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STOCHASTIC THEORY FOR STRENGTH OF CONCRETE

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Summary: *Stochastic theory for fracture of concrete materials is applied. The fracture process can be described not only under monotonically increasing load but also under time-dependent loading conditions. Theoretical predictions are compared with earlier published data. Special accent is placed on the effect of rate of loading on the mean strength and corresponding variability. Specimens of high strength mortar, low strength mortar, lightweight and normal concrete were tested under compressive and bending load. The experimental results materially verify the theoretical concept.*

1. Assumptions

In general, majority of the previous studies taking an interest in the fracture characteristics of concrete may be divided into three groups from the view point of the criterion (eg. Mihashi, 1983).

1. Macroscopis scale:

Characteristic length in the order of 100 mm or more. Typical materials properties to be studied: average stress and strain, strength, nonlinearity of mechanical properties.

2. Submacroscopic scale:

Characteristic length in the order of 1 mm or 10 mm. Typical materials properties to be studied: local stres and strain, crack formation, failure process, fracture mechanism.

3. Microscopic scale:

Characteristic length in the order of 10^{-1} mm or less. Typical materials properties to be studied: microstress and microstrain, hydration, porosity, structure.

The typical macroscopic events are caused by submacroscopic failure and remarkably influenced by the factors on the microscopic level. Concrete materials i.e. cement paste, mortar and concrete contain enough submacroscopic material defects such as voids (entrapped air), flaws, shrinkage cracks and interfacial cracks. Therefore the stress distribution in the solids is remarkably disturbed by these defects. And it is well known that the failure process is highly affected by some bigger material defects among them. Under uniaxial load the failure behaviour around each defect is rather independent of the rest of the materials structure with the exception of direct neighbouring pores. The failure process can be assumed to be the

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same of many microprocesses. Until fracture occurs, a series of typical states of crack propagation is followed.

The failure process highly depends on the material and the loading condition. Cement paste under tensile load behaves just like porous rock. Fracture occurs in a quite brittle manner and there are not stable states with crack initiation. The specimen fractures immediately when a crack initiates from a pre-existing micro crack.

Concrete and mortar under compressive load, however, are not fractured in such a way. There are some stable states with submacroscopic cracks and the failure is caused by the accumulation of many cracks.

From the view point of the microscopic level, the fracture of concrete materials may by caused by a series of local failure processes in the phase of hydratation products of cement and interfaces. When a failure criterion is satisfied in one part of the phase, a crack is created. Its extension and the connection with other cracks cause eventually fracture. This holds true even under tensile load.

a. The concrete system may be considered to consist of a group of *m* elements with two or three phases which are linked in series (in the case of fracture under multi-axial compressive load, the structural element must be reconsidered according to the failure process). This situation is shown schematically in Fig 1.

