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ABOUT STABILITY AN AEROELASTIC SYSTEM

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Summery: *Natural frequencies and the thresholds for loosing the stability of thin-walled cylindrical shell conveying by flowing fluid are theoretically studied. Potential flow theory for fluid and semi-membrane (half bending) theory of shells are used. The shells of finite length are considered for different cases of boundary conditions at the edges of the shell. Arising of the aerodynamic damping in the studied aeroelastic system due to the flowing fluid, and properties of adjoint solution (i.e. the solution corresponded to the opposite orientation of the boundary conditions of the cylinder) are showed. Obtained values of the derivatives of aerodynamic damping (over flow velocity U) in some analytical form are compared with numerical results published before within the used semi-membrane theory of shells and within the 3D shell theory.*

1. Semi-membrane (half bending) theory of shells

Semi-membrane (half bending) theory of shells is a simpler theory for vibration of the thin shells ([11], [12], [14], [15]) and it may be used in many technical applications enabling more lucid solution of the coupled shell-fluid problems. Equation of motion for the cylindrical shell may be written as

$$\frac{Eh^3}{12\left(1-\nu^2\right)}\frac{1}{R^8\partial\theta^4}\left(\frac{\partial^2}{\partial\theta^2}+1\right)^2 w+\frac{Eh}{R^6}\frac{\partial^4 w}{\partial s^4}-\frac{P_1}{R^5}\frac{\partial^6 w}{\partial\theta^6}+$$
$$+\frac{\rho_s h}{R^4}\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2}{\partial s^2}+\frac{\partial^2}{\partial\theta^2}\right)^2 w=\frac{1}{R^4}\frac{\partial^4\tilde{p}(w)}{\partial\theta^4}, \tag{1}$$

where h, R, L, ρ_s, E and ν are the thickness, radius, length, density, Young's modulus and Poisson's ratio of the shell; P_1 is the external static pressure causing a static pre-stress of the shell; w is the radial displacement of the middle surface of the shell, and $\tilde{p}(w)$ is the perturbation pressure exerted by the internal flows on the shell and t is time; θ and s are dimensionless cylindrical coordinates: $\theta\in[0,2\pi]$, $s=x/R\in[1,l=L/R]$.

This equation of motion can be used for studying the lowest natural frequencies of thin cylindrical shells of medium length L, when the radial motion of the shell is predominant and the modes of vibration are associated with higher circumferential wavenumbers, $n\geq 2$ (see, for example, Zolotarev & Popov [18]).

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Two conditions on each edge ($s = 0$ and $s = L$) of the shell must be satisfied:

a) for clamped edge

$$w = \frac{\partial w}{\partial s} = 0\,; \tag{2}$$

b) for simply supported edge

$$w = \frac{\partial^2 w}{\partial s^2} = 0\,; \tag{3}$$

c) for free edge

$$\left[\frac{\partial^2}{\partial s^2} + \nu\left(\frac{\partial^2}{\partial \theta^2} + 1\right)\right] w = 0;\ \left[\frac{\partial^3}{\partial s^3} + (2-\nu)\frac{\partial}{\partial s}\left(\frac{\partial^2}{\partial \theta^2} + 1\right)\right] w = 0\,. \tag{4}$$

2. Equations for potential flow of ideal uncompressible fluid

On the right-hand side of Eq. (1) is the unsteady (perturbation) pressure $\tilde{p}(w)$ originated from the flowing fluid with the density ρ_t, mean flow velocity U_i and speed of sound c_0. The pressure is determined by solving the equations for the potential flow of an ideal, generally compressible fluid [17,19]:

$$\frac{1}{\xi}\frac{\partial}{\partial \xi}\left(\xi\frac{\partial \Phi}{\partial \xi}\right) + \frac{1}{\xi^2}\frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\partial^2 \Phi}{\partial s^2} - \frac{1}{c^2}\left(\frac{\partial}{\partial \tau} + U\frac{\partial}{\partial s}\right)^2 \Phi = 0\,, \tag{5}$$

$$p(w) = -k_n\left(\frac{\partial \Phi}{\partial \tau} + U\frac{\partial \Phi}{\partial s}\right)\bigg|_{\xi=1}\,, \tag{6}$$

