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EXPERIMENTAL EVALUATION OF THE DRAG ROTATION COEFFICIENT OF THE SPHERICAL PARTICLE

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Summary: *Saltatory solid particles conveyed by fluid impact a channel bed from time to time. As a result of the collision the particles receive angular velocity, which gradually decreases with time. For numerical simulation of saltation it is necessary to know values of the drag rotation coefficient. In this paper experimental results of the rotating spherical particles moving in water are described. The rubber spherical balls with density near that of water were used; each of them was speeded up in a special chute that ensured that the particle rotated in a given plane. Standard video system was used to capture particle motion. Values of the drag coefficient of the rotating spherical particle were determined in a dependence on rotation particle Reynolds number.*

1. Introduction

A rotating solid sphere moving freely in a viscous fluid is a subject of interest in many engineering applications. Knowledge of its behavior is also important for the numerical simulation of the saltation process, one of the modes of the particle transport in an open channel. The saltatory movement of particles near the channel bed is often modeled by solid spheres, and it is needful to know the particle behavior, values of forces and of the moment of forces acting on the particle in dependence on the time and particle position.

The drag moment M of a sphere rotating around its diameter D in the fluid depends on the particle angular velocity ω and radius r, density ρ and viscosity ν of the fluid. *Sawatzki (1970)* described the reliable experimental and theoretical data for this case in the dimensionless form. However, the data were obtained for the relatively large scale sphere of diameter D = 240 mm rotating with a constant angular velocity around fixed axis in a viscous boundless fluid, which was motionless in infinity. But the particle conveyed by fluid in a channel has not only angular velocity but also a translational one, which is usually different from a local velocity of the fluid in the channel. Therefore the data must be examined for the case of the rotating particle moving in the fluid with different translational velocity.

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Between two subsequent collisions with the channel bed the saltating particle moves and rotates in a fluid freely suspended. Its translational and angular velocities change slowly hence we can consider the quasi-steady state, assuming that at any time the value of the drag moment reaches the same value as that for the steady case with the same value of the angular velocity. However, the effect of the translation velocity on the value of the drag moment must be studied.

2. Dimensional analysis

Let us consider a rotating spherical particle of radius *r* moving in the fluid with density ρ and with coefficient of kinematics viscosity ν. The absolute value of the particle translational velocity is V and of the particle angular velocity is ω. The angle between the vector of the translational velocity and the direction of the angular velocity vector is α. If M denotes the drag moment which fluid acts on the particle with then it follows from the π-theorem

$$C_\omega = -\frac{M}{\frac{\rho}{2}\omega^2 r^5} = f\left(\mathrm{Re}_\omega, \mathrm{Re}, \alpha\right), \tag{1}$$

where

C_ω - drag rotation coefficient ;

$\mathrm{Re}_\omega = \dfrac{\omega r^2}{\nu}$ - rotational Reynolds number of the particle,

$\mathrm{Re} = \dfrac{Vr}{\nu}$ - translational Reynolds number of the particle.

The aim of the present experimental investigation is a determination of the drag moment M and its dimensionless form – the drag rotation coefficient C_ω as the function $C_\omega = f\left(\mathrm{Re}_\omega, \mathrm{Re}, \alpha\right)$. The data of Sawatzki (1970) correspond to the case of Re = 0. Lamb (1932) proved that also in the case of $\mathrm{Re}_\omega << 1$ the dependence of drag moment on angular velocity is linear

$$M = -8\pi\mu r^3\omega, \tag{2}$$

where $\mu = \nu\rho$ is coefficient of dynamic viscosity.

For saltating particles values of the rotational Reynolds number can vary from a few tens for the sand particles less than 1 mm to a few tens of thousands for the gravel particles with size in the range of centimeters. For this reason the expression (1) supposes the more general quadratic dependence of M on ω.

3. Experimental study

The experiments were carried out in a rectangular glass vessel of dimensions 300x200x800 mm. Water surface was kept on the level 730 mm. The rubber spherical balls were used as model particles, diameter and mass of the balls are shown in Table 1. The hairlines were drawn on the balls along two perimeters of the ball with angle of 90^0 between them to make possible an estimation of particle rotation in the fluid. Each measured particle was speeded up in the special chute that was inclined to water surface at angle from 50 to 60 degrees. This chute ensures particle rotation in a given plane. Different levels of the initial high of the particles in the chute were used to provide different values of the initial translational and angular velocities of the individual particle. Immediately after the particle entries to the water the values of the translational Reynolds number were below $Re < 20\ 000$ and the values of the rotational Reynolds number were $Re_\omega < 40\ 000$.