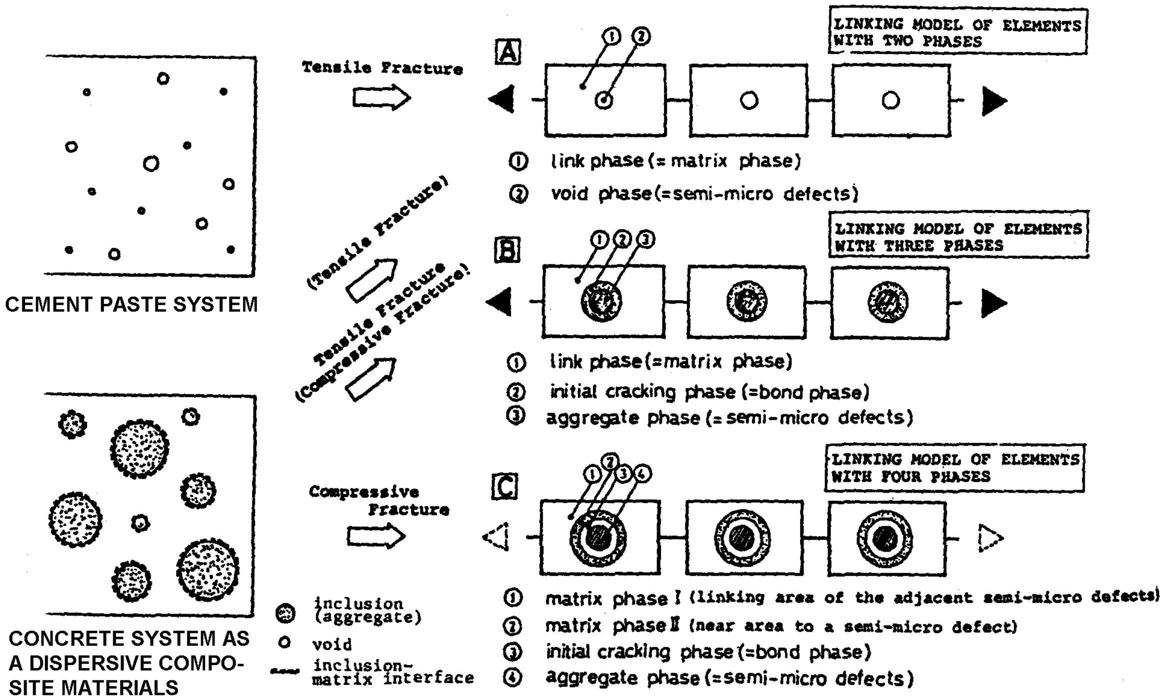

Fig. 1 Linking model of elements

b. Each phase consists of *n* units which contain a circular crack. This model may be representative for hardened cement paste and is shown in Fig 2.

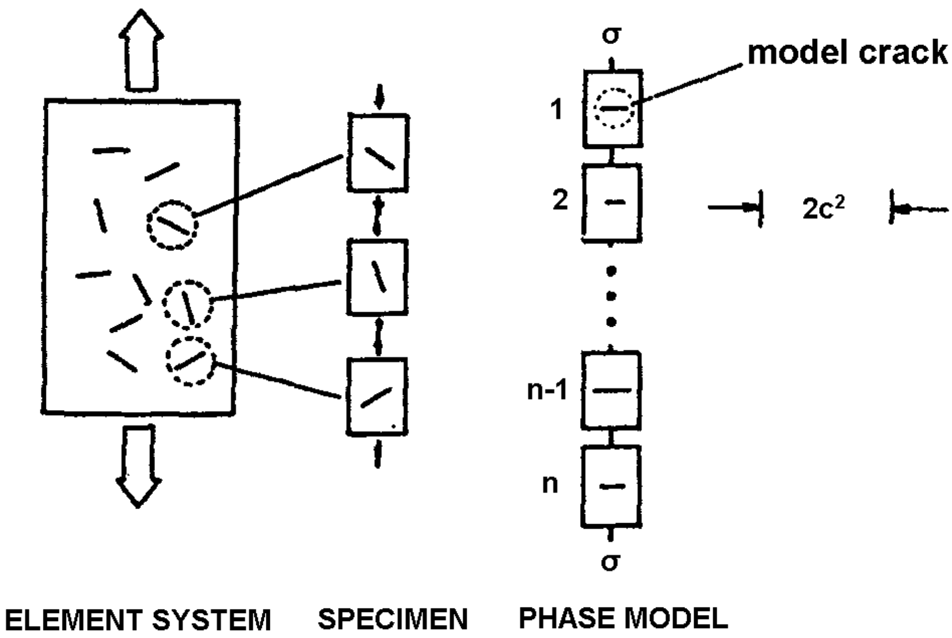

Fig. 2 Hardened cement paste phase model

c. The dimensions of a circular crack may be evaluated by the equivalent crack length $2c^2$: where c is a fracture factor