with the impermeability condition on the vibrating surface of the shell

$$\frac{\partial \Phi}{\partial \xi}\bigg|_{\xi=1} = \left(\frac{\partial w}{\partial \tau} + U\frac{\partial w}{\partial s}\right), \tag{7}$$

where the following dimensionless quantities were introduced:

$$\tau = t\sqrt{E/\rho_s R^2};\ k_n = (1-\nu^2)R\rho_t/h\rho_s,\quad p(w) = \tilde{p}(w)(1-\nu^2)R/Eh\,,\ \xi = r/R,$$

$$\rho = \frac{\rho_t}{\rho_s},\ P_1 = p_1/E\,,\ U = U_i\sqrt{\rho_s/E},\quad c = c_0\sqrt{\rho_s/E}\,.$$

3. Method of solution

The solution of the Eq. (1) for the radial displacement w of the cylindrical shell is assumed as:

$$w = W\, \mathrm{e}^{i(\Omega\tau - \alpha s - n\Theta)}, \tag{8}$$

where $\Omega = \omega R\sqrt{\rho_s/E}$ is the dimensionless frequency of the disturbance; α is the axial wave number and n is the circumferential wave number.

From the potential flow theory for inviscid incompressible fluid [15] the perturbation pressure p can be written as

$$p = -k_n \frac{I_n(\alpha)}{\alpha I_n^{'}(\alpha)}\left(\frac{\partial}{\partial \tau} + U\frac{\partial}{\partial s}\right) w\,, \tag{9}$$

where I_n is the Bessel function in usual notation and $I_n^{'} = dI_n / d\,\xi$.

Assuming long waves in the axial direction and using the approximation of the Bessel functions for small argument [38] we can write

$$\frac{I_n(\alpha)}{\alpha I_n^{'}(\alpha)} \approx \frac{1}{n}\left[1 - \frac{\alpha^2}{2n(n+1)}\right]. \tag{10}$$

Substituting Eqs. (8), (9) and (10) into Eq. (1) will yield a characteristic equation as the algebraic equation for the axial wave number α:

$$\begin{aligned} F(\alpha,U,\Omega) = {} & \alpha^4\left[(1-\Omega^2)\frac{h}{R} + \rho U^2\frac{n^2}{2(n+1)}\right] - \alpha^3\left[\rho U\Omega\frac{n^2}{(n+1)}\right] + \\ & -\alpha^2\left[2\Omega^2 n^2\frac{h}{R} + \rho n^3\left(U^2 - \frac{\Omega^2}{2n(n+1)}\right)\right] + \alpha\left[2\rho U\Omega n^3\right] + \\ & + \frac{n^4(n^2-1)^2}{12(1-\nu^2)}\left(\frac{h}{R}\right)^3 - P_1 n^6 - \Omega^2 n^3\left(\rho + n\frac{h}{R}\right) = 0\,. \end{aligned} \tag{11}$$

For a given n the solution of the Eq. (11) is assumed as a linear combination of four waves propagating in the longitudinal direction:

$$w_n = \cos(n\theta)\sum_{j=1}^{4} W_{n\,j}\,\mathrm{e}^{-i\,\alpha_j s}\,, \tag{12}$$

where the coefficients $W_{n\,j}$ must be chosen so that they satisfy two boundary conditions at each edge (s = 0 and s = L) of the shell and the wave numbers α_j, (j = 1,…,4) are given by the roots of the Eq. (11).

The wave numbers α_j must satisfy, for example, next boundary conditions:

a) for the supported (s = 0) - supported (s = L) edges

$$\det\begin{bmatrix} 1 & 1 & 1 & 1 \\ \alpha_1^2 & \alpha_2^2 & \alpha_3^2 & \alpha_4^2 \\ e^{-iL\alpha_1} & e^{-iL\alpha_2} & e^{-iL\alpha_3} & e^{-iL\alpha_4} \\ \alpha_1^2 e^{-iL\alpha_1} & \alpha_2^2 e^{-iL\alpha_2} & \alpha_3^2 e^{-iL\alpha_3} & \alpha_4^2 e^{-iL\alpha_4} \end{bmatrix} = 0, \tag{13}$$

