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STRAIN LOCALIZATION IN STRUCTURES UNDER BENDING

L. Brdečko[1]

Summary: *For the establishment of new more rigorous models of concrete structures it is usually necessary to take development of cracks into account. If strain softening is considered in the smeared crack model, the results of numerical solutions are influenced by mesh size of the finite element model. The paper is focused on the application of some methods, usually called localization limiters, on beams in bending and plates. This paper presents mesh adjusted softening model and nonlocal continuum model for plasticity and damage.*

1. Introduction

The post peak modelling of concrete behaviour is at this time a very usual phenomenon in the analysis of reinforced concrete structures by finite element methods. Different models with discrete or smeared cracks are used. The smeared crack model, when strain softening is considered, often leads to instability of solutions and the results are dependent on the finite element mesh size. The failure is concentrated to the most stressed element. If the size of this element decreases to zero, the total dissipated energy of damage is decreased to zero too. If strain softening is to be considered, this circumstance must be treated somehow.

Two approaches are presented in this paper, both of which reduce the shortcomings introduced above. Both methods are well-known from analysis of simpler problems, their application on beams in bending will be shown here and extension of one of these approaches to plates will be outlined.

2. Mesh adjusted softening modulus

The mesh adjusted modulus is known as crack band model in 2D problems. The purpose of this model is to ensure, for different sizes of finite elements, the same amount of dissipated energy if material is damaged. The softening branch of the stress-strain diagram is modified according to element size in the direction orthogonal to the crack.

The basic condition is that the energy which is necessary for crack opening is constant. This process reduces the dependency of load-displacement diagram and dissipated energy

[1]Ing. Luděk Brdečko, Ph.D.: Technical University of Brno, Faculty of Civil Engineering, Department of Structural Mechanics; Veveří 95; 662 37 Brno; tel.: +420.541 147 368; e-mail: brdecko.l@fce.vutbr.cz

on finite element mesh. The damage remains localized into one element, respectively into a band of elements, the width of which is related to size of one element. A realistic load-displacement diagram is achieved, but the strain profile in the vicinity of failure remains unrealistic.

3. Nonlocal continuum

In contrast to the previously discussed model, the goal of the nonlocal continuum model is to spread damage into certain region, whose size is given by characteristic length, basically a material constant. One of the variables in constitutive relations is considered as nonlocal, i.e. it is related to values of this variable in the vicinity of examined point. If an appropriate variable is considered as nonlocal, the model ensures:

- the dissipation of energy does not tend to zero, if the size of finite element decreases,
- the load-displacement diagram is realistic and not influenced by mesh size,
- the strains in the vicinity of failure are realistic,
- the size of the damaged region is correct,
- elastic solution of structure is not affected by nonlocal averaging.

In 1D case it is possible to express nonlocal averaging in the following way. The dependency of nonlocal variable on local variable decreases with increasing distance r of another point (co-ordinate ξ) from examined point (co-ordinate x)

$$r = |x - \xi|. \tag{1}$$

This relation is expressed by the function

$$\alpha_0(r) = \begin{cases} \left(1 - \frac{r^2}{R^2}\right)^2 & r < R \\ 0 & r \geq R \end{cases}, \tag{2}$$

which is bell-shaped. The variable R is called interaction radius and is related to characteristic length of material. It is possible to normalize the equation (2) over its definition domain

$$\alpha(x, \xi) = \frac{\alpha_0(x, \xi)}{\int\limits_L \alpha_0(x, \zeta) d\zeta}. \tag{3}$$

Because definition domain of function α_0 is limited by R, the integration length is limited to $\langle x - R,\, x + R \rangle$. The equation (3) normalizes α_0 also near the structure boundaries. The nonlocal variable $\overline{v}$ can be obtained from local v by the relation

$$\overline{v}(x, \xi) = \int\limits_L \alpha(x, \xi) v(\xi) d\xi. \tag{4}$$

Application of nonlocal continuum model to two types of constitutive relations - plasticity and damage model - are presented in the paper. The constitutive relation for plasticity can be expressed as

$$\sigma = E(\varepsilon - \varepsilon_{pl}), \tag{5}$$

were σ is stress, ε is total strain, ε_{pl} is plastic strain and E is Young's modulus. The process of application of nonlocal averaging is following:

1. computation of local increment of plastic deformation $\delta\varepsilon_{pl}$ the usual way,
2. computation of nonlocal increment of plastic deformation $\overline{\delta\varepsilon_{pl}}$ from local by nonlocal averaging,
3. computation of nonlocal plastic deformation $\overline{\varepsilon_{pl}}$ from increments $\overline{\delta\varepsilon_{pl}}$,
4. computation of stresses from nonlocal plastic deformation and local total deformation. by using eq. (5).