$$c = g\left(s, l, t\right) \tag{1}$$

c depends on s, which is a micro-stress disturbance coefficient caused by material defects, on $2l$ the microcrack length (which may be related to micropore size) and on time t, s and l are random variables and mutually independent. When the relative fracture factor of a microcrack under a given stress is estimated, the following equation is obtained on the basis of elementary fracture mechanics:

$$c = ks\sqrt{\frac{l}{Ev}} \tag{2}$$

where E is Young's Modulus, v is the surface energy and k is a constant. If only a specific crack length such as the expected maximum micro crack length ($2\bar{l}$) is considered, the density function of the fracture factor is described as follows.

$$f_c\left(c\right) = \frac{1}{k}\sqrt{\frac{Ev}{\bar{l}}}\, f_s\left(s\right) \tag{3}$$

d. Various failure methods are indicated in Fig. 3. For each condition only the microstress distribution is changed by submicroscopic cracking. In type A, there are no stable states for crack formation. In type B, the internal micro stress distribution of state 1 is different from the initial state. Fracture of the specimen is dominated not only by the initial state but also by the state 1. Finally in type C, the failure process is treated much more comprehensive.

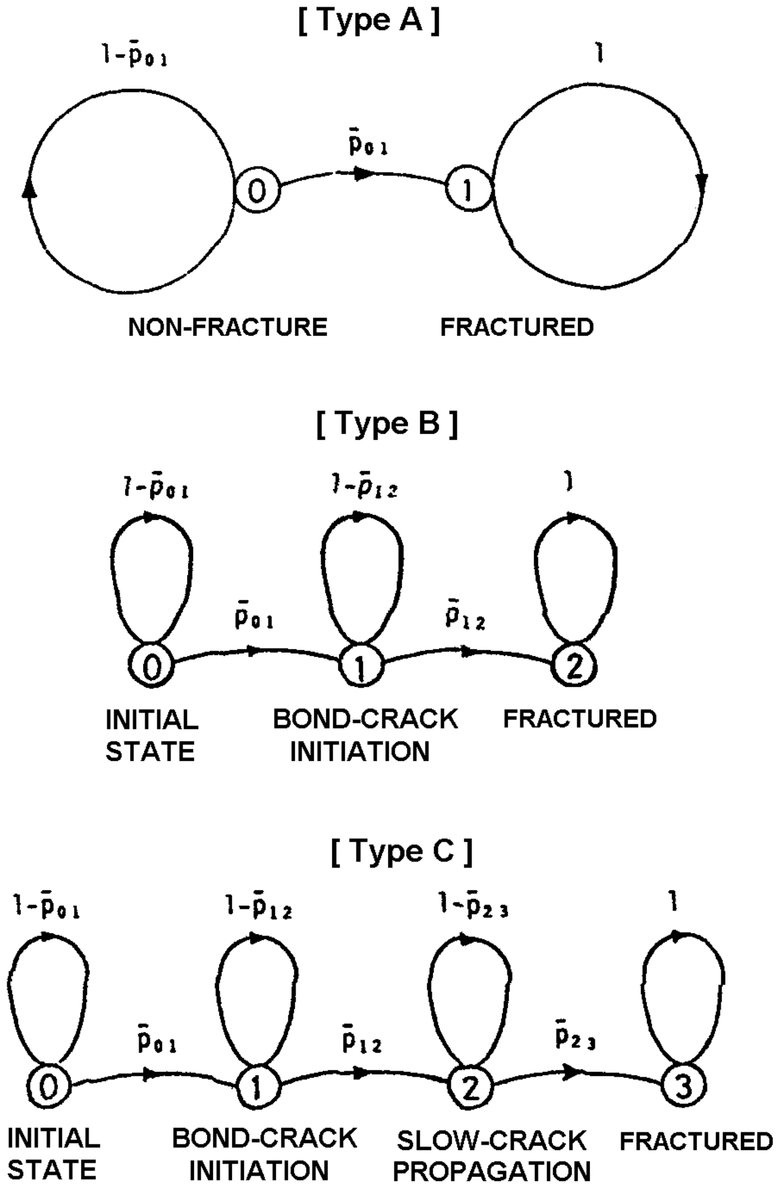

Fig. 3 Mathematical models – Transition line graph

e. The distribution of material defects and the characteristic properties of each element are statistically equivalent over the whole zone.

