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LARGE DEFORMATION ANALYSIS OF INFLATED AIR-SPRING SHELL BY THE FINITE ELEMENT METHOD

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Abstract: *Nonlinear hyperelastic constitutive equations for large deformations of orthotropic composite material are incorporated into the finite strain analysis by FEM. The parameters of material are determined from experiments. Results for the deformation of the inflated air-spring shell made of composite with rubber matrix reinforced by textile cords are given.*

1. Introduction

The intensive research of air-springs behavior is pursued in Laboratory of hydrodynamics at TU of Liberec in order to determine their damping properties. Air-springs form an example of layered multiphase flexible composite structures that consist of rubbery matrix and stiff reinforcement made of textile cords. The high modulus, low elongation cords carry most of the load, and the low modulus, high elongation rubber matrix preserves the integrity of the composite and transfers the load. The primary objective of this type composite is to withstand large deformation and fatigue loading while providing high load carrying capacity.

A phenomenological constitutive model that is capable of predicting the large deformations of composites with rubber matrix has been presented by Marvalova & Urban (2001). The behavior of the rubber matrix was described by Ogden model and the influence of the cords was embodied by the exponential function that was developed by Holzapfel (2000) and applied to the finite strain calculation of a fiber reinforced rubber tube.

The material model verification by juxtaposition of the air-spring deformations measured experimentally and computed numerically by the integration of the system of the basic differential equations of the problem was the subject of the previous paper of the authors Marvalova & Nam (2003).

In the present paper the orthotropic composite model of the material with the rubber matrix reinforced by textile cords is incorporated into the FEM analysis of the large deformations of the inflated cylindrical air-spring. The derivation of the axisymetric membrane element kinematics and of the constitutive matrix is presented. The tangent stiffness matrix and the external force vector are also formulated. The computation was carried out in Matlab. Intermediate stages of inflated membrane and limit points were computed by the combination of modified Newton-Raphson method with load increments controled by the iteration count of previous convergence and by the arc-length method.

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2. Description of structural material and experimental analysis

The experimental investigation of loading characteristics and vibro-isolational properties of air-springs is a part of the research of the Hydrodynamic laboratory of TU Liberec. For this purpose it is necessary to know the material properties.

The sheet of an air-spring is usually made up of four layers – the inner and the outer layer of calandered rubber and the two plies of cord reinforced rubber in which the two families of cords are arranged symmetrically with respect to the circumferential direction and have a specific bias angle (Figure 1). The angle α of fibers is 48.8^{o}.

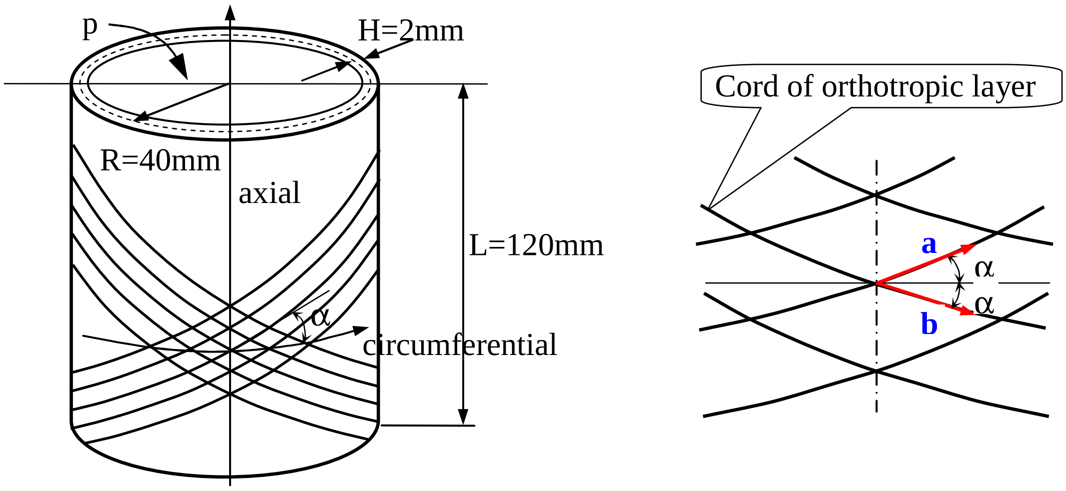

Figure 1. Textile cord reinforced circular tube of cylindrical air-spring. Continuum model for the structure the orthotropic layer (with double-helically arranged fibers).