b) for the clamped ($s = 0$) - support ($s = L$) edges

$$\det\begin{bmatrix} 1 & 1 & 1 & 1 \\ \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ e^{-iL\alpha_1} & e^{-iL\alpha_2} & e^{-iL\alpha_3} & e^{-iL\alpha_4} \\ \alpha_1^2 e^{-iL\alpha_1} & \alpha_2^2 e^{-iL\alpha_2} & \alpha_3^2 e^{-iL\alpha_3} & \alpha_4^2 e^{-iL\alpha_4} \end{bmatrix} = 0, \tag{14}$$

c) for the clamped ($s = 0$) - clamped ($s = L$) edges

$$\det\begin{bmatrix} 1 & 1 & 1 & 1 \\ \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ e^{-iL\alpha_1} & e^{-iL\alpha_2} & e^{-iL\alpha_3} & e^{-iL\alpha_4} \\ \alpha_1 e^{-iL\alpha_1} & \alpha_2 e^{-iL\alpha_2} & \alpha_3 e^{-iL\alpha_3} & \alpha_4 e^{-iL\alpha_4} \end{bmatrix} = 0, \tag{15}$$

d) for the clamped ($s = 0$) - free ($s = L$) edges

$$\det\begin{bmatrix} 1 & 1 & 1 & 1 \\ \cdots & \alpha_j & \cdots & \cdots \\ \cdots & (\alpha_j^2 + \nu(n^2+1))\,e^{-iL\alpha_j} & \cdots & \cdots \\ \cdots & \alpha_j(\alpha_j^2 + (2-\nu)(n^2+1))\,e^{-iL\alpha_j} & \cdots & \cdots \end{bmatrix} = 0, \tag{16}$$

The determination of the complex natural frequencies $\Omega(U)$ is reduced to the problem of finding zeros of the corresponded function $G(\alpha_1,\alpha_2,\alpha_3,\alpha_4) = \det$.

Eq. (11) it is possible obtain

$$\frac{\mathrm{d}F}{\mathrm{d}U} = \frac{\partial F}{\partial U} + \frac{\partial F}{\partial \alpha}\frac{\mathrm{d}\alpha}{\mathrm{d}U} + \frac{\partial F}{\partial \Omega}\frac{\mathrm{d}\Omega}{\mathrm{d}U} = 0 \ , \tag{17}$$

from which it is follows

$$\frac{\mathrm{d}\alpha}{\mathrm{d}U} = -\left[\frac{\partial F}{\partial U} + \frac{\partial F}{\partial \Omega}\frac{\mathrm{d}\Omega}{\mathrm{d}U}\right]\Big/\frac{\partial F}{\partial \alpha} \ . \tag{18}$$

Corresponding boundary conditions give

$$\frac{\mathrm{d}G}{\mathrm{d}U} = \sum_{j=1}^{4}\frac{\partial G}{\partial \alpha_j}\frac{\mathrm{d}\alpha_j}{\mathrm{d}U} = 0 \ , \tag{19}$$

and after substituting

$$\sum_{j=1}^{4}\frac{\partial G}{\partial \alpha_j}\cdot\left[\frac{\mathrm{d}F}{\mathrm{d}U}_{|\alpha=\alpha_j} + \frac{\mathrm{d}F}{\mathrm{d}\Omega}_{|\alpha=\alpha_j}\cdot\frac{\mathrm{d}\Omega}{\mathrm{d}U}\right]\Big/\frac{\mathrm{d}F}{\mathrm{d}\alpha}_{|\alpha=\alpha_j} = 0 \ , \tag{20}$$

or finally

$$\frac{\mathrm{d}\Omega}{\mathrm{d}U} = -\frac{\sum_{j=1}^{4}\frac{\partial G}{\partial \alpha_j}\cdot\frac{\mathrm{d}F}{\mathrm{d}U}_{|\alpha=\alpha_j}\Big/\frac{\mathrm{d}F}{\mathrm{d}\alpha}_{|\alpha=\alpha_j}}{\sum_{j=1}^{4}\frac{\partial G}{\partial \alpha_j}\cdot\frac{\mathrm{d}F}{\mathrm{d}\Omega}_{|\alpha=\alpha_j}\Big/\frac{\mathrm{d}F}{\mathrm{d}\alpha}_{|\alpha=\alpha_j}} \ . \tag{21}$$

Now, we formulate a very important aeroelastic stability theorem in the most general form.