2. Basic philosophies of the stochastic theory

It can be erroneous that all types of cracking identically influence the failure process. Some of them are very stable and the other fast propagate. Before the final stage of failure process, most of the cracking is limited to around some larger defects. It is well known that the failure process is highly affected by some larger material defects. Round larger material defects, the highly stressed region may be wider and the possibility that the some weaker defects are acted upon higher tensile stress may increase (Brož, 2004).

In the model presented, the following experimental results are intended to create the theoretical version.

1) The broken parts in normal concrete systems are the cement paste phase and cement-aggregate interface. They include a lot of fine cracks, flaws and voids.
2) Aggregates and voids play a role to cause local stress concentration and sometimes even to change the sort of stress such as local tensile stress under compressive load.
3) In the tensile stress field, micro-cracks easily grow into mesolevel cracks in order to release the locally stored strain energy.
4) Until fracture of the specimen occurs, a series of characterical conditions of crack growth are ensued. Basic aspects of the stochastic theory are constituted by three characteristics.
    1. "Statistical Approach" is taken to describe the random properties of local strength caused by "Geometrical Random".
    2. "Stochastic Process Theory" is introduced to present the random properties of the rate of cracking based on the kinematic and thermodynamic properties of a solid.
    3. "Changeable Elements" are employed to stand for the imperfectly brittle failure course of concrete.

3. Data matching

3.1 Stress rate effect

The influence of rate of loading may be described as follows:

$$\frac{\bar{f}}{\bar{f}_0} = \left(\frac{\dot{\sigma}}{\sigma_0}\right)^{\frac{1}{\beta+1}} \tag{4}$$

and $\bar{f}_0$ stand for strength under a high rate of loading (dynamic) and a low rate of loading (static), respectively. The corresponding rates of loading are described by $\dot{\sigma}$ and $\dot{\sigma}_0$. In Figs. 4 and 5, some experimental results presented by Mihashi are indicated.

Table 1 Experimental program to study the influence of rate of loading

Group	Material	Loading Condition	Different Rates of Loading	Number of Tests
I	Mortar (W/C=0.45)	Bending	5	148
II	Mortar (W/C=0.65)	Bending	5	149
III	Mortar (W/C=0.45)	Compression	6	188
IV	Mortar (W/C=0.65)	Compression	6	190
V	Concrete	Compression	2	60