The constitutive relation for damage can be expressed with

$$\sigma = (1 - \omega)E\varepsilon, \tag{6}$$

where ω is a parameter of damage which increases from zero to one with growing damage. The process of application of nonlocal averaging is following:

1. computation of local deformation ε the usual way,
2. computation of nonlocal deformation $\overline{\varepsilon}$ from local ε by nonlocal averaging,
3. computation of damage parameter ω from nonlocal deformation, $\overline{\varepsilon}$
4. computation of stresses from nonlocal stiffness and local deformation by using eq. (6).

When nonlocal continuum model is practically implemented the integrals in (3) a (4) are calculated numerically, e.g. using integration points of stiffness matrix of each element. The values of function α are the weight coefficients, which are not changed through the iterations and it is possible to compute them, for every two points, before the actual solution. With regard to the certain dependencies of deformations over the cross-section, which is introduced by Navier hypothesis, the averaging over height is omitted.

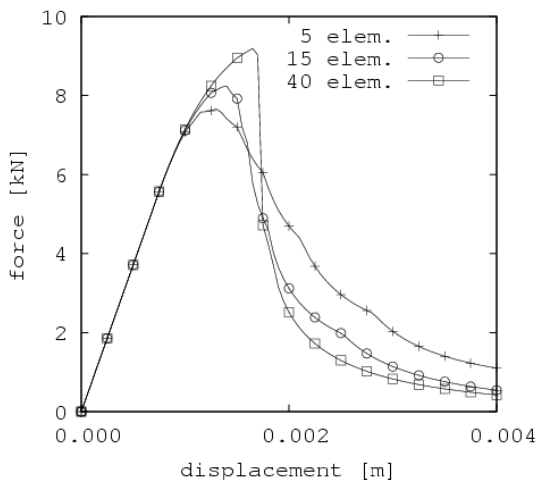

Figure 1: Beam deflection – mesh dependent softening modulus

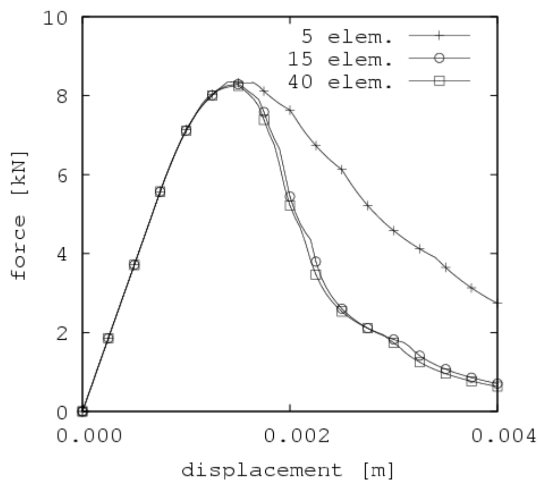

Figure 2: Beam deflection – nonlocal continuum

4. Plates

Because the nonlocal model of plates is under development at this time, this paper will only outline some differences from the beam model and will present local model with some results.

Firstly, the integration in (3),(4) runs over a 2D area. The definition domain of function (2) is a circle with radius R, and it is also the integration area. If the direction of cracks is not respected in averaging, the weight coefficients α are not changed through the iterations and it is possible to compute them before the solution too. Considering a linear relation of deformation through the height of cross-section the averaging of total deformation can run through the deformation of cross-section. For other nonlocal variables it has to run through variables of layers.

A finite element model of reinforced concrete plate was made. The model is formed by plain-plate eight-node isoparametric elements. The layered concept is used for assembling of stiffness matrix and integration of internal forces over cross-section. The model includes plasticity of steel, plasticity of concrete with Chen yield function and multiply hardening in the element. The cracks are implemented by fixed crack model. Residual stiffness of crack in normal direction by softening of stress-strain diagram in tension is considered. The details of this model can be found in lit. (Brdečko, 2001).

