ENGINEERING MECHANICS 2004
NATIONAL CONFERENCE
with international participation
Svratka, Czech Republic, 10 - 13 May 2004

THE CUTTING VELOCITY MODEL OF MULTIPLE DISC MILLING

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Summary: *The paper presents analytic model of the cutting velocity model of multiple disc milling. To determine the equation of the particle's relative motion in the inter-disc space i.e. dependence between its distance x from the axis of rotation and time t, the forces acting for a single piece were defined: gravity force, horizontal component reaction of the milled material, vertical component reaction of the disc surfaces, fictitious internal centrifugal force, and fictitious Coriolis force. The new developed model gives accounts the following features and phenomena characteristic of milling: tool eccentricity, insert throw, uneven insert spacing, spindle tilt – either steady or variable, spindle error motion, changes in the effective disc geometry due to machining parameters, type of operation (up- or down-milling) and spindle tilt.*

Keywords: *multiple disc milling, mathematical model*

1. Introduction

One and all available analytic models of velocity are restricted to ideal milling processes with equal insert spacing, no insert throw, no tool eccentricity, no spindle tilt and infinitely rigid systems. Moreover, almost all assume circular trajectories of cutting points. On the other hand, only one of the above limitations (i.e., circular paths of the cutting velocity applies to the numerical models. However, the attractiveness of the latter models is diminished by their inherent limitation, viz. incompatibility with analytical algebra and calculus. More specifically, the numerical, models are capable of providing a specific solution for a chosen set of input parameters. When generalization is required, extensive computer simulations are needed. Conclusions from such simulations can thus be biased.

In this paper an analytic model free of the limitations discusses above is presented. The new model is accurate, versatile and simple.

2. Velocity of the Milled Particles

To solve the problem for the work purposes the simplifying assuptions were made (Fig. 1):

1. pieces are moving on the smooth surface, in the hypothetical passage AB, lenght $2r_i$ formed between two discs seated on the vertical axis CD and rotating with the constant angular velocity ω. Inside the passage made of the surfaces of discs, a spherical piece K with the mass m, is moving. At the initial instant the idealized piece K is at rest in the distance $OK_0 = a$ from the axis of rotation,

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2. piece motion K will consist of relative motion which is straight – line motion along the passage and of transportation which is rotary motion around axis CD with the constant angular velocity.

To determine the equation of the particle's relative motion in the inter-disc space i.e. dependence between its distance x from the axis of rotation and time t, the forces acting for a single piece were defined:

– gravity force $Q = m \cdot g$,

– horizontal component reaction of the milled material R_r,

– vertical component reaction of the disc surfaces N,

– fictitious internal centrifugal force

$$B_u = m \cdot p_u = m \cdot x \cdot \omega^2,$$

– fictitious Coriolis force

$$B = m \cdot a = 2 \cdot m \cdot \omega \frac{dx}{dt}$$

where $\frac{dx}{dt} = w$ is the relative velocity of a pieces.