Motion of sand or gravel particles is too quick and a high-speed video system must be used for its study. However, as it follows from the dimensional analysis, the drag rotation coefficient is independent on the particle density and hence it can be chosen arbitrarily. For this reason the balls density was chosen nearly to that of water what makes possible to visualize the particle motion by a standard video system with image recording at the rate 50 frames per second. For each experiment 150-200 images were obtained. From the images geometric and kinematic properties of the particle motion were found. However, only the parts of the images out of the unsteady entrance region, were used for analysis of the experimental data. Fig. 1 shows an example of the realized experiments.

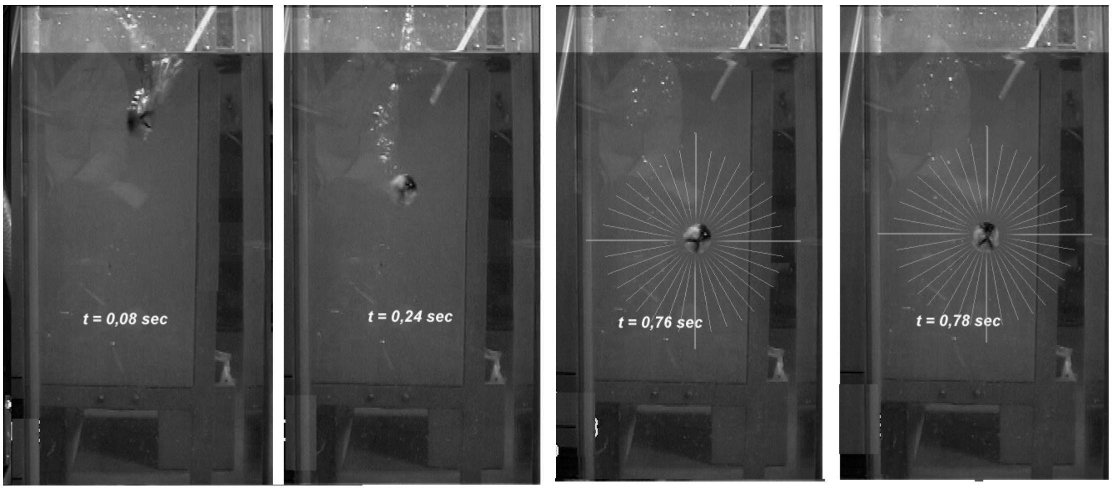

Fig. 1 Images of particle motion

4. Method of analysis

The equation of rotational motion of a solid body is

$$J \frac{d\omega}{dt} = M , \tag{3}$$

where J is the moment of inertia of the body. For a sphere of the mass m the moment of inertia is $J = 0.4mr^2$. The equation (3) allows to find the dynamic parameters of the body if its kinematics parameters are known and vice versa.

Table 1. The balls parameters

Particle No.	diameter d [mm]	mass m [g]	balls density ρ_s [kg /m^3]
1	36.4	25.3	1 000+
2	36.2	24.4	980
3	25.6	8.8	1 000+
4	21.6	5.6	1 060
5	21.5	5.2	1 000
6	21.3	4.8	950
7	14.6	1.6	980

A typical time dependence of the particle angular displacement $\Delta\varphi$ for time interval Δt = 0,02 s between two consecutive video images is illustrated in Fig. 2. The value of the angular velocity ω can be obtained as $\omega = \Delta\varphi / \Delta t$. To solve the equation (3) it is necessary to know also the angular acceleration – i.e. derivation $d\omega / dt$, which is hardly possible to find from the data in Fig. 2 with appropriate accuracy. Thus two different ways were used to analyze the experimental data.

a) From Eqs. (1) and (3) after integration follows

$$\widetilde{C}_\omega = \frac{0.8m}{\rho r^3}\left(\frac{1}{\omega} - \frac{1}{\omega_0}\right)\frac{1}{t - t_0}. \tag{4}$$

Let us now consider that $\widetilde{C}_\omega$ is average value of the drag rotation coefficient C_ω in the time interval from t_0 to t:

$$\widetilde{C}_\omega = \frac{1}{t - t_0}\int_{t_0}^{t} C_\omega dt. \tag{5}$$

If C_ω remains constant over the time interval from t_0 to t then $\widetilde{C}_\omega = C_\omega$. The expression (4) does not contain the derivation of angular velocity ω in time.

b) The dependence of $\Delta\varphi$ on t can be fitted by a polynomial of the fourth order (see the curve 2 in Fig. 2). Since the angular velocity is $\omega = \Delta\varphi / \Delta t$ the particle angular acceleration $d\omega / dt$ can be set as the derivation of angular velocity in time. In this case the drag rotation coefficient C_ω is obtained as a smooth function of time.