4. Stability theorem

The complex natural frequency of the studied aeroelastic system Ω depends on the flow velocity U such, that value of its derivation $\frac{\mathrm{d}\Omega}{\mathrm{d}U}$ at the beginning (i.e. for $U = 0$) is equal zero for any symmetrical boundary conditions and is pure imagine value for every unsymmetrical boundary conditions.

The proof of this statement (by analytic calculating of the value $\frac{\mathrm{d}\Omega}{\mathrm{d}U}$) we will show for the case b) clamped ($s = 0$) - support ($s = L$) edges, nevertheless all other cases a), c) and d) can be proved by similar way and results of they all will be presented in summary table.

Let Ω_0 and $\alpha_{0,j}$, ($j = 1,\ldots,4$) satisfy the characteristic equation at the beginning ($U = 0$), i.e. are solutions of the equation

$$F(\alpha_{0,j}, U = 0, \Omega_0) = 0 . \tag{22}$$

Then, it is possible to involve next relations for derivatives:

$$\frac{\partial F}{\partial U}_{|U=0} = 2\rho n^3 \Omega_0 \alpha_{0,j} \left(1 - \frac{\alpha_{0,j}^2}{2n(n+1)}\right) , \tag{23}$$

$$\frac{\partial F}{\partial \Omega}_{|U=0} = -2\Omega_0 \frac{h}{R}\left(\alpha_{0,j}^4 + 2\alpha_{0,j}^2 n^2 \left(1 - \frac{\rho R}{4h(n+1)}\right) + n^4 \left(1 + \frac{\rho R}{hn}\right)\right) , \tag{24}$$

$$\frac{\partial F}{\partial \alpha}_{|U=0} = 4\alpha_{0,j}^3 (1-\Omega_0^2)\frac{h}{R} - 2\alpha_{0,j}\Omega_0^2 n^2 \left(2\frac{h}{R} - \frac{\rho}{2(n+1)}\right) . \tag{25}$$

After substitution $U = 0$ into the Eq. (11) it is possible we can write

$$\alpha^4 - \alpha^2 B - C = 0 , \tag{26}$$

where

$$B = n^2\Omega^2 \frac{\left(2 - \frac{1}{2(1-\mathrm{n}^2)} \cdot \frac{\rho R}{h}\right)}{(1-\Omega^2)}$$

and

$$C = \frac{-\frac{\mathrm{n}^4(\mathrm{n}^2\text{-}1)^2}{12(1-\nu^2)}\left(\frac{h}{R}\right)^2 + n^6 \frac{R}{h} P_1 + n^3\Omega^2\left(n + \frac{\rho R}{h}\right)}{1-\Omega^2} .$$

It is naturally to suppose $\Omega^2 << 1$, $\dfrac{h}{R} << 1$, and $C >> B^2/4 > 0$.
Then, we obtain the solutions of the Eq. (26) in the form:

$$\begin{aligned} \alpha_{0,1} &= -\alpha_{0,2} = \lambda \\ \alpha_{0,3} &= -\alpha_{0,4} = i\beta \ , \end{aligned} \tag{27}$$

where λ and β are positive real numbers.

Now we will show how it is look the expression (21), for example, for the boundary conditions clamped ($s = 0$) - support ($s = L$) edges.
After substitution the wave numbers $\alpha_{0,j}$, which have the structure (27) into derivatives of the corresponded boundary function G (14) we obtain

$$\frac{dG}{d\alpha}_{|\alpha=\alpha_{0,1}} = -\overline{\frac{dG}{d\alpha}_{|\alpha=\alpha_{0,2}}} \quad \text{and} \quad \frac{dG}{d\alpha}_{|\alpha=\alpha_{0,3}} = -\overline{\frac{dG}{d\alpha}_{|\alpha=\alpha_{0,4}}} \ .$$

Analogously, it is possible to involve same relations for $\dfrac{dF}{dU}_{|\alpha=\alpha_{0,j}}$, $\dfrac{dF}{d\Omega}_{|\alpha=\alpha_{0,j}}$ and $\dfrac{dF}{d\alpha}_{|\alpha=\alpha_{0,j}}$.