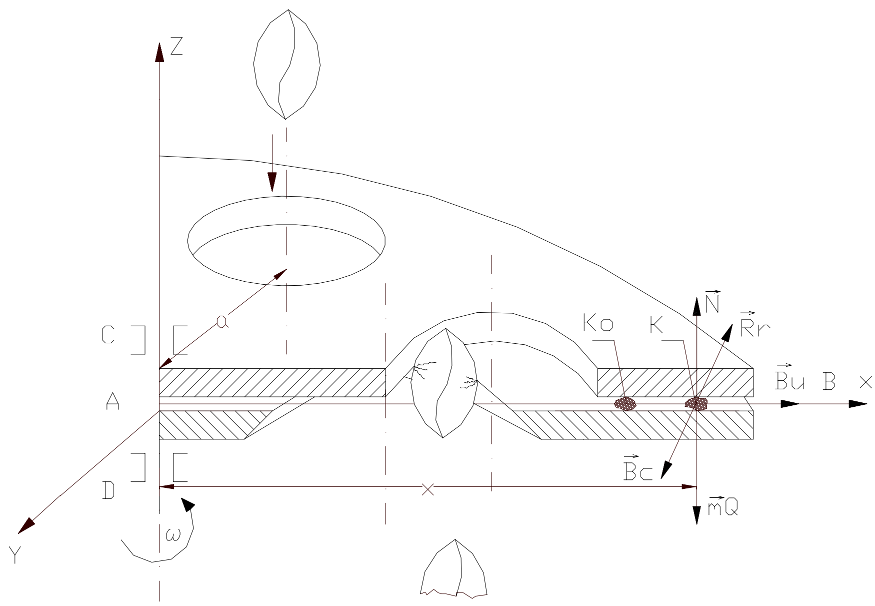

Fig. 1. A piece of material in the inter-disc space of the mill [1]

Dynamic equation of particle relative motion is in the form

$$m \cdot a_w = m \cdot g + N + R_1 + B_u + B_c \tag{1}$$

Summing the forces for the different directions in the Cartesian coordinates axes (x, y, z) gives:

x direction: $$m \cdot \frac{d^2x}{dt^2} = m \cdot \omega^2 \cdot x \tag{2}$$

y direction: $$0 = 2 \cdot m \cdot \omega \cdot \frac{dx}{dt} - R_r \tag{3}$$

z direction: $$0 = N - m \cdot g \tag{4}$$

The equation leading to determination of dependence $x = f(t)$, was obtained by the integration of equation (2). The equation after simple conversion has the form

$$\frac{d^2x}{dt^2} - \omega^2 \cdot x = 0 \tag{2a}$$

It is quadratic linear differential eguation with constant factors. The solution of the equation is function

$$x = C_1 \cdot e^{-\omega \cdot t} + C_2 \cdot e^{\omega \cdot t} \tag{5}$$

The integration constants were determined from the initial conditions, for $t = 0$

$$x = a, \tag{6a}$$

and

$$w = \frac{dx}{dt} = 0 \tag{6b}$$

The relative velocity of a piece is

$$w = \frac{dx}{dt} = \left(-C_1 \cdot e^{\omega \cdot t} + C_2 \cdot e^{\omega t}\right) \cdot \omega \tag{7}$$

Substituting initial condition (6a, 6b) to the equations (5) and (7) we obtained $C_1 = C_2 = \frac{a}{2}$

The equation of a particle moving has the form

$$x = \frac{a}{2} \cdot \left(e^{\omega \cdot t} + e^{-\omega \cdot t}\right) = a \cdot \cosh(\omega \cdot t,) \tag{8}$$

or:

$$x = C_1 \sinh \cdot \omega \cdot t + C_2 \cosh \cdot \omega \cdot t + \frac{B_u}{m\omega^2}\left(1 - \cosh \cdot \omega \cdot t\right), \tag{9}$$

with:

$$\frac{dx}{dt} = C_1\omega \cdot \cosh \cdot \omega \cdot t + C_2\omega \cdot \sin\omega \cdot t - \frac{B_u}{m \cdot \omega^2}\omega \cdot \sin\omega \cdot t \tag{10}$$

and for $x = a$, $\frac{dx}{dt} = 0$ integration constants are $C_2 = a$, and $C_1 = 0$:

So

$$x = a \cdot \cosh\omega \cdot t + \frac{B_u}{m\omega^2}\left(1 - \cosh\omega \cdot t\right) \tag{11}$$

after rearranging

$$x = \left(a - \frac{B_u}{m\omega^2}\right)\cosh\omega \cdot t + \frac{B_u}{m\omega^2} \tag{12}$$

for $\varphi = \omega t$ and $t = \frac{\varphi}{\omega}$:

$$x = \left(a - \frac{B_u}{m\omega^2}\right)\cosh\varphi + \frac{B_u}{m\omega^2} \tag{13}$$

and relative velocity:

$$w = \frac{dx}{dt} = \frac{a\omega}{2} \cdot \left(e^{\omega \cdot t} + e^{-\omega \cdot t}\right) = a \cdot \sinh(\omega \cdot t) \tag{14}$$