The experimental tests were carried out at five different positions of the air-spring. First the end plates of the unloaded air-spring were fixed at the distance by 15, 20, 30, 40 or 50 mm shorter than the free height of the sheet. Then the air-spring was loaded and unloaded gradually by step 0.05MPa in the range 0.1–0.5MPa. Photographs of the deformed sheet were recorded by digital camera, the axial force and the inner pressure were measured and stored at every stage of loading. The deformed shape of the air-spring surface was measured from the photographic records through digital image processing techniques and the main geometric features of the inflated membrane were determined (Marvalova & Nam, 2003).

3. Identification of material parameters

The hyperelastic material in this paper is considered incomprresible locally orthotropic composite. Let's assume the isochoric deformation and neglect the dissipation due to irreversible effects. The strain energy function of the orthotropic hyperelastic material can be supposed as the sum of the energy stored in matrix material (isotropic part exprimed by Ogden's model) and in cords (anisotropic part depending on the cord elongation)

$$\Psi(\lambda_1,\lambda_2)=\sum_{i=1}^{3}\frac{\mu_i}{\alpha_i}\left(\lambda_1^{\alpha_i}+\lambda_2^{\alpha_i}+\lambda_1^{-\alpha_i}\lambda_2^{-\alpha_i}-3\right)+\frac{k_1}{k_2}\left\{\exp[k_2(\lambda_2^2\cos^2\alpha+\lambda_1^2\sin^2\alpha-1)^2]-1\right\}, \quad (1)$$

where λ_1 and λ_2 are the axial and circumferential stretches respectively, and α is the angle of the two families of reinforcing fibers. The parameters μ_i and α_i (i = 1, 2, 3) of Ogden's model of rubber (Holzapfel, 2000) are

$$\mu_1 = 630 \text{ kPa}, \mu_2 = 1.2 \text{ kPa}, \mu_3 = -10 \text{ kPa}, \alpha_1 = 1.3, \alpha_2 = 5, \alpha_3 = -2. \quad (2)$$

The stress-like parameter k_1= 4.187e+04 kPa and the non-dimensional parameter k_2= -23.775 are determined from the experimental measurement and from the 2D cylindrical membrane approximation.

4. Deformation analysis of cylindrical membrane by FEM

4.1 The axisymmetric membrane element

The axisymmetric membrane element that has length of L and thickness of H in the reference configuration and length of l in the current configuration is presented in Figure 2. This element has two nodes and the vector of node displacement is $\mathbf{u} = \{u_{R1}, u_{Z1}, u_{R2}, u_{Z2}\}^T$.

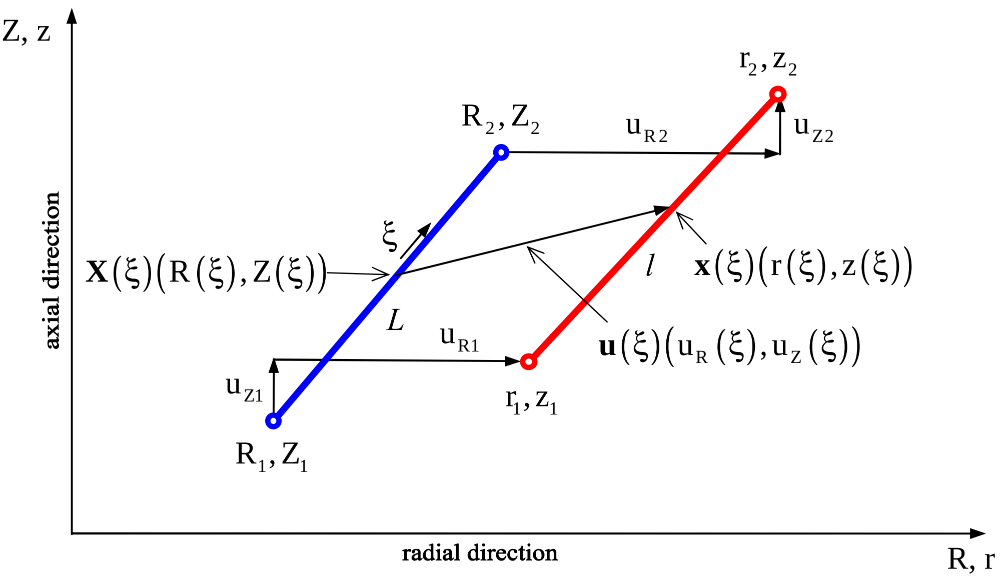

Figure 2. An axisymmetric membrane element

The coordinates in reference and current configuration and displacement vector are interpolated linearly as the functions of the isoparametric coordinate ξ