Finally

$$\left[\frac{dG}{d\alpha}_{|\alpha=\alpha_{0,1}}, \frac{dG}{d\alpha}_{|\alpha=\alpha_{0,2}}, \frac{dG}{d\alpha}_{|\alpha=\alpha_{0,3}}, \frac{dG}{d\alpha}_{|\alpha=\alpha_{0,4}}\right] = \left[g_1^{re} + ig_1^{im}, -g_1^{re} + ig_1^{im}, ig_3^{im}, ig_4^{im}\right] , \tag{28}$$

$$\left[\frac{dF}{dU}_{|\alpha=\alpha_{0,1}}, \frac{dF}{dU}_{|\alpha=\alpha_{0,2}}, \frac{dF}{dU}_{|\alpha=\alpha_{0,3}}, \frac{dF}{dU}_{|\alpha=\alpha_{0,4}}\right] = \left[f_u^{re}, -f_u^{re}, if_u^{im}, -if_u^{im}\right] , \tag{29}$$

$$\left[\frac{dF}{d\Omega}_{|\alpha=\alpha_{0,1}}, \frac{dF}{d\Omega}_{|\alpha=\alpha_{0,2}}, \frac{dF}{d\Omega}_{|\alpha=\alpha_{0,3}}, \frac{dF}{d\Omega}_{|\alpha=\alpha_{0,4}}\right] = \left[f_o^1, f_o^1, f_o^3, f_o^3\right] , \tag{30}$$

$$\left[\frac{dF}{d\alpha}_{|\alpha=\alpha_{0,1}}, \frac{dF}{d\alpha}_{|\alpha=\alpha_{0,2}}, \frac{dF}{d\alpha}_{|\alpha=\alpha_{0,3}}, \frac{dF}{d\alpha}_{|\alpha=\alpha_{0,4}}\right] = \left[f_\alpha^{re}, -f_\alpha^{re}, if_\alpha^{im}, -if_\alpha^{im}\right] , \tag{31}$$

where $g_1^{re}, g_1^{im}, g_3^{im}, g_4^{im}$, f_u^{re}, f_u^{im}, f_o^1, f_o^3, $f_\alpha^{re}, f_\alpha^{im}$ are real values. Substituting these relations for corresponded derivatives of G and F into formula for $\dfrac{\mathrm{d}\Omega}{\mathrm{d}U}$ follows

$$\frac{\mathrm{d}\Omega}{\mathrm{d}U}_{|U=0} = i \cdot \frac{2f_u^{re} f_\alpha^{im} g_1^{im} + (g_3^{im} + g_4^{im}) f_u^{im} f_\alpha^{re}}{2f_o^1 f_\alpha^{im} g_1^{re} + (g_3^{im} - g_4^{im}) f_o^3 f_\alpha^{re}} \ . \tag{32}$$

The relation (32) shows that the natural frequency Ω changes at the beginning ($U = 0$) its ***imaginary part only***.

The Tab. 1 presents the analytically involved derivatives (anal.) in comparison with numerical results obtained within 1D (num.-1D, see: [11], [12], [14]) and 3D (num.-3D, see: [13], [17]-[23]) shell theory for several types of boundary conditions and next input parameters:

$h = 0.0002$ m, $R = 0.15$ m, $l = 1$ m, E=$7.2\cdot10^{10}$ Pa, $\nu = 0.34$, ρ_s =2800 kg·m^{-3},
$p_1 = 500$ Pa, ρ_t =12 kg·m^{-3}, $n = 6$.

	m	$\mathrm{Im}\left(\frac{\mathrm{d}\Omega}{\mathrm{d}U}\right)$ (anal.)	$\frac{\mathrm{d}\delta}{\mathrm{d}U}$ (num.-1D)	$\frac{\mathrm{d}\delta}{\mathrm{d}U}$ (num.-3D)
Supp-supp	1	0	0	0
	2	0	0	0
Clamp-supp	1	-0.22685e-3	-0.2246e-3	-0.2160e-3
	2	-0.67742e-3	-0.6762e-3	-0.6500e-3
Clamp-clamp	1	0	0	0
	2	0	0	0
Clamp-free	2	2.80e-3	-	88.66e-3

Tab. 1. The comparison of the analytically involved derivatives (anal.) with numerical results obtained within 1D and 3D shell theory for several types of boundary conditions and shape modes $n = 6$ and $m = 1$ and 2 .