As we know the value of the angular particle velocity ω in the given time t, the value of the particle rotational Reynolds number Re_ω can be determined and relationship between C_ω and Re_ω becomes known.

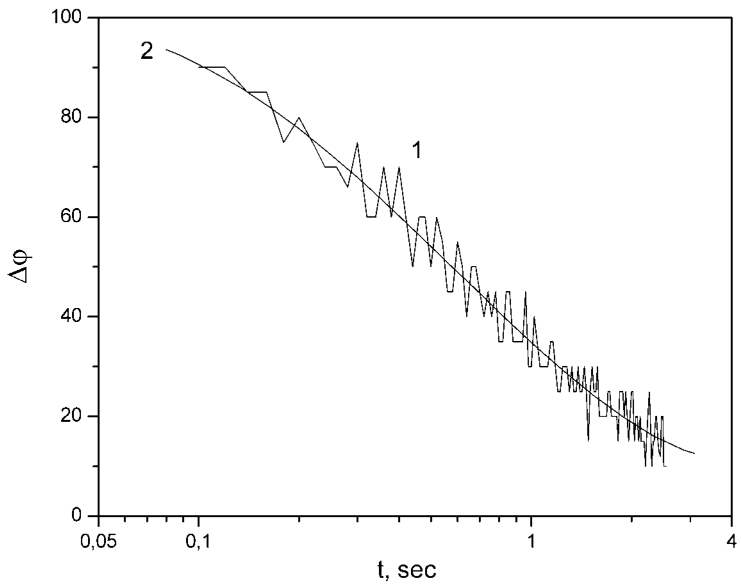

Fig. 2 The dependence of the particle angular displacement $\Delta\varphi$ between two consecutive video images on time *t*. 1 - experimental data; 2 – polynomial approximation.

5. Experimental results

The experimental data were processed using both above mentioned ways of analysis. Results of the processing for the particle No. 2 are shown in Fig. 3. The scattering of the experimental data is large but the fitted curve is near to the results of Sawatzki (1970) especially for relatively large values of the time interval $(t - t_0)$.

Result of the experimental data processing according to the first method of analysis for all measured particles (No. 1- 6) is shown in Fig. 4. The mark 1 corresponds to the data of Sawatzki (1970) for the drag rotation coefficient of the large sphere (D=240 mm) rotating with constant angular velocity in an infinite motionless fluid. Three straight lines approximate his data:

1. $\mathrm{Re}_\omega \approx 10$, $\quad C_\omega \approx \mathrm{Re}_\omega^{-1}$;

2. $1\,000 < \mathrm{Re}_\omega < 40\,000$, $\quad C_\omega \approx \mathrm{Re}_\omega^{-1/2}$;

3. $400\,000 < \mathrm{Re}_\omega < 10^7$, $\quad C_\omega \approx \mathrm{Re}_\omega^{-1/5}$.