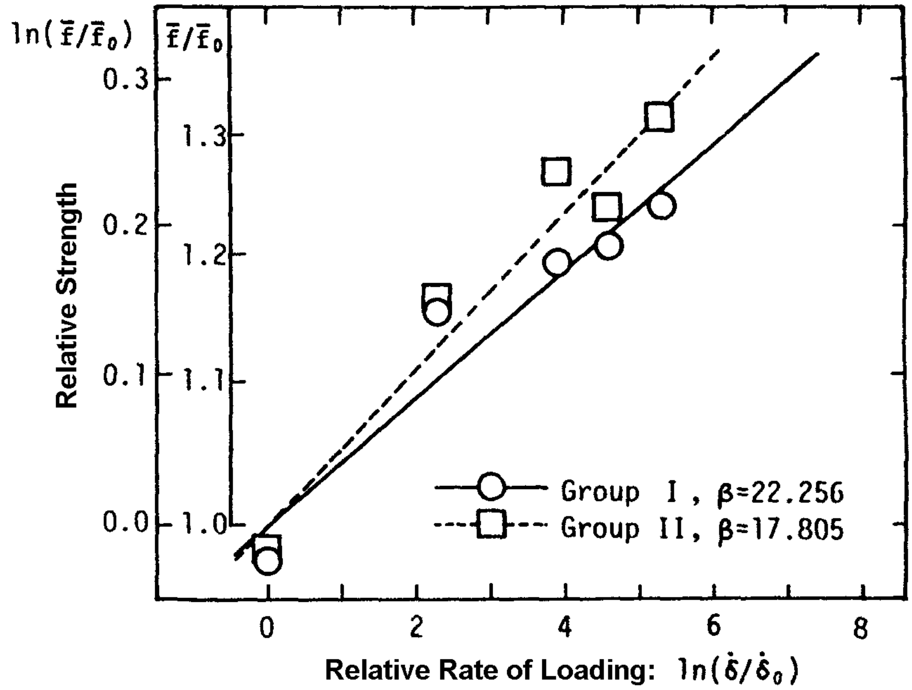

Fig. 4 Relation between the related rate of loading and the related mean value of flexural strength

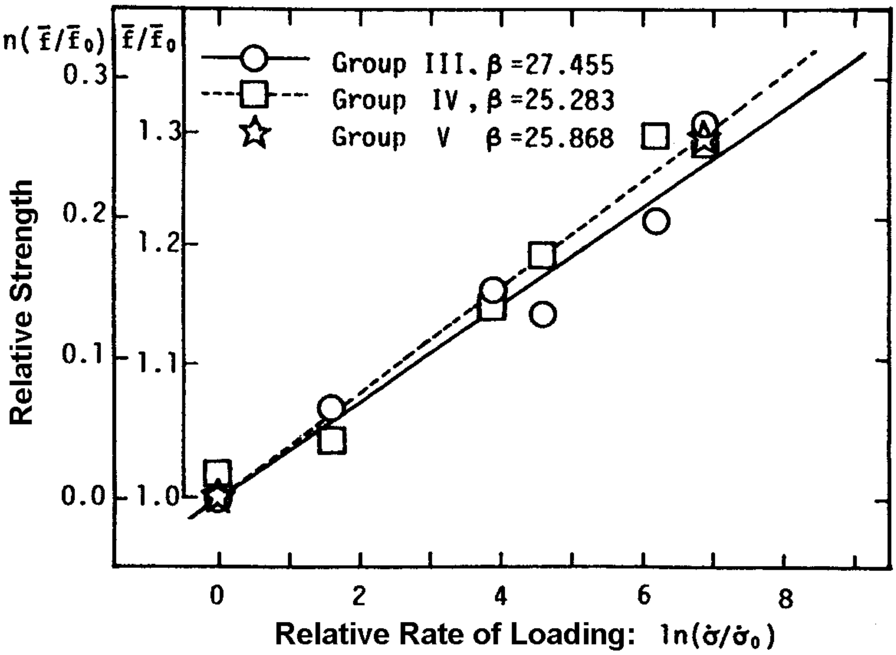

Fig. 5 Relation between the related rate of loading and the related mean value of compressive strength

It should be put down that equation expresses sufficiently the dependence of strength under high rate of loading in a very wide scope.

3.2 Variability of strength

Some experimental results of the probability of fracture of mortar prisms under bending loads and of normal concrete prisms under compressive loads are indicated in Figs. 6 and 7 respectively. Solid lines demonstrate the probability of fracture F(σ).