The formulated above theorem together with next properties of adjoint solution give complete notion about stability (if you like instability) of studied aeroelastic systems.

5. Properties of adjoint solutions

If the set quantities $\{\lambda_j(j=1,\ldots,8),U,\Omega\}$ is a solution of Eqs. (3.18)-(3.24) with the boundary conditions (2.8), then the sets

$$\begin{aligned} &\text{a)} \quad \{\lambda_j(j=1,\ldots,8),-U,-\Omega\},\\ &\text{b)} \quad \{-\lambda_j^*(j=1,\ldots,8),-U,\Omega^*\},\\ &\text{c)} \quad \{-\lambda_j^*(j=1,\ldots,8),U,-\Omega^*\} \end{aligned} \tag{33}$$

are solutions too. The asterisks denote the complex conjugated values.

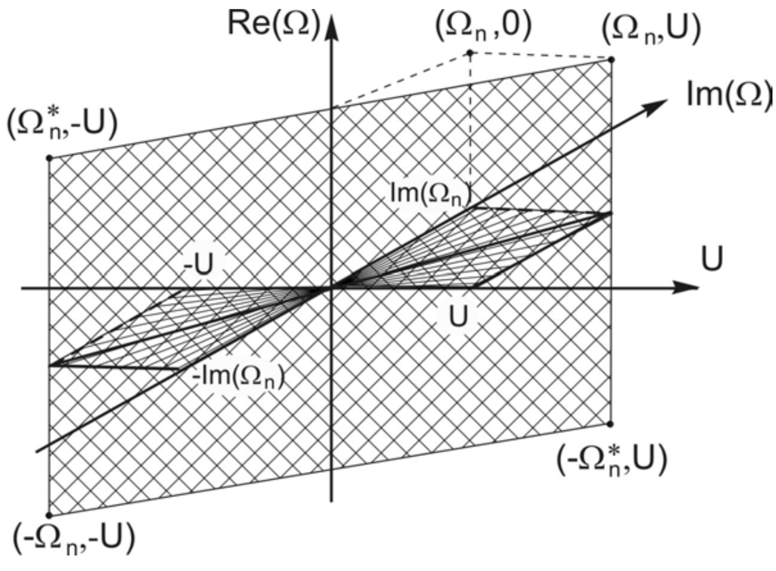

Fig. 1.

Proof of this property follows from holomorphic properties of all expressions presented in Eqs. (11)-(16). Graphic visualization of this property in the 3D space (U, ReΩ, ImΩ) is shown in Fig. 1. This property is valid in the more general case 3D shell theory too (see [13]).
We note that a sign change of the flow velocity U can be interpreted as a change in the orientation of the boundary conditions with respect to the flow velocity. Hence from the Eq. (33) it follows that if a certain vibration mode of the shell in a certain flow regime (at a specific value of U) is stable (i.e., $\mathrm{Im}(\Omega)>0$, $\mathrm{Re}(\Omega)\neq 0$), then under the same conditions the vibrations of the shell with the boundary conditions at the entrance and exit switched will be unstable (i.e., $\mathrm{Im}(\Omega)<0$, $\mathrm{Re}(\Omega)\neq 0$).

6. Conclusion

The presented theoretical results correspond to the previous studies, where neither the internal nor the external friction of the shell in fluid has been taken into account. In that case the cylindrical shell containing fluid flowing from the clamped to the supported edge or from the free to the clamped edge is unstable for any arbitrarily low fluid flow velocity.

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REFERENCES

[1] Bolotin, V. V. (1963) Nonconservative Problems of the Theory of Elastic Stability. New York: MacMillan Co.

[2] Warburton, G. B. (1961) Vibration of a cylindrical shell in an acoustic medium. I. Mech. E. Journal Mechanical Engineering Science, Vol. 3, 69-79.

[3] Chen, S. S. (1987) Flow-induced vibration of circular cylindrical structures. Hemisphere, Washington.

[4] Horáček, J. & Zolotarev, I. (1981) Stability and bending wave propagation in a long cylindrical shell containing a flowing fluid. Strojnícky časopis, Vol. 32, 687-700.

