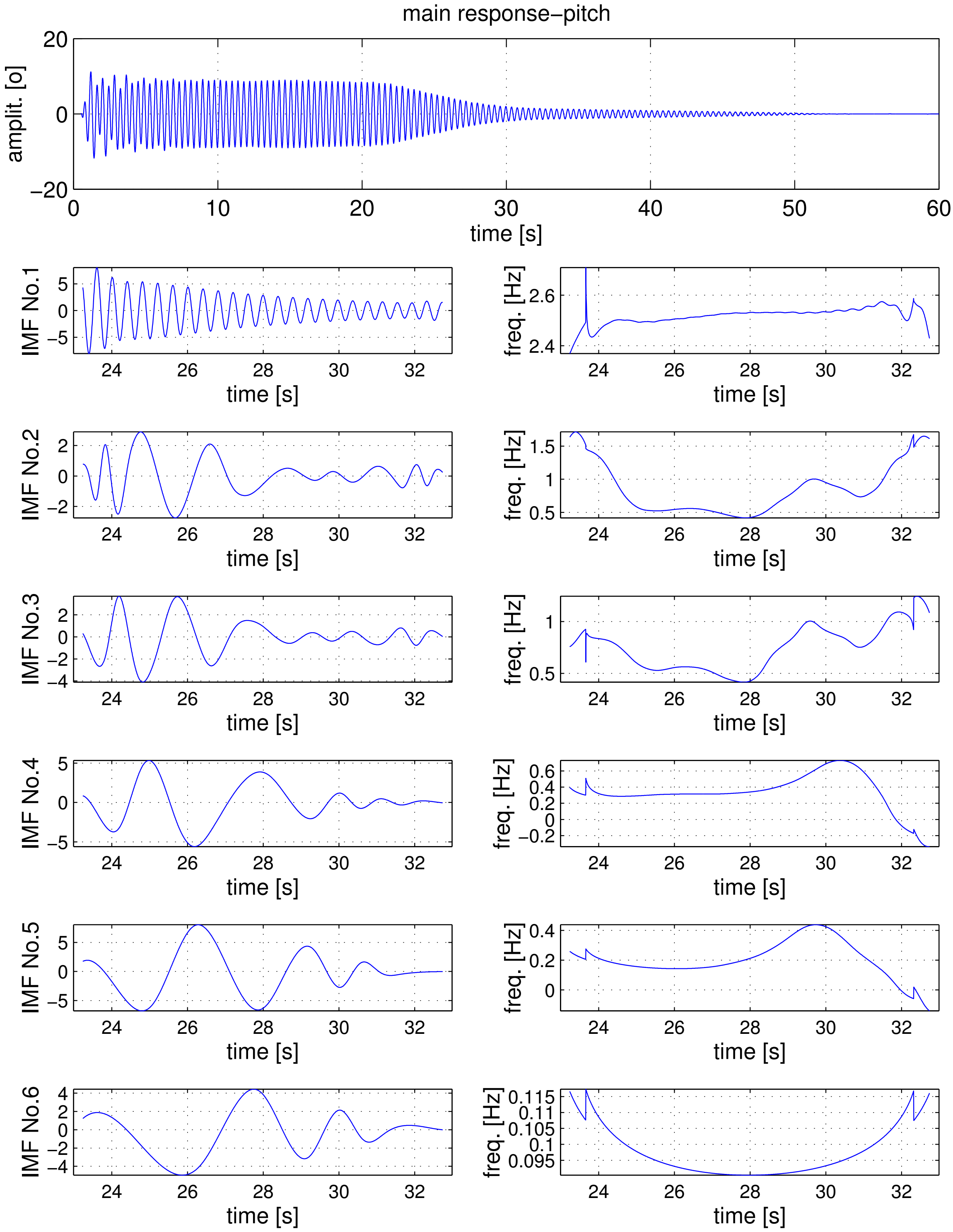

Figure 2. Empirical mode decomposition of a signal. The more significant mode No. 1 has the frequency oscillating around the value 2.5 Hz. It has been identified as the flutter frequency.

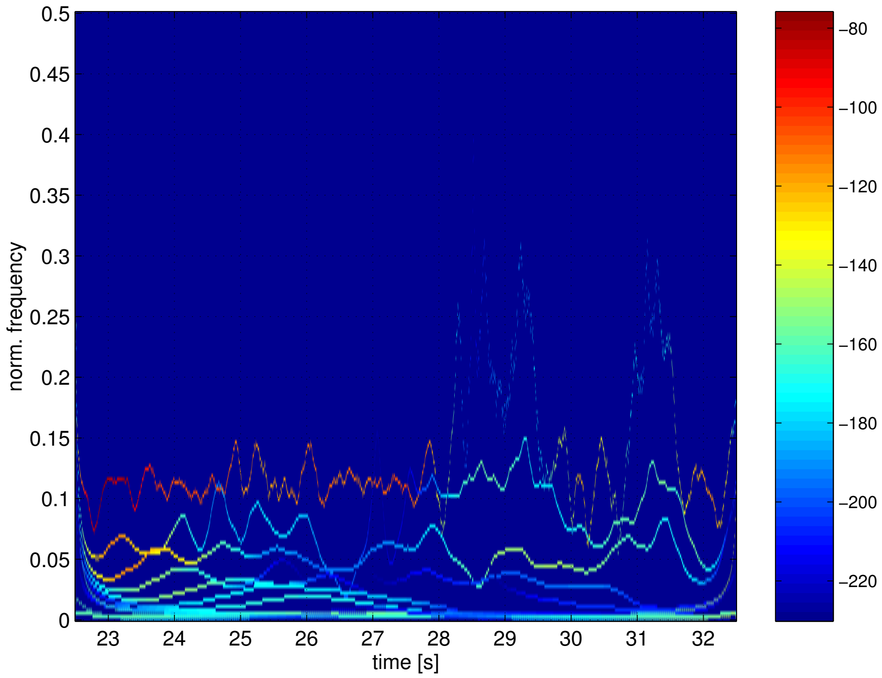

Figure 3. The Hilbert energy spectrum of a signal from the Figure 2. The frequency is normalized. Graphical representation is given by $20 \log H^2$.

6. Conclusion

The new method of how to decompose a signal representing a wind excited structural response into different oscillatory modes is outlined and applied here. If we represent all these frequency functions in the time-frequency plane, we obtain a complete description of the signal taht is not possible with other standard methods. The individual modes have particular features which allows to analyzed them by means of Hilbert transform and assure that they are orthogonal, so that there is no leakage of the energy. Since all data is required to decompose it and the calculate the derivatives, one can conclude that the phase and amplitudes of the resultants have really global properties.

7. Acknowledgement

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