$$\boldsymbol{X}(\xi) = \mathbf{N} \cdot \{R_1, Z_1, R_2, Z_2\}^T, \quad \boldsymbol{x}(\xi) = \mathbf{N} \cdot \{r_1, z_1, r_2, z_2\}^T, \quad (5)$$

$$\boldsymbol{u}(\xi) = \boldsymbol{x}(\xi) - \boldsymbol{X}(\xi) = \mathbf{N} \cdot \mathbf{u}, \quad \mathbf{u} = \{u_{R1}, u_{Z1}, u_{R2}, u_{Z2}\}^T, \quad (6)$$

where **N** is the matrix of shape functions

$$\mathbf{N} = \begin{bmatrix} \mathbf{N}_1 \\ \mathbf{N}_2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1-\xi & 0 & 1+\xi & 0 \\ 0 & 1-\xi & 0 & 1+\xi \end{bmatrix}. \quad (7)$$

Green-Lagrange (GL) deformation tensor and second Piola-Kirchhoff (PK2) strain tensor are used as the conjugate pair in order to express the strain energy in the Lagrangian description.

The axial Green strain can be found from the change of length element

$$E_{11} = \frac{l^2 - L^2}{2L^2} = (\mathbf{b}_{1l} + \mathbf{b}_{1n}(\mathbf{u})) \cdot \mathbf{u}\,, \tag{8}$$

$$\mathbf{b}_{1l} = \frac{1}{L^2}\{R_1, Z_1, R_2, Z_2\} \cdot \mathbf{A}, \quad \mathbf{b}_{1n}(\mathbf{u}) = \frac{1}{2L^2}\mathbf{u}^T \cdot \mathbf{A}, \qquad \mathbf{A} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \tag{9}$$

The hoop component of GL is determined from the change of the circumference length:

$$E_{22} = \frac{o^2 - O^2}{2O^2} = \frac{u_R(\xi)}{R(\xi)} + \frac{1}{2}\left(\frac{u_R(\xi)}{R(\xi)}\right)^2 = e_l + \frac{1}{2}e_l^{\,2}, \quad e_l = \frac{1}{R(\xi)}\mathbf{N}_1 \cdot \mathbf{u}, \tag{10}$$

The variation $\delta\mathbf{E}$ can be expressed by variation $\delta\mathbf{u}$ in the following form

$$\delta\mathbf{E} = \begin{Bmatrix} \delta E_{11} \\ \delta E_{22} \end{Bmatrix} = \mathbf{B}_{nl} \cdot \delta\mathbf{u}, \;\; \mathbf{B}_{nl} = \begin{bmatrix} \mathbf{B}_{nl1} \\ \mathbf{B}_{nl2} \end{bmatrix} = \begin{bmatrix} \frac{1}{L^2}(\{R_1, Z_1, R_2, Z_2\} + \mathbf{u}^T) \cdot \mathbf{A} \\ \left(1 + \frac{1}{R(\xi)}\mathbf{N}_1 \cdot \mathbf{u}\right)\frac{1}{R(\xi)}\mathbf{N}_1 \end{bmatrix} \tag{11}$$

The components of PK2 stress tensor can be deduced from strain energy function according to the relation

$$\mathrm{S}_{\mathrm{ii}} = \frac{1}{\lambda_{\mathrm{i}}}\frac{\partial \Psi}{\partial \lambda_{\mathrm{i}}}, \quad \mathrm{i} = 1, 2 \tag{12}$$

Furthermore, the components E_{11} a E_{22} of GL can be obtained in the term of principal stretches as

$$E_{ii} = \frac{\lambda_i^{\,2} - 1}{2} \Rightarrow \frac{\partial \lambda_i}{\partial E_{ii}} = \frac{1}{\lambda_i}, \quad i = 1, 2 \tag{13}$$

