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ON THE CHOICE OF AN ADEQUATE PHENOMENOLOGICAL MODEL FOR VISCOPLASTIC FLUIDS

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Summary: *At present a series of phenomenological models are used for the description of flow curves of viscoplastic materials. Their application for the given materials hitherto subjects to the two following factors: number of entering parameters and agreement with the experimental data. However, for the case of viscoplastic fluids there is still one point playing a crucial role: determination whether a mutual ratio of viscous and plastic effects generated by the proposed model corresponds to the physical characteristics of the material studied. This contribution aims to analyse partially this topic and to provide a hint for a choice of a corresponding model.*

1. Introduction

This contribution shows that for basic explicit constitutive equations commonly used for description of viscoplastic behaviour (as e.g. Bingham, Herschel-Bulkley, Casson, Robertson-Stiff, Shul´man models) there is possible to convert the traditional rheograms (shear stress against shear rate of strain) to newly defined graphs. The axes in these graphs represent relative viscous and plastic stresses in the whole range of shear rate (expressed as the ratio of the viscous and plastic part of shear stress to the total stress, respectively). By this conversion the flow curves in classical rheograms are transformed onto the segments of straight lines in newly defined ones. If the experimental data are transformed in the same sense, then it is possible to decide (according to which axis they are closer in the new rheogram) which effects (viscous or plastic) are prevailing for the given case.

2. Analysis

Prior to the analysis outlined above, the following six-parameter model is proposed

$$\tau \quad = \quad k\left[\frac{a+\left(k|\dot{\gamma}|\right)^{p}}{b+\left(k|\dot{\gamma}|\right)^{q}}\right]^{r}\cdot\dot{\gamma} \quad . \tag{1}$$

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Choosing b=0 and qr=1 the traditional visco-plastic models (such as Bingham, Herschel-Bulkley, Casson, Robertson-Stiff, Shul'man) can be obtained as special sub-cases.

Let us denote (for the sake of simplicity we use $\dot{\gamma}$, τ instead of $|\dot{\gamma}|$, $|\tau|$)

$$N = \frac{d(\log \tau)}{d(\log \dot{\gamma})} \quad . \tag{2}$$

Substituting from the rel.(1) we obtain

$$N = 1 + \frac{pr(k\dot{\gamma})^p}{a + (k\gamma)^p} - \frac{qr(k\dot{\gamma})^q}{b + (k\dot{\gamma})^q} \quad . \tag{3}$$

Denoting

$$N' = \frac{d(\log N)}{d(\log \dot{\gamma})} \tag{4}$$

we get

$$N' = \frac{p^2 r \dfrac{a(k\dot{\gamma})^p}{\left[a + (k\dot{\gamma})^p\right]^2} - q^2 r \dfrac{b(k\dot{\gamma})^q}{\left[b + (k\dot{\gamma})^q\right]^2}}{1 + pr\dfrac{(k\dot{\gamma})^p}{a + (k\dot{\gamma})^p} - qr\dfrac{(k\dot{\gamma})^q}{b + (k\dot{\gamma})^q}} \quad . \tag{5}$$

If we analyse the course of the model (1) in the coordinates N, N' we can derive that for selected sub-cases mentioned above the rheograms consist of linear segments with the end points at the axes N (representing the relative magnitude of the viscous component, i.e. $(\tau-\tau_0)/\tau$) and N' (representing the relative magnitude of the plastic component, i.e. τ_0/τ) where τ_0 stands for a yield stress. From here it is possible to consider which of these two components is prevailing. For better comparison of the individual models it is suitable to transform the corresponding segments in the coordinates N, N' to a unique one, for the sake of simplicity the segment connecting the points (0,1) and (1,0) was chosen.

3. Example

Let us consider a set of experimental data representing behaviour of a viscoplastic fluid (Fig.1). For simplification we chose in the model (1) b=0, qr=1. A deviation of the theoretical value of the corresponding flow curve - applied to the given set of experimental data – from the experimental one is (Fig.2)

$$\varepsilon = \frac{\tau_{\exp} - \tau_{\text{mod}}}{\tau_{\text{mod}}} \quad . \tag{6}$$

This deviation is expressed in the transformed coordinates as (Fig.3)

$$\delta = \sqrt{2X_{\text{mod}}^2 - 2X_{\text{mod}} + 1} \cdot \left[1 - (1 + \varepsilon)^{-1/r}\right] \quad . \tag{7}$$