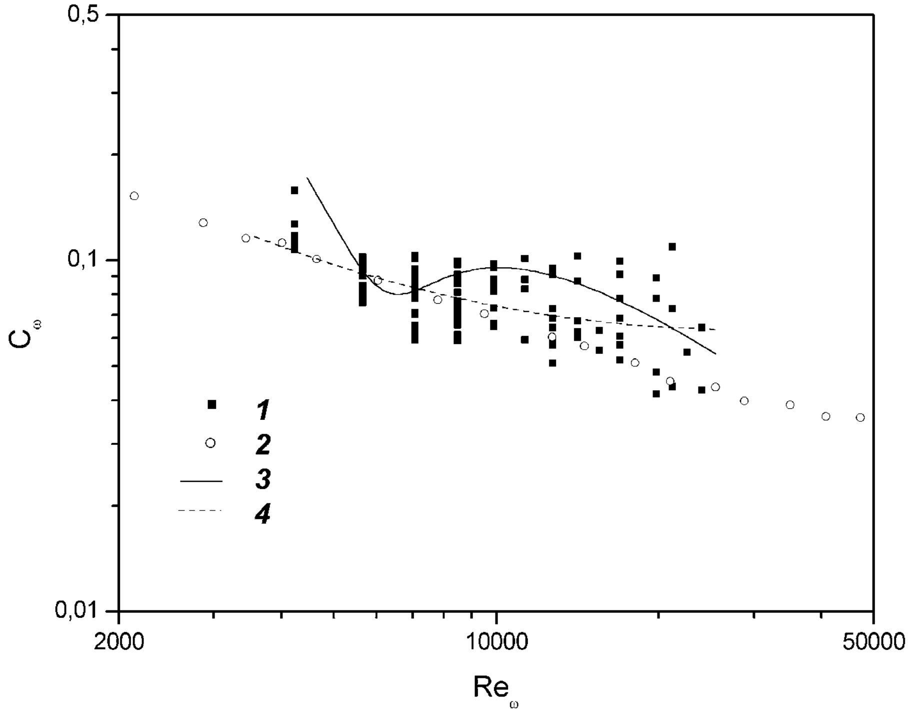

Fig. 3 Variation of the drag rotation coefficient C_ω with Re_ω for particle No. 2
1- method a) of the experimental data analysis; 2 - Sawatzki (1970); 3 – method b) of the experimental data analysis; 4 - curve fitting of the method a) of the experimental data analysis

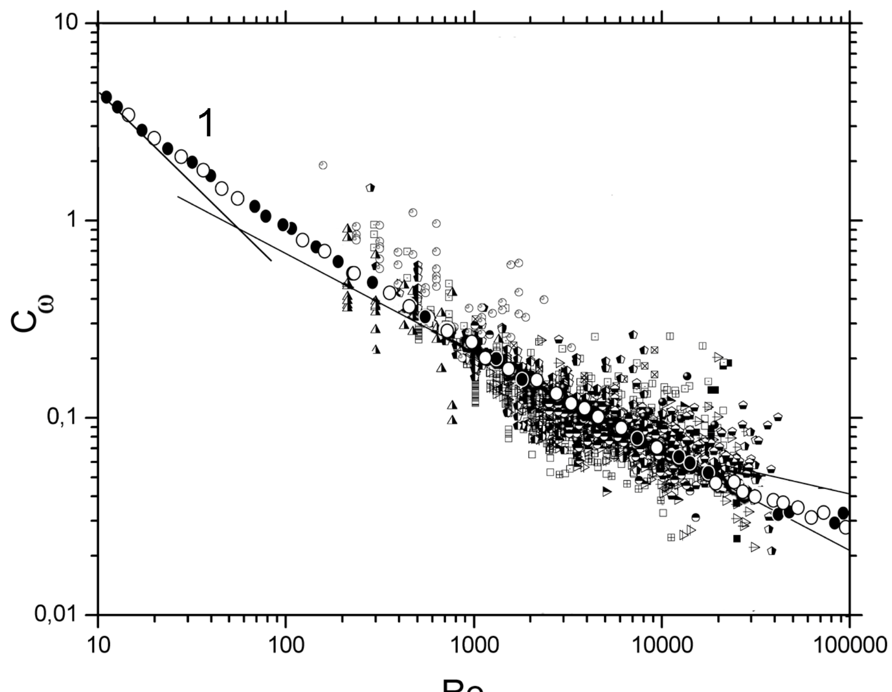

Fig. 4 Dependence of the drag rotation coefficient C_ω on the Reynolds number of the rotating particle Re_ω. Comparison of the experimental data with the data of Sawatzki (lines 1)

The present experiments were performed in the range of the particle rotational Reynolds number Re_ω from 200 to 40 000. A considerable scattering of the experimental data in Fig. 4 in comparison with the results of Sawatzki (1970) is due to the imperfection of the particle shape – actually the particles were not exactly spherical with a smooth surface, but they are closer to the real particles in the natural saltation process. Also the translational movement influences the accuracy of the drag rotation coefficient determination. However, it is shown in Fig. 4 that in the studied range of the translational velocity ($\mathrm{Re}<20\ 000$) the effect of translational motion on the drag rotation coefficient is relatively small. Therefore the data of Sawatzki (1970) for the drag rotation coefficient of the sphere that rotates with constant angular velocity in the motionless fluid can be used for the numerical simulation of the saltation process with a sufficient accuracy.