The elastic tensor $\mathbb{C}$ is determined from principal components of PK2

$$\mathbb{C} = \frac{\partial \mathbf{S}}{\partial \mathbf{E}}, \quad \mathbb{C} = \begin{bmatrix} \frac{1}{\lambda_1}\frac{\partial S_{11}}{\partial \lambda_1} & \frac{1}{\lambda_2}\frac{\partial S_{11}}{\partial \lambda_2} \\ \frac{1}{\lambda_1}\frac{\partial S_{22}}{\partial \lambda_1} & \frac{1}{\lambda_2}\frac{\partial S_{22}}{\partial \lambda_2} \end{bmatrix}, \quad \mathbf{S} = \mathbb{C}\mathbf{E}\,. \tag{14}$$

$$\mathbb{C}_{11} = \frac{1}{\lambda_1^4}\sum_{a=1}^{3}\mu_a\left[(\alpha_a - 2)\lambda_1^{\alpha_a} + (\alpha_a + 2)(\lambda_1\lambda_2)^{-\alpha_a}\right] + 8k_1\exp\left(k_2 m^2\right)\sin^4\gamma\left(1 + 2k_2 m^2\right)$$

$$\mathbb{C}_{12} = \mathbb{C}_{21} = \frac{1}{\lambda_1^2\lambda_2^2}\sum_{a=1}^{3}\mu_a\alpha_a(\lambda_1\lambda_2)^{-\alpha_a} + 8k_1\exp\left(k_2 m^2\right)\sin^2\gamma\cos^2\gamma\left(1 + 2k_2 m^2\right)$$

$$\mathbb{C}_{22} = \frac{1}{\lambda_2^4}\sum_{a=1}^{3}\mu_a\left[(\alpha_a - 2)\lambda_2^{\alpha_a} + (\alpha_a + 2)(\lambda_1\lambda_2)^{-\alpha_a}\right] + 8k_1\exp\left(k_2 m^2\right)\cos^4\gamma\left(1 + 2k_2 m^2\right)$$

$$m = \lambda_2^2\cos^2\gamma + \lambda_1^2\sin^2\gamma - 1$$

Note that the elasticity tensor is not constant, but depends on the deformations and then must be updated in every iterative step.

4.2 Principle of virtual work and its linearization

Principle of virtual work in given problem can be written in the following form:

$$R(\mathbf{u},\delta\mathbf{u},p)=\int_{\Omega_0}\boldsymbol{S}:\delta\boldsymbol{E}\,dV-p\int_{\partial\Omega}\mathbf{n}^{\mathrm{T}}\cdot\delta\mathbf{u}\,ds=0\,, \tag{15}$$

in which p is the internal pressure, $\delta\mathbf{u}$ stands for a virtual displacement vector, Ω_0 is undeformed volume, $\partial\Omega$ represents the deformed membrane surface and $\mathbf{n}$ is the normal vector of deformed membrane surface.

The principle of virtual work can be expressed through introduction of the external load factor λ as

$$R(\mathbf{u},\delta\mathbf{u},\lambda)=\mathbf{R}(\mathbf{u},\lambda)^{T}\cdot\delta\mathbf{u}=(\mathbf{f}_{\mathbf{int}}(\mathbf{u})-\lambda\mathbf{f}_{\mathbf{ext}}(\mathbf{u}))^{T}\cdot\delta\mathbf{u}\,, \tag{16}$$

In this equation $\mathbf{u}$ is the nodal displacement vector (6), $\mathbf{R}(\mathbf{u},\lambda)$ represents the out-of-balance force which must be equal to zero to ensure equilibrium. Moreover $\mathbf{R}(\mathbf{u},\lambda)$ can be written under the following form

$$\mathbf{R}(\mathbf{u},\lambda)=\mathbf{f}_{\mathbf{int}}(\mathbf{u})-\lambda\mathbf{f}_{\mathbf{ext}}(\mathbf{u}) \tag{17}$$

The internal force vector of element can be introduced from the first integral of principle of virtual work (15) by replacing $\delta\mathbf{E}$ from (11) and $\mathrm{d}V=2\pi\mathrm{R}(\xi)\mathrm{H}\,\mathrm{L}\,\mathrm{d}\xi$

$$\mathbf{f}_{\mathbf{int}}{}^{\mathrm{e}}=\int_{-1}^{1}\mathbf{B}_{\mathrm{nl}}^{\mathrm{T}}\mathbf{S}\,2\pi\mathrm{R}(\xi)\mathrm{H}\,\mathrm{L}\,\mathrm{d}\xi,\quad , \tag{18}$$