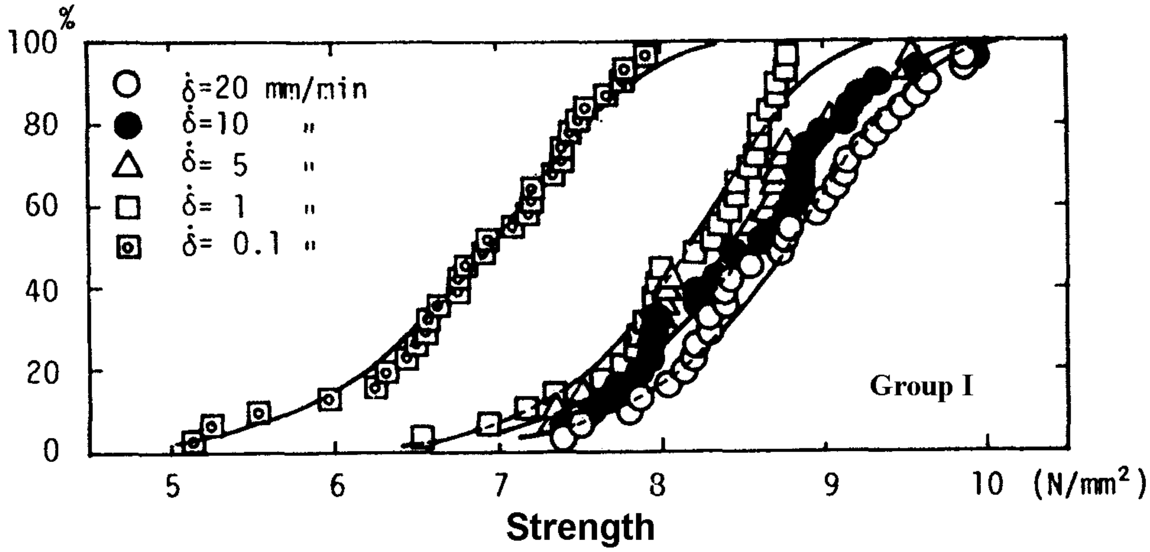

Fig. 6 Probability of fracture of mortar under bending load

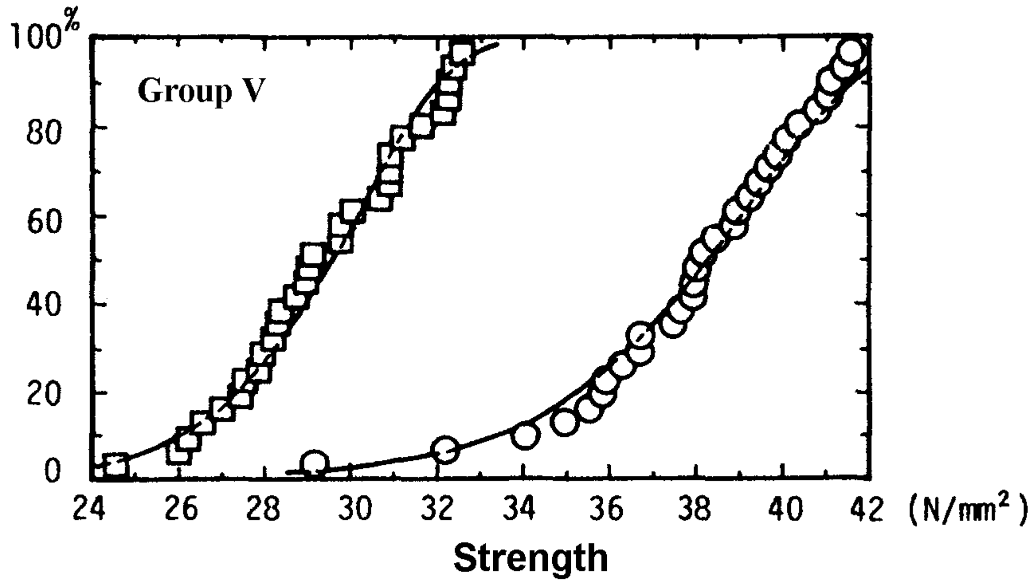

Fig. 7 Probability of fracture of concrete under compressive load

In the case of material of Type A, the survival probability is as follows:

$$\ln\{-\ln P(\sigma)\} = \ln\{mL/(\beta+1)\dot{\sigma}\} + (\beta+1)\ln\sigma \quad (5)$$

Equation shows that the relationship between ln{-ln$P(\sigma)$} and ln σ is linear and that the slope of that line is (β+1). In the case of a material of Type B, however, these equations become nonlinear.

