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DETERMINATION OF THE CORRELATION FUNCTION OF THE ROTARY VIBRATION IN PLANE MOTION

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Summary: *The aim of the present research was to investigate the method of identifying the function of rotary vibration of the object in plane motion. The body motion was defined in experiment with three sensors measuring displacement against three axes. Matrix was developed which factored in the dependence of linear displacement of the sensors on generalized coordinates, including polar coordinate. Based on the analytical formula, a relationship was defined which is correlation function of the polar coordinate.*

Origin of the problem

The problem which will be analyzed in the present work generally refers to the question of the isolation of the flexible multibodies system from environment vibrations, Fig.1.

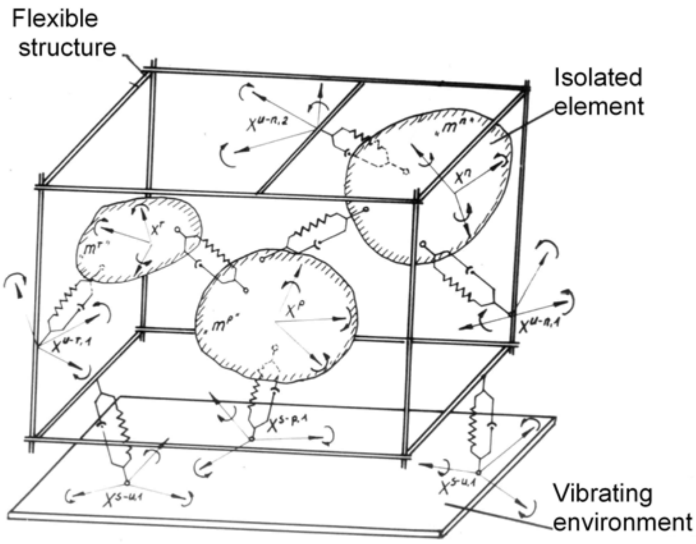

Fig. 1. Model multibodies object

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The objects examined are situated inside a flexible construction which is characterized by its receptances. However, the flexible elements which insulate all the construction from external oscillations and each object from the construction are described by their stiffness and damping. The system described is kinematically forced by stationary and ergodic random process. For the theoretical model described in such a way, equations of motion were obtained. In order to verify the correctness of the formula obtained, an experimental model was developed, Fig. 2, which was solved theoretically and then, to verify the results, investigated in the experiment.

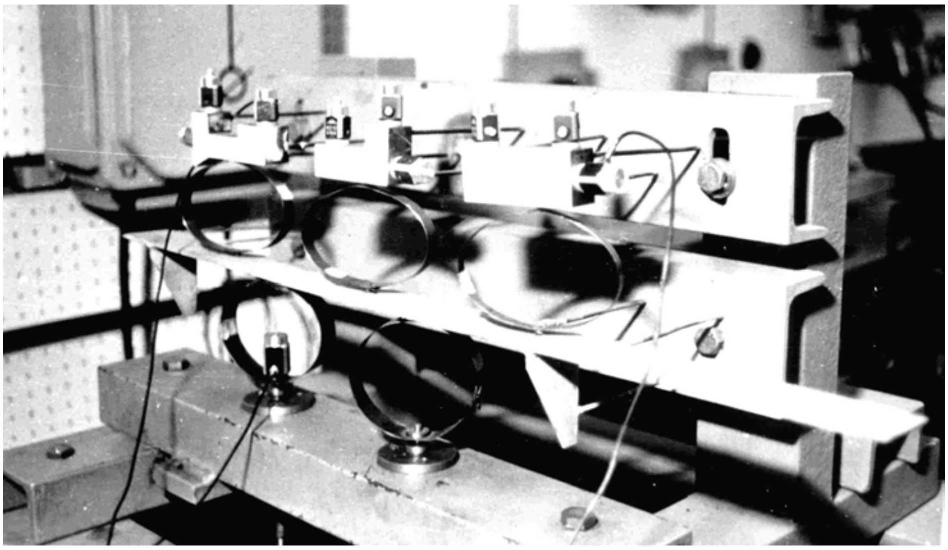

Fig. 2. Photo of the model

That model has some discrete masses each with three degrees of freedom-plane motion, Fig. 3. The main objective of the work is how to determine the correlation function of the rotary vibration by means of the knowledge of correlation of the longitudinal vibration.