As it results from the equation (3) and (14) the horizontal reaction of a piece of material to the walls of the hypothetical passage is (Example 1.0):

$$R_r = 2 \cdot m \cdot \omega \cdot \frac{dx}{dt} = m \cdot a \cdot \omega^2 \cdot \left(e^{\omega \cdot t} - e^{-\omega \cdot t}\right) = 2 \cdot m \cdot a \cdot \omega^2 \cdot \sinh(\omega \cdot t) \tag{15}$$

The time t_0, after which the milled piece is going to leave the interdisc space was calculated from the equation (8) assuming that for $t = t_0$ is $x = r_i$.

Thus the equation was obtained:

$$r_i = \frac{a}{2} \cdot \left(e^{\omega \cdot to} + e^{-\omega \cdot to}\right) \tag{16}$$

After substitution $e^{\omega \cdot to} = u$ as a consequence of simple conversion such an equation was obtained:

$$a \cdot u^2 - 2 \cdot r_i \cdot u + a = 0 \tag{17}$$

which root is

$$u = \frac{r_i + \left(r_i^2 - a^2\right)^{1/2}}{a}$$

Thus the dependence was obtained

$$e^{\omega t_0} = \frac{r_i + \left(r_i^2 - a^2\right)^{1/2}}{a}$$

from the equation we received the searched time t_0 after which a particle will leave the fictitious passage created between the surface of disc and hypothetical walls of milled material:

$$t_o = \frac{1}{\omega} \cdot \ln \frac{r_i + \left(r_i^2 - a^2\right)^{1/2}}{a} \tag{18}$$

Relative velocity w_0 of a milled piece at the moment of leaving of inter–disc space was calculated directly from the equation (14), replacing $t = t_0$:

$$w_o = \frac{a \cdot \omega}{2} \cdot \left(e^{\omega \cdot t_0} - e^{-\omega \cdot t_o}\right) = \omega \cdot \left(r_1^2 - a^2\right)^{1/2} \tag{19}$$

The distance from the rotation axis $OK_0 = a$ is variable for the disc pack, it depends on the radius of hole spacing in the examined discs. A quasi cutting pieces of material force acts on this radius.

The situation is complicated when there are more than one row of holes in the discs. In such a situation it is assumed that the distance OK_0 is related to the furthest – from the rotation centre–milling edge.

Computed example

During computation based on developed analytic model there were used following data:

$\omega = 1, 2, 3, 4, \text{and } 5 \; [\mathrm{rad} \cdot \mathrm{s}^{-1}]$ – angular velocity

$a = 0{,}03 \; [\mathrm{m}]$ – distance

$r_i = 0.06 \div 0.15 \; [\mathrm{m}]$

The equation (19) were used: $w_o = \frac{a \cdot \omega}{2} \cdot \left(e^{\omega \cdot t_0} - e^{-\omega \cdot t_o}\right) = \omega \cdot \left(r_1^2 - a^2\right)^{1/2}$