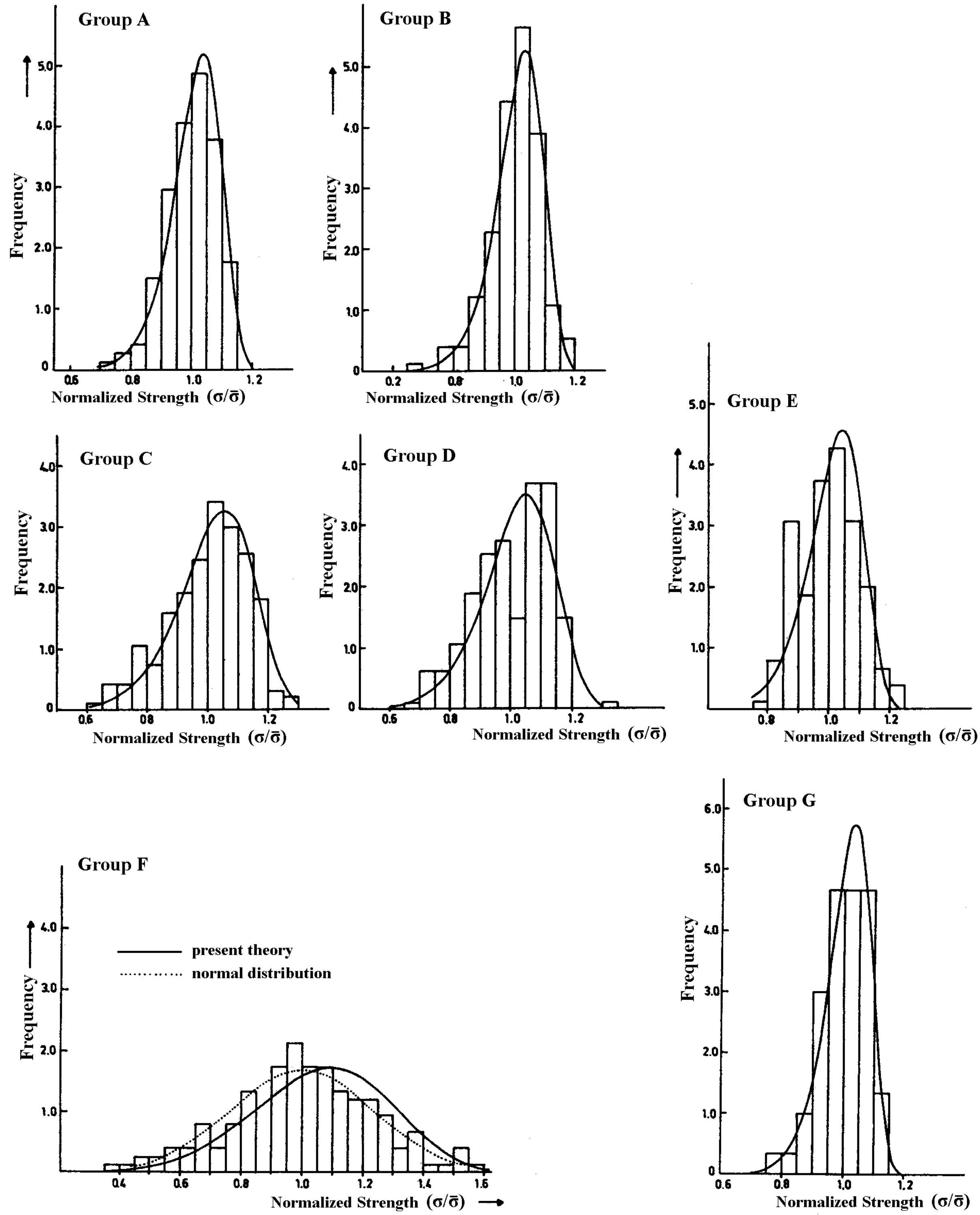

Fig. 8 Histograms for Normalized Strength and the Theoretical Density Function for groups A to G