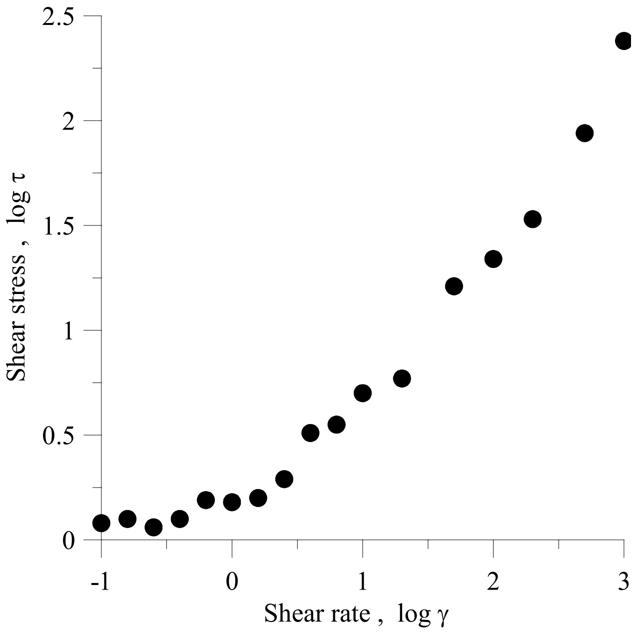
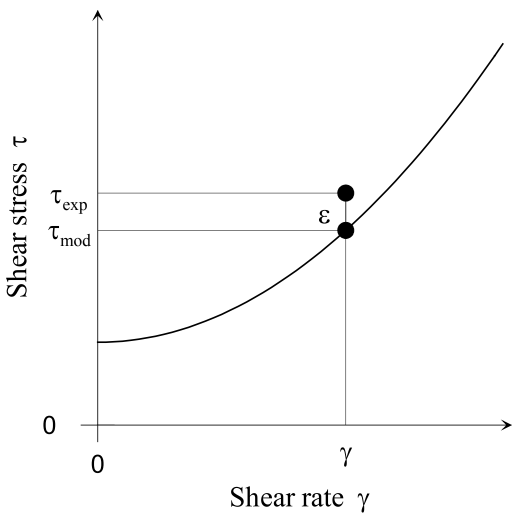


Fig.1 Set of experimental data.



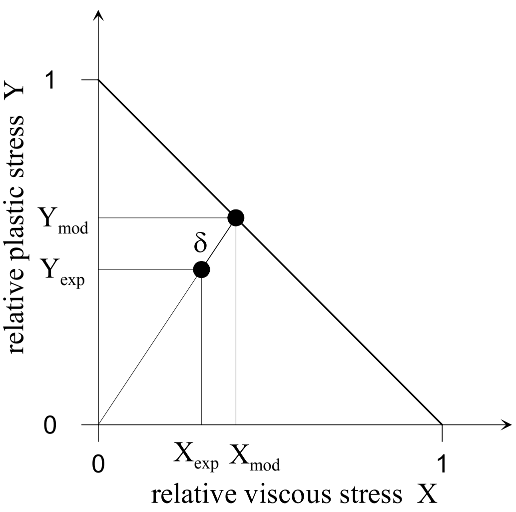


Fig.2 Deviation of experimental value from that given by a chosen flow curve

Fig.3 Deviation of experimental value from that given by a chosen flow curve after transformation

The following figures document the participation of viscous and plastic components for the individual classical viscoplastic models (experimental data used are the same for all presented cases, see Fig.1).

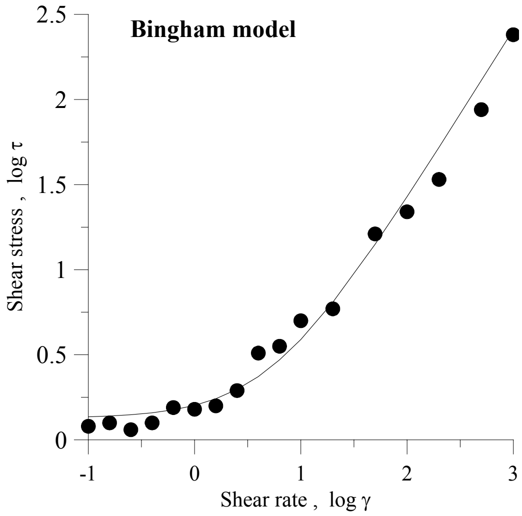


Fig.4a Classical rheogram
(Bingham model).