[5] Païdoussis, M. P. & Denise, J. P. (1972) Flutter of thin cylindrical shells conveying fluid. Journal of Sound and Vibration, Vol. 20 (1), 9-26.

[6] Weaver, D. S. & Unny, T. E. (1973) On the dynamic stability of fluid conveying pipes. Trans. ASME. Ser. E, Vol.40 (1), 48-52.

[7] Weaver, D. S. & Myklatun, B. (1973) On the stability of thin pipes with an internal flow. Journal of Sound and Vibration, Vol. 31 (4), 399-410.

[8] Shayo, L. K. & Ellen, C. H. (1974) The stability of finite-length circular cross-section pipes conveying inviscid fluid. Journal of Sound and Vibration, Vol. 37 (4), 535-545.

[9] Kornecki, A. (1974) Static and dynamic instability of panels and cylindrical shells in subsonic potential flow. Journal of Sound and Vibration, Vol. 32 (2), 251-263.

[10] Païdoussis, M. P. (1975) Flutter of conservative systems of pipes conveying incompressible fluid. J. Mech. Eng. Sci., Vol. 17 (1), 19-28.

[11] Horáček, J. & Zolotarev, I. (1983) Natural frequencies, damping and stability of a structurally damped cylindrical shell conveying fluid. Strojnicky casopis, 34, 189-204.

[12] Horáček, J. & Zolotarev, I. (1984) Influence of fixing the edges of a cylindrical shell conveying fluid on its dynamic characteristics. Soviet Applied Mechanics, 20, 756-765.

[13] Zolotarev, I. (1987) Optimization of aerodynamic damping of a cylindrical shell with flowing fluid with the aid of boundary conditions. Soviet Machine Science, Mashinovedenie, No 1, 49-55.

[14] Horáček, J. & Zolotarev, I. (1991) Vibration of cylindrical shells containing flowing fluid. In Dynamics of Bodies in Interaction with Surroundings (eds A. N. Guz, L. Půst & Š. Markuš), pp. 215-272. Kiev: Naukova dumka.

[15] Novozhilov, V. V. (1964) The Theory of Thin Elastic Shells. Groningen, The Netherlands: P. Noordhoff.

[16] Zolotarev, I. and Popov, A.L. (1986) Limits of validity of approximate methods for the vibrations of fluid-filled shells. Strojnicky casopis, 37 (6), 728-739.

[17] Zolotarev, I. (1987) Free vibrations of a cylindrical shell conveying by fluid with elastically supported edges. Strojnícky časopis, Vol. 38 (5), 533-550 (in Czech).

[18] Misra, A.K., Wong, S.S.T. & Païdoussis M.P. (2001) Dynamics and stability of pinned-clamped and clamped-pinned cylindrical shells conveying fluid. Journal of Fluid and Structures, 15, 1153-1166.

[19] Zolotarev I. (2001) Vibration cylindrical shells containing flowing fluid. Proceedings of the Seminar on Interaction and Feedbacks '2001, Prague, 27-28 November 2001, pp. 249-252.

[20] Zolotarev I., Pelletier D. (2000) On the solution of a natural vibration of cylindrical shells. Proceedings of the Seminar on Interaction and Feedbacks '2000, Prague, 28-29 November 2000, pp. 287-296.

[21] Zolotarev I. and Kruntcheva M. (1995) Vibration of cylindrical shells involving coupling with flowing acoustical medium. Proc. of Sixth Int. Conf. on Flow-Induced Vibration, London, 10-12 April 1995, A.A.Balkema Rotterdam, p. 535-544.

[22] Zolotarev I. (2002). Vibration and stability of cylindrical shells containing flowing fluid. ASME´2002, 5th International Symposium on Fluid-Structure Interactions, Aeroelasticity, Flow-Induced Vibration and Noise, November 17-22, 2002, New Orleans, USA.

[23] Horáček J., Zolotarev I. (2002). Free vibration and stability of cylindrical shells in interaction with flowing fluid. PROCEEDINGS of the NATO CLG GRANT REPORT № PST.CLF. 977350 „Nonlinear dynamics of shells with fluid-structure interaction“, IT AV CR, Prague 2002, pp. 45-82.