Similarly the external force vector of element can be introduced from the second integral of principle of virtual work (15) by replacing $\mathrm{d}s=2\pi r(\xi)\,l\,d\xi$

$$\mathbf{f}_{\mathbf{ext}}{}^{\mathrm{e}}=\int_{-1}^{1}\mathbf{N}^{\mathrm{T}}\cdot\mathbf{n}(\xi)2\pi r(\xi)\,l\,d\xi,\quad \mathbf{n}(\xi)=\frac{1}{l}\begin{Bmatrix} z_2-z_1\\ -(r_2-r_1)\end{Bmatrix}, \\ r(\xi)=\mathbf{N}_1\cdot\{r_1,\mathrm{z}_1,r_2,\mathrm{z}_2\}^{\mathrm{T}} \tag{19}$$

Now a system of nonlinear equation is obtained in the following form

$$\mathbf{f}_{\mathbf{int}}(\mathbf{u})-\lambda\,\mathbf{f}_{\mathbf{ext}}(\mathbf{u})=\mathbf{0}\,, \tag{20}$$

In order to solve the nonlinear equations (20), the classical tangent stiffness matrix $\mathbf{K_t}$ has to be defined. The stiffness matrix $\mathbf{K_t}$ is expressed in the following form

$$\mathbf{K_t}=\left[\frac{\partial\mathbf{f}_{\mathbf{int}}}{\partial\mathbf{u}}-\lambda\frac{\partial\mathbf{f}_{\mathbf{ext}}}{\partial\mathbf{u}}\right] \tag{21}$$

By the derivative of internal force vector (18) with respect to the displacements, it follows that

$$\frac{\partial\mathbf{f}_{\mathbf{int}}{}^{e}}{\partial\mathbf{u}}=\mathbf{K_G}{}^{e}+\mathbf{K_M}{}^{e}\,, \tag{22}$$

$$\mathbf{K_G}^e = \int_{-1}^{1} \mathbf{B}_{nl}^T \, \mathbb{C} \mathbf{B}_{nl} \; 2\pi R(\xi) H \, L \, d\xi, \tag{23}$$

$$\mathbf{K_M}^e = \int_{-1}^{1} \left(\frac{S_{11}}{L^2} \mathbf{A} + \frac{S_{22}}{R^2(\xi)} \mathbf{N}_1^T \, \mathbf{N}_1 \right) 2\pi R(\xi) H \, L \, d\xi \tag{24}$$

in which $\mathbf{K_G}^e$ is the standard stiffness matrix obtained from the first term of (22) and $\mathbf{K_M}^e$ is the geometric or initial stress stiffness matrix, derived from the second term of (22).

Similarly by the derivative of external force vector (19) with respect to the displacements will be obtained the following matrix

$$\mathbf{K_p}^e = \frac{\partial \mathbf{f_{ext}}^e}{\partial \mathbf{u}} = \frac{2\pi}{3} \begin{bmatrix} 2\mathrm{z}_{21} & -(2r_1 + r_2) & \mathrm{z}_{21} & 2r_1 + r_2 \\ 4r_1 - r_2 & 0 & -(r_1 + 2r_2) & 0 \\ \mathrm{z}_{21} & -(r_1 + 2r_2) & 2\mathrm{z}_{21} & r_1 + 2r_2 \\ 2r_1 + r_2 & 0 & r_1 - 4r_2 & 0 \end{bmatrix}, \tag{25}$$

$$\mathrm{z}_{21} = z_2 - z_2, \quad \{r_1, z_1, r_2, z_2\} = \{R_1, Z_1, R_2, Z_2\} + \{u_{R1}, u_{Z1}, u_{R2}, u_{Z2}\}$$

The total stiffness matrix for one element is re-expressed as:

$$\mathbf{K_t}^e = \mathbf{K_G}^e + \mathbf{K_M}^e - \lambda_n \mathbf{K_p}^e . \tag{26}$$

5. Nonlinear numerical solution

The basic approach to solve the nonlinear responses is the incremental-iterative method or the continuation method, also called path-following method determining the equilibrium points on the load (pressure)-displacement paths or equilibrium paths. The incremental-iterative approach described here is based on a combination of the modified Newton-Raphson iterative method and the arc-length method. The tangent stiffness matrix and internal force vector and external force vector are updated not only at the commencement of every iterative cycle, but also at each load step (Figure 3).