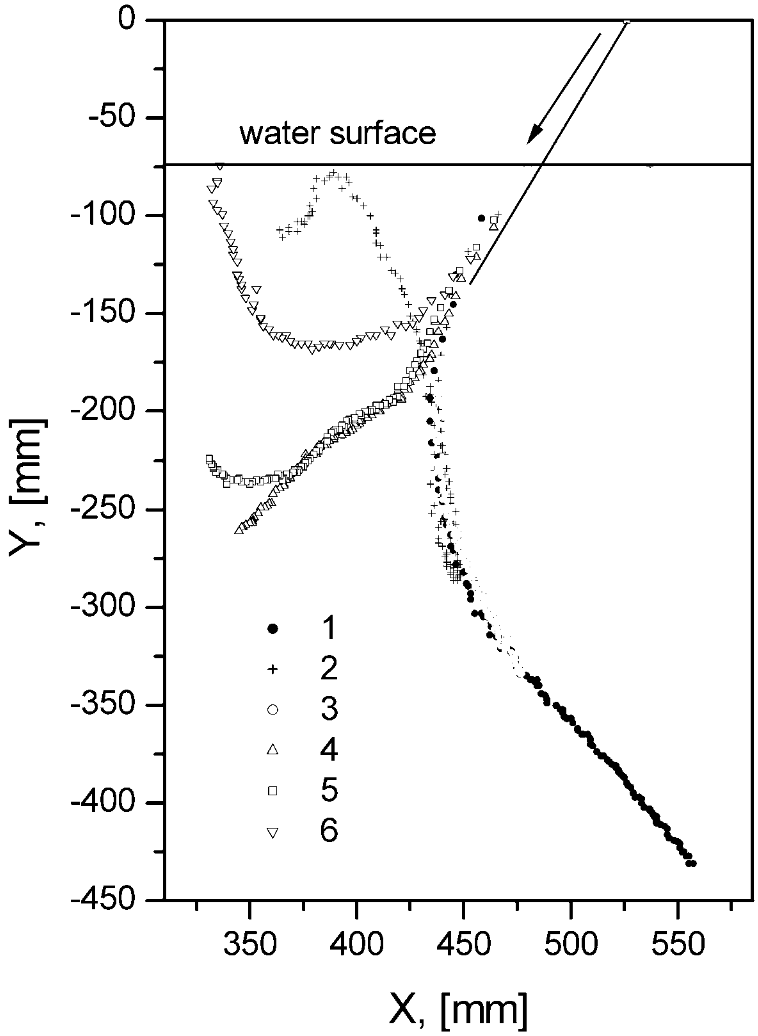

Fig.5 Trajectories of the all measured particles

The trajectories of the particles No. 1 – 6 are shown in Fig. 5. The Magnus force $\overline{F}_M$ acts on the rotating particle in the direction normal to both the translational velocity vector $\overline{V}_R$ and the angular velocity vector $\overline{\varpi}$. It can be expressed from the following equation

$$\overline{F}_M = \rho\, C_M \left(\overline{\varpi} \times \overline{V}_R\right), \tag{6}$$

where C_M is the dimensionless coefficient of Magnus force.

The influence of Magnus force on the particle trajectory is visible for the particles with larger diameter and greater mass (No. 1, 2, and 3). The small particles (No. 4, 5, and 6) lose the most part of the energy in the unsteady entrance region, therefore their translational and angular velocities are small and the effect of the Magnus force is also small.

6. Conclusions

The results of experiments with rotating spherical particles moving in the water are described. The experiments were performed for the rotational Reynolds number Re_ϖ< 40 000 and the translational Reynolds number Re < 20 000. It is shown that the influence of the translational particle motion on the values of the drag rotational coefficient is small in the studied range of Reynolds numbers. Therefore the data of Sawatzki (1970) for the drag rotation coefficient of the large sphere (D = 240 mm) that rotates with constant angular velocity in an infinite motionless fluid can be used for the numerical simulation of the saltation process with a sufficient accuracy.

7. Acknowledgement

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8. Notation

C_ϖ - drag rotation coefficient

C_M - dimensionless coefficient of Magnus force

$\overline{F}_M$ - Magnus force

J - particle moment of inertia

M - drag moment of the rotating particle in fluids

m - particle mass

r - particle radius

Re - translational Reynolds number of the particle

Re_ϖ - rotational Reynolds number of the particle

t - time

t_0 - initial time
Δt - time between two consecutive video images;
$\overline{V}_R$ - particle slip velocity vector (difference between translational velocity of the particle and fluid),
V - absolute value of the particle translational velocity
α - angle between the translational velocity and the angular velocity vectors
$\Delta\varphi$ - particle angular displacement between two consecutive video images
μ - coefficient of dynamic viscosity
ν - coefficient of kinematic viscosity
ρ - density of fluid
ρ_s - density of solids
$\overline{\omega}$ - particle angular velocity vector
ω - absolute value of the particle angular velocity
ω_0 - absolute value of the particle angular velocity in the time $t = t_0$.

9. References

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