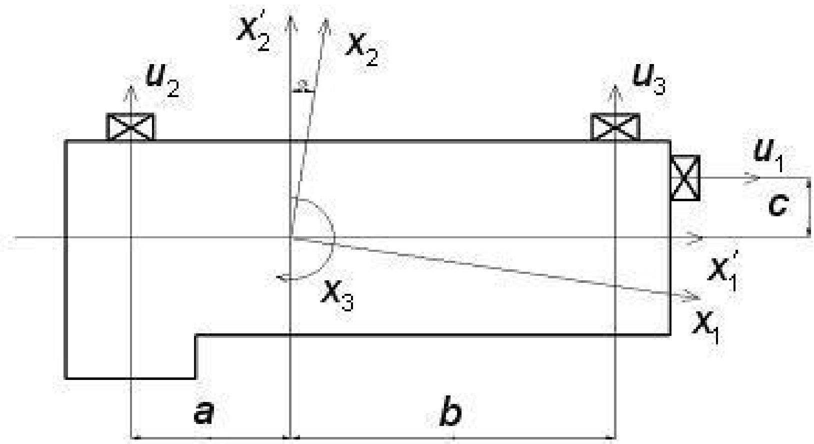

Fig. 3. Model of the body

Problem solution

Let us consider any element in plane motion. x_1, x_2, x_3 are assumed as the generalized unknown coordinates. The first two coincide with the main central axes of the body inertia. The third coordinate represents the rotary motion.

To define the three coordinates, it is enough to know the linear displacement of three body points; and so three vibration meters are installed to the body investigated to measure the displacements along the U_{1-3} axes.

$$\begin{aligned} U_1 &= x_1 \cos\lambda + x_2 \sin\alpha - x_3 \cdot c \\ U_2 &= -x_1 \sin\alpha + x_2 \cos\alpha - x_3 \cdot a \\ U_3 &= -x_1 \sin\lambda + x_2 \cos\alpha + x_4 \cdot b \end{aligned} \tag{1}$$

Formula (1) can be described in a form of matrix

$$\{U_{1-3}\} = \begin{bmatrix} \cos\alpha & +\sin\alpha & -c \\ -\sin\alpha & +\cos\alpha & -a \\ -\sin\alpha & +\cos\alpha & +b \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} \tag{2}$$

or

$$\mathbf{U}_{1-3} = \mathbf{A} \cdot \mathbf{X}_{1-3} \tag{3}$$

If matrix A is a nonsingular matrix, then we can determine A^{-1} matrix and equation (3) multiplied by A^{-1} gives:

$$\{X_{1-3}\} = [A]^{-1} \cdot \{U_{1-3}\} = \frac{\hat{A}}{|A|} \cdot \{U_{1-3}\} = [Z]\,\{U_{1-3}\} \tag{4}$$

where $\hat{A}$ stands for adjoint matrix of matrix A

$$\hat{A} = \begin{bmatrix} \Delta_{11} & \Delta_{21} & \Delta_{31} \\ \Delta_{12} & \Delta_{22} & \Delta_{32} \\ \Delta_{13} & \Delta_{23} & \Delta_{33} \end{bmatrix} = \begin{bmatrix} \hat{A}_{11} & \hat{A}_{12} & \hat{A}_{13} \\ \hat{A}_{12} & \cdot & \cdot \\ \hat{A}_{31} & \cdot & \hat{A}_{33} \end{bmatrix} \tag{5}$$

Matrix $\hat{A}$ is then the transpose matrix of algebraic complements matrix. Finally the formula which defines $x_{1\text{-}3}$ values searched for, assumes the following form (4)

$$\begin{aligned} x_1 &= Z_{11} \cdot U_1 + Z_{12} \cdot U_2 + Z_{13} \cdot U_3 \\ x_2 &= Z_{21} \cdot U_1 + Z_{22} \cdot U_2 + Z_{23} \cdot U_3 \\ x_3 &= Z_{31} \cdot U_1 + Z_{32} \cdot U_2 + Z_{33} \cdot U_3 \end{aligned} \tag{6}$$

where, as seen from equations (4), (5) and (6)

$$Z_{11} = \frac{\hat{A}_{11}}{|A|} \text{ and in a more general form } Z_{ij} = \frac{A_{ji}}{|A|}$$

Now, based on the formulae obtained, we can define the parameters of the random process which are interesting for us, however but we focus on the class of stationary and ergodic processes.

The function of autocorrelation assumes the form of

$$R_x(\tau) + = E[x(t)\cdot x(t+\tau)], \tag{7}$$

where E is a symbol of averaging inside square brackets.

Substituting (6) to (7), we obtain

$$\begin{aligned} R_{xn} &= E[(Z_{n1}U_1(t) + Z_{n2}U(t) + Z_{n3}\cdot U_3(t)) \\ &= (Z_{n1}\cdot U_1(t+\tau) + Z_{n2}\cdot U_2(t+\tau) + Z_{n3}\cdot U_3(t+\tau)] \end{aligned} \tag{8}$$

multiplying

$$\begin{aligned} R_{xh} = E\big[&Z_{n1}{}^2(U_1(t)\cdot U_1(t+\tau) + Z_{n1}\cdot Z_{n2}U_1(t)\cdot U_2(t+\tau) + \\ &+ Z_{n1}Z_{n3}U_1(t)\cdot U_3(t+\tau) + Z_{n2}Z_{n1}U_2(t)\cdot U_1(t+\tau) \\ &+ Z_{n2}{}^2U_2(t)\cdot U_2(t+\tau) + Z_{n2}Z_{n3}\cdot U_2(t)U_3(t+\tau) \\ &+ Z_{n3}\cdot Z_{n1}U_3(t)\cdot U_1(t+\tau) + Z_{n3}\cdot Z_{n2}U_3(t)\cdot U_2(t+\tau) \\ &+ Z_{n3}^2U_3(t)\cdot U_3(t+\tau)\big] \end{aligned}$$