5.1 Predictor and corrector solutions

Consider a particular equilibrium point at an instant $\mathrm{t_n}$ on the equilibrium path which is defined by the nodal displacement $\mathbf{u}_\mathrm{n}$ and load factor λ_n. The purpose of the numerical solution is to find one new equilibrium point on the path. The new point at subsequent instant $\mathrm{t_{n+1}}$ is defined by displacement and load factor increments denoted $\Delta\mathbf{u}$ and $\Delta\lambda$, respectively, and it satisfies simultaneously the following equations

$$\begin{cases} \mathbf{R}\left(\mathbf{u}_\mathrm{n} + \Delta\mathbf{u}, \lambda_\mathrm{n} + \Delta\lambda\right) = 0 \\ \mathrm{F}\left(\Delta\mathbf{u}, \Delta\lambda\right) = 0 \end{cases} \tag{27}$$

The first equation is the residual equation and the second one is extra equation which is normally called arc-length constraint associated with path-following procedures. For the more widely used cylindrical arc-length method, the constraint equation is given in the following form (Crisfield, 1997; De Souza Neto and Feng, 1999)

$$\Delta \mathbf{u}^T \Delta \mathbf{u} = ds^2 \tag{28}$$

where ds is arc-length which is the control parameter.

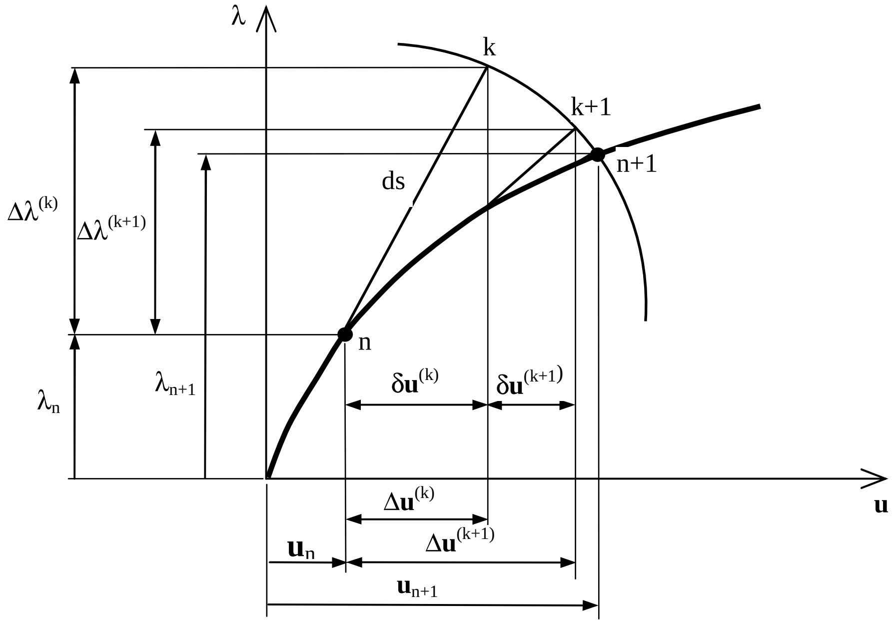

Figure 3. The combination of modified Newton-Raphson and arc-length method

Here the Newton-Raphson iterative method is applied to the previous system (27) is now examined. Consider the algorithm at a given iteration, the system to be solved is

$$\begin{cases} \mathbf{R}^{(k+1)} = \mathbf{R}^{(k)} + \dfrac{\partial \mathbf{R}}{\partial \Delta \mathbf{u}} \delta \mathbf{u} + \dfrac{\partial \mathbf{R}}{\partial \Delta \lambda} \delta \lambda = 0 \\ F^{(k+1)} = F^{(k)} + \dfrac{\partial A}{\partial \Delta \mathbf{u}} \delta \mathbf{u} + \dfrac{\partial A}{\partial \Delta \lambda} \delta \lambda = 0 \end{cases} \tag{29}$$

where subscripts k+1 and k, respectively, are for the current iteration and for the previous iteration. For the solution displacement vector is obtained as

$$\mathbf{u}_{n+1}^{(k+1)} = \mathbf{u}_n + \Delta \mathbf{u}^{(k+1)} \tag{30}$$

$$\Delta \mathbf{u}^{(k+1)} = \Delta \mathbf{u}^{(k)} + \delta \mathbf{u} \tag{31}$$