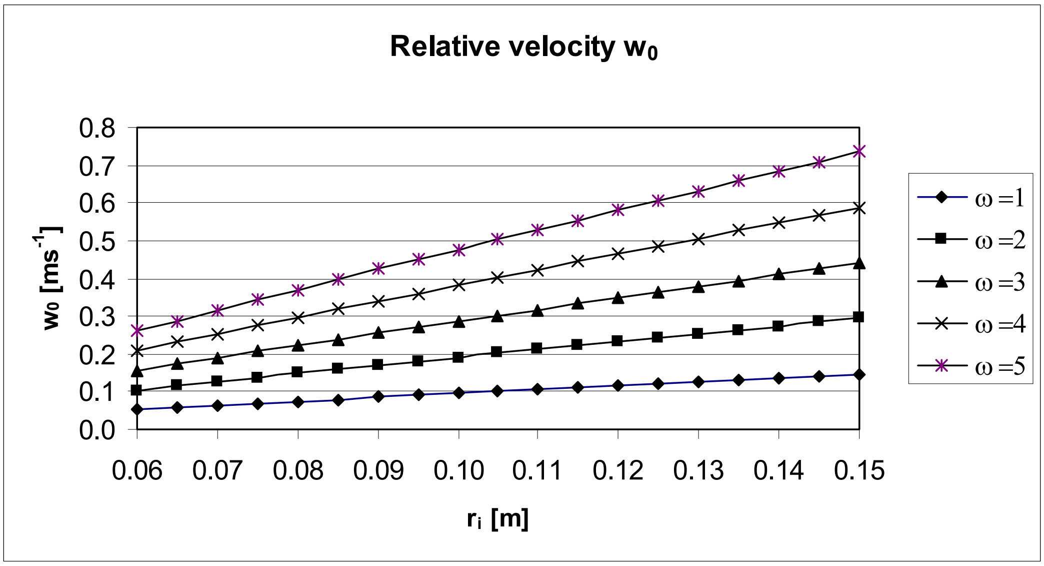

Fig. 2. Computed relative velocity

3. Discussion

The modelling methodology and equations of speed cutting presented in this paper are based upon the results obtained by several researchers over a period of the last years. This methodology represents a unified approach, in which the essential features of previously developed analytic and numerical models are integrated. The new model given by Eqs. (18) and (19) accounts for the following features and phenomena characteristic of milling:

- (i) Tool eccentricity.
- (ii) Insert throw.
- (iii) Uneven insert spacing.
- (iv) Spindle tilt, either steady or variable.
- (v) Spindle error motion.
- (vi) Changes in the effective disc geometry due to machining parameters, type of operation (up- or down-milling) and spindle tilt.
- (vii) Deflections of the machine, disc and workpiece caused by cutting forces.

The proposed modeling methodology results from a systematic approach to the milling process. It facilitates, therefore, an orderly derivation of versatile models, most appropriate for the particular investigated processes. The development of models begins with the selection of coordinate systems shown in Fig. 1. The result is a small number of equations in simple form. The same general form of final equations represents models of varying complexity.

Another important attribute of the employed systematic approach is the separation of various features and phenomena related to disc milling.

The systematic approach underlying the proposed methodology assures the coherence and compatibility of various derived models. Thus, these models can be developed incrementally, by increasing their sophistication until the desirable level of agreement with the actual process, or performance to cost ratio are achieved.

A motivation to develop the presented model came from the research of indirect, on-line tool condition monitoring methods. It was observed that the measured milling forces did not correlate well with forces predicted by the available models. Discrepancies between the measured and predicted forces were particularly strong at small chip thicknesses, i.e., during entries and exits of cutting inserts from the machined material.

The proposed modeling methodology involves three steps. First, the process is systematically decomposed and all involved physical phenomena are arranged as independent. This leads to defining several coordinate systems shown in Fig. 1. A logical consequence of this decomposition is the implementation of homogeneous transformation matrices, as an elegant and efficient means of managing relationships between variables in all involved coordinates. Finally, to enhance the effectiveness of the introduced matrix notation, the information about the tool is formulated in a matrix form. Two essential ingredients of this information, viz. ideal tool geometry and departures from this geometry, are separated to simplify model derivation.

This is apparent in the last part of the paper, in which tool geometry and surface texture are estimated on the basis of in-process measured forces.

4. Acknowledgments

Financial support of this research and advanced laboratory facilities provided by the Ministerstwo Nauki i Informatyzacji (bf. Komitet Badań Naukowych) in years 2002-2004, are gratefully acknowledged.

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