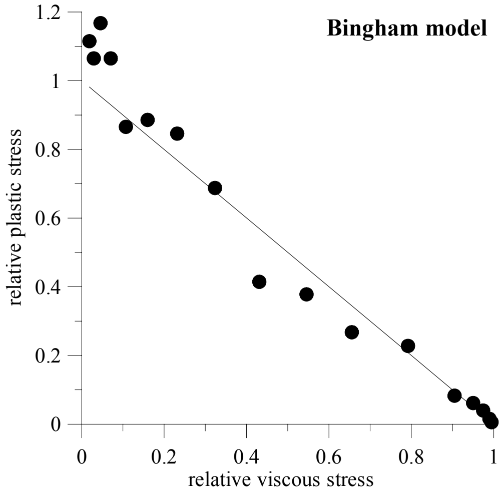


Fig.4b Transformed rheogram
(Bingham model).

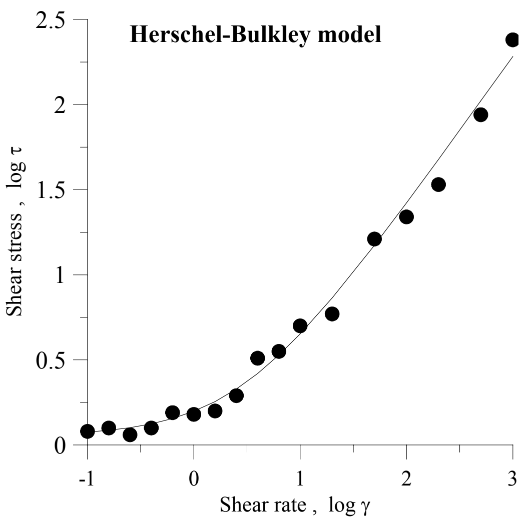


Fig.5a Classical rheogram
(Herschel-Bulkley model).

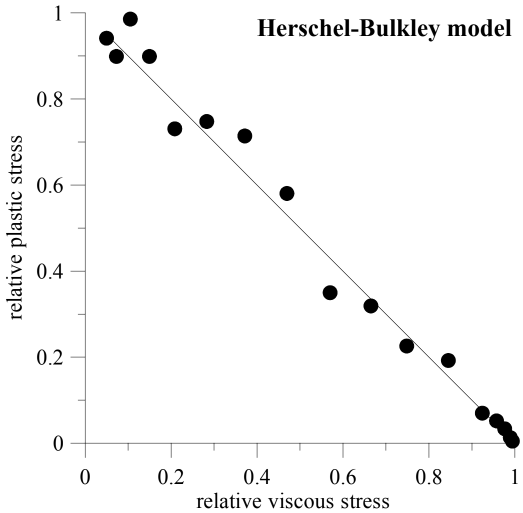


Fig.5b Transformed rheogram
(Herschel-Bulkley model).

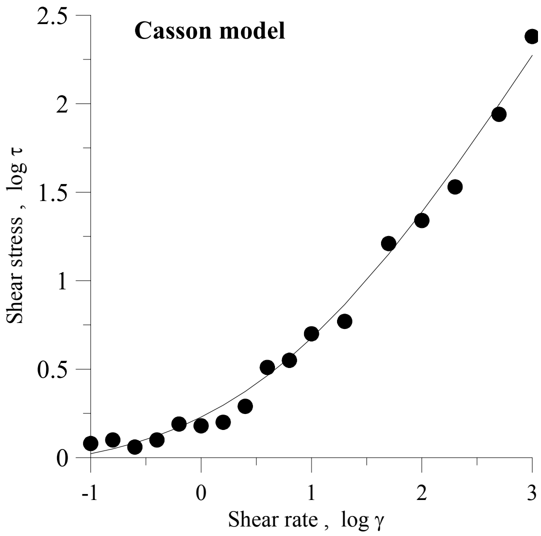


Fig.6a Classical rheogram
(Casson model).

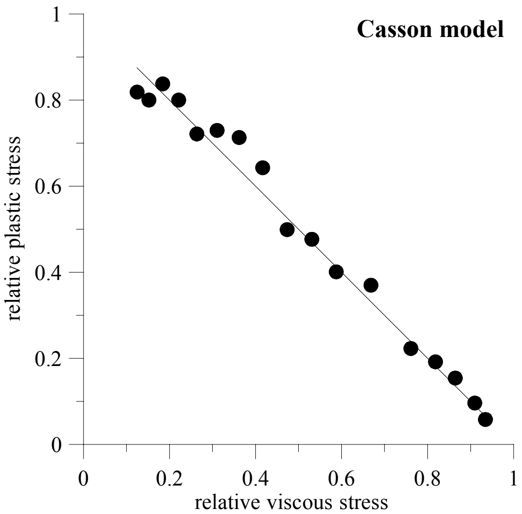


Fig.6b Transformed rheogram
(Casson model)