where $\Delta \mathbf{u}^{(k)}$ and $\Delta \mathbf{u}^{(k+1)}$ are the incremental displacement guesses for the previous and current iteration, respectively, and $\delta \mathbf{u}$ is the iteractive displacement. The incremental load factor, $\Delta \lambda$, is updated according to

$$\Delta \lambda^{(k+1)} = \Delta \lambda^{(k)} + \delta \lambda \tag{32}$$

Using the previous definition of the tangent stiffness matrix (21) the system (29) becomes

$$\begin{cases} \mathbf{R}^{(k+1)} = \mathbf{R}^{(k)} + \mathbf{K}_t \left(\mathbf{u}^{(k)} \right) \delta \mathbf{u} - \delta \lambda \mathbf{f}_{ext} \left(\mathbf{u}^{(k)} \right) = 0 \\ F^{(k+1)} = F^{(k)} + 2 \Delta \mathbf{u}^{(k)} . \delta \mathbf{u} = 0 \end{cases} \tag{33}$$

The system of nonlinear equations for iterative solutions $\delta\mathbf{u}$ and $\delta\lambda$ (the corrector solution) is derived by simply linearising the residual equation and cylindrical arc-length equation using Taylor series. The system of equations (33) is written as

$$\begin{bmatrix} \mathbf{K}_\mathbf{t}\left(\mathbf{u}^{(k)}\right) & -\mathbf{f}_{ext}\left(\mathbf{u}^{(k)}\right) \\ 2\Delta\mathbf{u}^{(k)T} & 0 \end{bmatrix} \begin{Bmatrix} \delta\mathbf{u} \\ \delta\lambda \end{Bmatrix} = -\begin{Bmatrix} \mathrm{R}\left(\mathbf{u}^{(k)}, \Delta\lambda^{(k)}\right) \\ \Delta\mathbf{u}^{T}\Delta\mathbf{u} - \mathrm{ds}^2 \end{Bmatrix} \tag{34}$$

where the subscript n+1 have been abandoned for notational convenience.

When k=0 (the predictor solution), the forward-Euler tangential predictor solution is adopted (Verron and Marckmann, 2001). The two predicted increments $\Delta\bar{\mathbf{u}}$ and $\Delta\bar{\lambda}$ are supposed to satisfy the first equilibrium equation of (33) with $\mathbf{R}^{(0)} = 0$

$$\Delta\bar{\mathbf{u}} = \Delta\bar{\lambda}\ \delta\bar{\mathbf{u}}. \tag{35}$$

where tangent displacement vector $\delta\bar{\mathbf{u}}$ is given by

$$\delta\bar{\mathbf{u}} = \mathbf{K}_t^{-1}\,\mathbf{f}_{ext}. \tag{36}$$

The possible iteractive load factor $\delta\lambda$ for the predictor solution is

$$\delta\lambda = \pm\frac{\mathrm{ds}}{\sqrt{\delta\bar{\mathbf{u}}^T\delta\bar{\mathbf{u}}}}. \tag{37}$$

and the success of the path-following technique depends crucially on the choice of the appropriate sign for iterative load factor.

In order to predict the continuation direction the sign of predictor load factor must be chossen. From the criterion of Feng et al the sign of the predictor load factor is made to coincide with the sign of the internal product between the previous converged incremental displacement, $\Delta\mathbf{u}_n$, and the current tangential solution, $\delta\bar{\mathbf{u}}$:

$$\mathrm{sign}(\delta\lambda) = \mathrm{sign}\left(\Delta\mathbf{u}_n^T\delta\bar{\mathbf{u}}\right). \tag{38}$$

5.2 Step-length control

For controlling the step length size the arc-length for use in the current increment (n+1) can be computed using the arc-length of previous increment (n) by

$$\mathrm{ds}_{n+1} = \mathrm{ds}_n\left(\frac{J_d}{J_n}\right)^{\gamma} \tag{39}$$

in which ds_n is arc-length for previous load step n, J_n is actual number of iterations required for convergence in the previous load step, J_d is a user-defined desired number of iterations for convergence, typically 3 to 5. The exponent γ usually lies in the range 0.5 to 1.0.