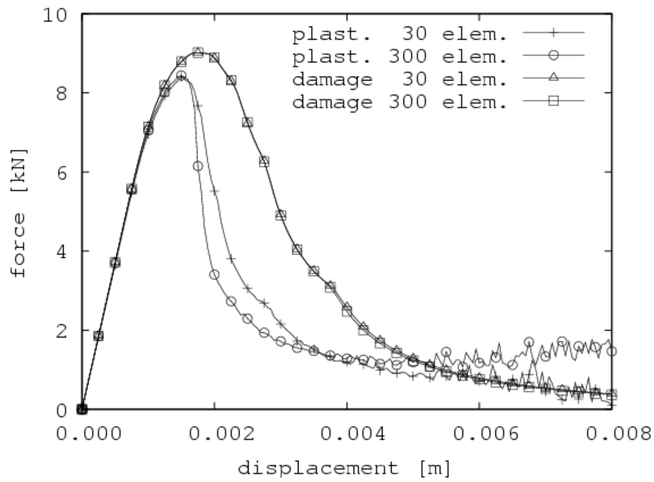

Figure 3: Beam deflection – plasticity and damage of nonlocal continuum

5. Results and conclusions

The behaviour of localization limiters is demonstrated on the examples of the cantilever beam with length of 1 m, which is loaded by increasing the lateral displacement at the end. The figures 1-3 display the load-displacement diagram of unsupported end of beam for different numbers of elements and different localization limiters.

Fig. 1 brings up the results for the model with mesh adjusted softening modulus. In contrast with beam in tension, in case of bending the shape of load-displacement diagram partially depends on the size of elements, but it is more realistic then with a local solution. The total dissipated energy is approximately constant. The model is usable even for large elements.

However, for nonlocal continuum model, it is necessary that the elements are smaller than damaged zone. Fig. 2 shows results for interaction radius 0.1 m. From certain (small enough) size of elements the load-displacement diagram is independent on the mesh. The strains are realistic.

Fig. 3 compares application on two types of constitutive relations. The interaction radius is set to 0.2 m. The presented damage model shows better behaviour in terms of load-displacement diagram, stability of solution and complete unloading.

The behaviour of local plate model is displayed on the square plate simply supported in corners. The plate is loaded by growing force in centre of plate. The plate is reinforced in two directions by two surfaces. The load-displacement diagram is compared with results of experiment from lit. (Duddeck at al., 1978).

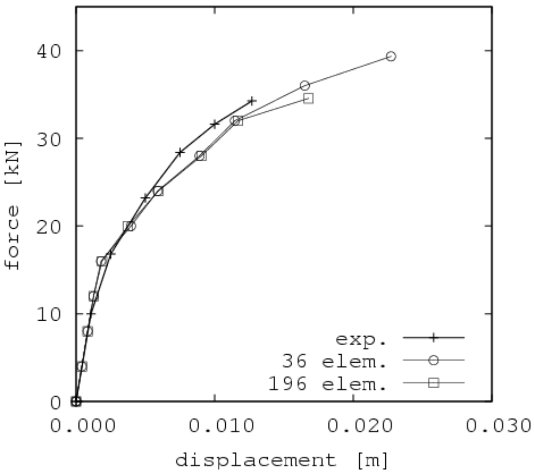

Figure 4: Deflection of plate for different mesh size

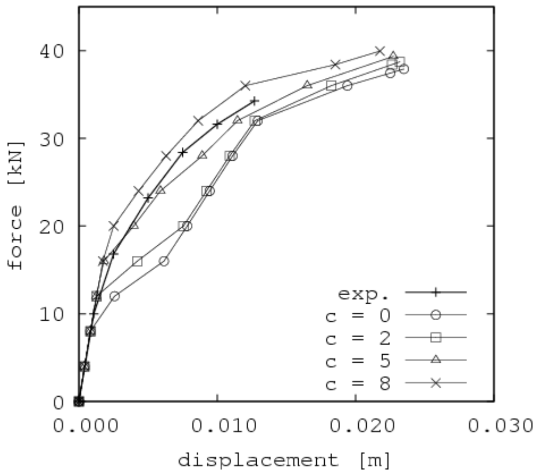

Figure 5: Deflection of plate for different softening modulus

Fig. 4 shows the results for different element sizes and fig. 5 for different softening branches of stress-strain diagram, which is represented by parameter $c = -E_z/E_0$, where E_0 is initial modulus and E_z is modulus of softening branch. It can be seen that if reinforcement is considered, the influence of mesh size and strain softening is smaller and it appears before the peak of load-displacement diagram.

6. Acknowledgement

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7. References

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