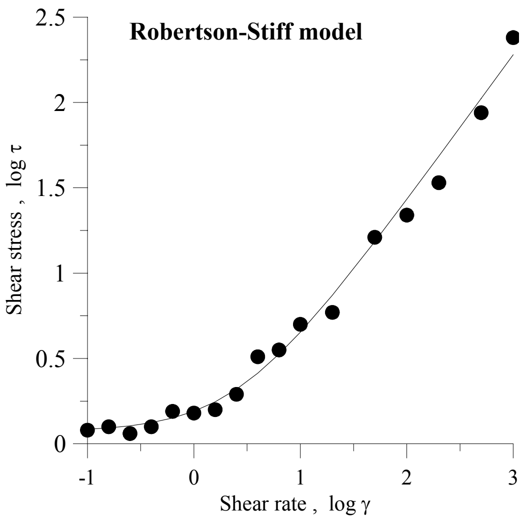


Fig.7a Classical rheogram
(Robertson-Stiff model).

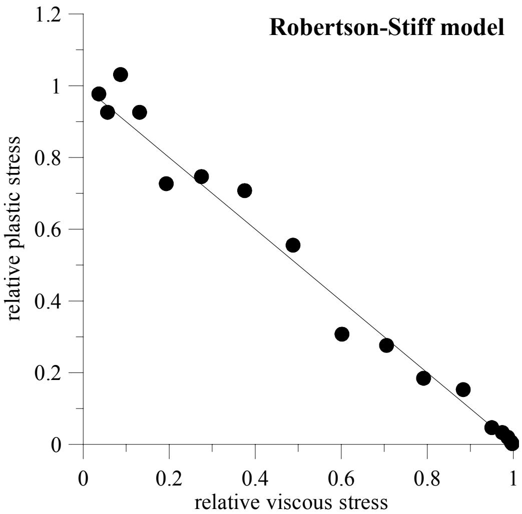
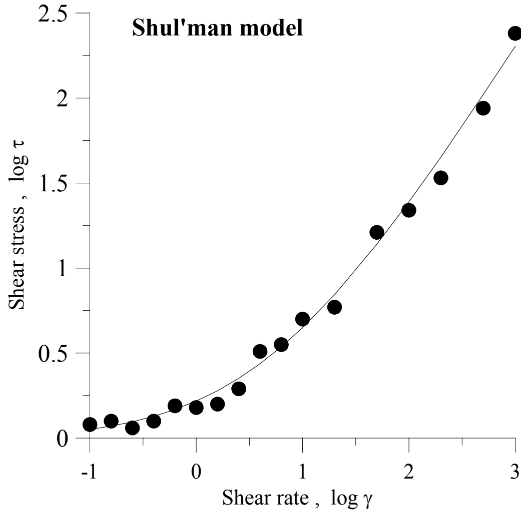


Fig.7b Transformed rheogram
(Robertson-Stiff model).



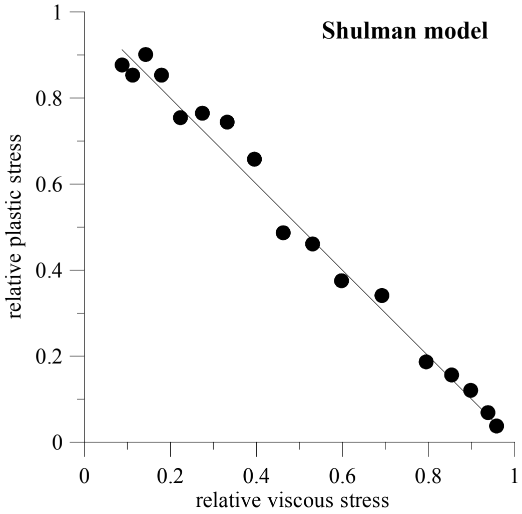


Fig.8a Classical rheogram (Shul'man.model).

Fig.8b Transformed rheogram (Shul'man.model).

Figs.4b-8b illustrate diverse characterization of viscous and plastic components for the individual constitutive equations.

4. Conclusions

The above transformations enable to compare the viscous and plastic components (that participate in the behaviour of the viscoplastic materials studied) according to the range of shearing parameters where they are measured, as the transformed rheograms are valid for the whole extent of these parameters.

As the analysis of a viscosity function against shear stress seems to be sometimes more advantageous in comparison against shear rate, the analogous consideration can be realised for the implicit six-parameter model

$$\tau \quad = \quad k\left[\frac{a+\left(k|\tau|\right)^{p}}{b+\left(k|\tau|\right)^{q}}\right]^{r}\cdot\dot{\gamma} \quad . \qquad (8)$$

Acknowledgement

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