5.3 Convergence Criteria

A some convergence criterion based on the incremental displacements or energy are presented (Clarke and Hancock, 1990). The residual convergence criteria is introduced here. The Euclidean norm of residual is compared with some predefined tolerance:

$$\left\|\mathbf{R}\left(\mathbf{u},\lambda\right)\right\| \le \varepsilon_r \tag{40}$$

where typically ε_r is in the range 10^{-2} to 10^{-5}, depending on the desired accuracy and the non-linear charateristics of the particular problem.

6. Numerical results

Numerical results are obtained after simulate the inflation of cylindrical membrane by predictor-corrector method using Matlab software. The equilibrium paths are obtained to overcome the limit points (Figure 5 and Figure 6).

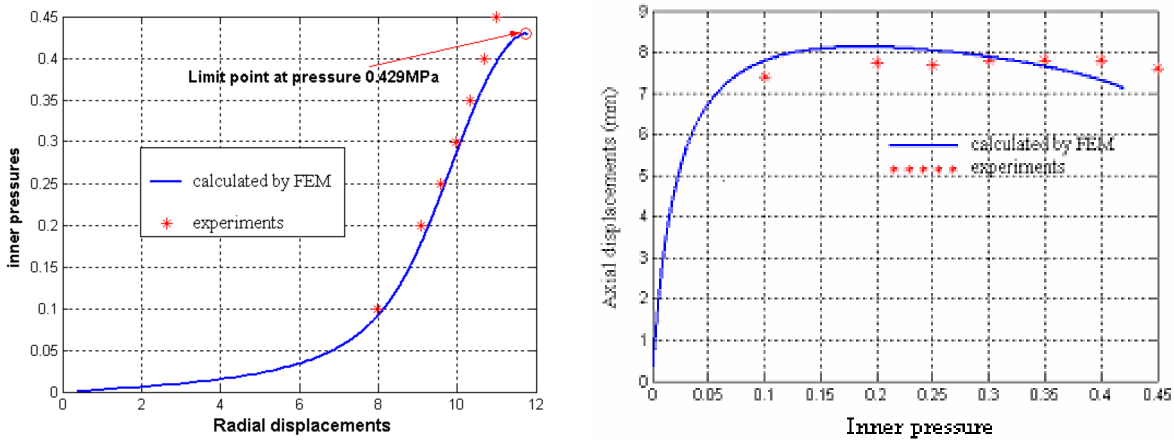

Figure 4. Equilibrium paths in the case of free heads

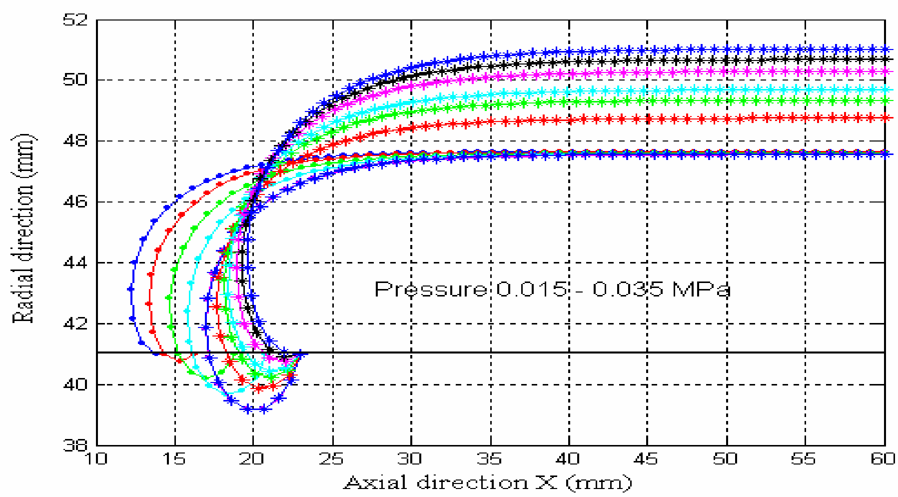

Figure 5. Profiles of deformed membrane at different stages of loading

7. Conclusions

The deformation of the nonlinear composite membrane was determined experimentally. The problem of the identification of the material parameters was solved. The deformation fields were determined by FEM in Matlab. Limit point is detected in the equilibrium path (following path) by using combination of modified Newton-Raphson and arc-length methods in FEA. Numerical results obtained correspond to the experimentally measured deformation of the inflated cylindrical air-spring.

8. Acknowledgement

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