The theory in question predicts that the coefficient of variability is not influenced by the rate of loading. Within the range of accuracy, this theoretical prediction is verified by an experimental study as demonstrated in Table 2. This result is especially interesting from the view point of a realistic reliability assessment of a concrete structure.

Table 2 Mean value, standard deviation and coefficient of variation of strength of concrete under some rates of loading

Rate of Loading	Mean Value of Strength (N/mm^2)	Standard Deviation	Coefficient of Variation
Group I : $\ln \sigma = 0.043 \ln \dot{\delta} + 2.050$; $\beta = 22.2$			
20.0 (mm/min)	8.72	0.677	0.078
10.0	8.48	0.713	0.084
5.0	8.39	0.685	0.082
1.0	8.11	0.585	0.072
0.1	6.87	0.729	0.106
Group II : $\ln \sigma = 0.053 \ln \dot{\delta} + 1.980$; $\beta = 17.8$			
20.0 (mm/min)	8.44	0.716	0.085
10.0	7.93	0.820	0.103
5.0	8.13	0.629	0.077
1.0	7.45	0.542	0.073
0.1	6.28	0.465	0.074
Group III : $\ln \sigma = 0.035 \ln \dot{\sigma} + 3.675$; $\beta = 27.4$			
50.505 (N/mm^2 sec)	46.51	5.075	0.109
25.253	43.37	6.348	0.146
5.173	40.63	5.993	0.147
.2.586	41.30	5.900	0.143
0.259	37.98	5.295	0.139
0.052	35.52	3.905	0.110
Group IV : $\ln \sigma = 0.038 \ln \dot{\sigma} + 3.361$; $\beta = 25.2$			
50.505 (N/mm^2 sec)	33.15	4.669	0.141
25.253	33.38	3.577	0.107
5.173	30.63	3.036	0.099
2.586	29.55	3.014	0.102
0.259	26.86	3.906	0.145
0.052	26.22	3.689	0.141
Group V : $\ln \sigma = 0.037 \ln \dot{\sigma} + 3.271$; $\beta = 25.9$			
17.341 (N/mm^2 sec)	37.86	2.805	0.074
0.018	29.31	2.151	0.073

Since the coefficient of variability is not influenced by rate of loading, it is reasonable to expect that the distribution function of the strength which is normalized by the mean value for each group, will be the same. Fig. 8 yields the histograms for normalized strength and the theoretical density function.

4. Conclusion

Compressive and bending strength of mortar and compressive strength of concrete enhance with increase rate of loading.

Compressive strength seems to be influenced less severely by rate of loading than bending strength. Strength of weaker specimens is more affected by rate of loading.

The influence of rate of loading is satisfactorily described by a power function: Light weight concrete shows a different behaviour in the range of high rates of loading.

The coefficient of variation is not influenced by rate of loading. On the basis of this, a distribution function of all strength values, normalized by the mean value for each rate of loading, can be obtained. The distribution function of strength is described satisfactorily by Weilbull's distribution function. This applies both for strength as determined under bending load and under compressive load.

The distribution of strength of light weight concrete may be aproximated by a Gausian distribution function.

The stochastic theory for fracture of concrete adequately gives an account of the experimental investigation of strength qualities that are attected by loading, environmental temperature, size of the specimen and age of the specimen. It is also possible to describe the failure process not only under a monotonically increasing load but under time dependent loading conditions such a sustained load and as a repeated load.

Besides, the present model provides a realistic basis for a mathematical statement of the inconstancy of aggregative materials like concrete.

5. Acknowledgement

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6. References

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