After multiplying (8) and placing constants Z_{n1}^2, Z_{n2}^2, Z_{n3}^2 and their mutual products before symbol of averaging E, we obtain:

$$\begin{aligned} R_{xn} &= Z_{n1}^2E[U_1(t)\cdot U_1(t+\tau)] + Z_{n1}\cdot Z_{n2}E[U_1(t)\cdot U_2(t+\tau)] + \\ &+ Z_{n1}Z_{n3}E[U_1(t)\cdot U_3(t+\tau) + Z_{n2}Z_{n1}E[U_2(t)\cdot U_1(t+\tau)] + \\ &+ Z_{n2}^2E[U_2(t)\cdot U_2(t+\tau)] + Z_{n2}Z_{n3}E[U_2(t)\cdot U_3(t+\tau)] + \\ &+ Z_{n3}Z_{n1}E[U_3(t)\cdot U_1(t+\tau)] + Z_{n3}Z_{n2}E[U_3(t)\cdot U_2(t+\tau)] + Z_{n3}^2E[U_3(t)\cdot U_3(t+\tau)] \end{aligned} \tag{9}$$

As products in 'E' represent the function of correlation and intercorrelation, then equation (9) can be presented as

$$\begin{aligned} R_{xn} &= Z_{n1}^2\cdot R_{u1}(\tau) + Z_{n1}\cdot Z_{n2}\cdot R_{u1u2}(\tau) + Z_{n1}\cdot Z_{n3}\cdot R_{u1\cdot u3}(\tau) + \\ &+ Z_{n2}\cdot Z_{n1}\cdot R_{u2u1}(\tau) + Z_{n2}^2R_{u2}(\tau) + Z_{n2}Z_{n3}\cdot R_{u2u3} + \\ &+ Z_{n3}\cdot Z_{n1}R_{u3\cdot u1}(\tau) + Z_{n3}\cdot Z_{n2}R_{u3u2}(\tau) + Z_{n3}^2R_{u3}(\tau) \end{aligned} \tag{10}$$

The above equation can be given in a form of matrix:

$$R_{xn} = \{Z_{n1}, Z_{n2}, Z_{n3}\}\begin{bmatrix} R_{u11} & R_{u12} & R_{u13} \\ R_{u21} & R_{u22} & R_{u23} \\ R_{u31} & R_{u32} & R_{u33} \end{bmatrix}\begin{Bmatrix} Z_{n1} \\ Z_{n2} \\ Z_{n3} \end{Bmatrix} \tag{11}$$

The function of autocorrelation of the sum of three processes requires the knowledge of function of intercorrelation and autocorrelation of component processes.

Let us consider the function of R_{x3} polar coordinate, equation (10). To determine it, the following elements of matrix $\mathbf{Z}$ must be found: Z_{31}, Z_{32}, Z_{33}.

From equations (2) and (5) we obtain:

$$Z_{31} = \frac{\hat{A}_{31}}{|A|} = \frac{\Delta_{13}}{|A|} = \frac{-\sin\alpha\cos\alpha + \sin\alpha\cos\alpha}{a+b} = 0$$

$$Z_{32} = \frac{\hat{A}_{32}}{|A|} = \frac{\Delta_{23}}{|A|} = \frac{-(\cos^2\alpha + \sin^2\alpha)}{a+b} = -\frac{1}{a+b} \tag{13}$$

$$Z_{33} = \frac{\hat{A}_{33}}{|A|} = \frac{\Delta_{33}}{|A|} = \frac{\cos^2\alpha + \sin^2\alpha}{a+b} = +\frac{1}{a+b}$$

Absolute value $|A| = a + b$.

Substituting (13) to (12), we obtain:

$$\begin{aligned} R_{x3} &= Z_{32}^2 \cdot R_{u2}(\tau) + Z_{32} \cdot Z_{33} \cdot R_{u2u3}(\tau) + \\ &+ Z_{33} \cdot Z_{32} R_{u3u2}(\tau) + Z_{33}^2 \cdot R_{u3}(\tau), \end{aligned} \tag{14}$$

and then

$$\begin{aligned} R_{x3} &= \left(\frac{1}{a+b}\right)^2 \cdot R_{u2}(\tau) - \left(\frac{1}{a+b}\right)^2 R_{u2u3}(\tau) + \\ &- \left(\frac{1}{a+b}\right)^2 R_{u3u2}(\tau) + \left(\frac{1}{a+b}\right)^2 R_{u3}(\tau) \end{aligned} \tag{15}$$

Due to the following relationships between U_2 and U_3 values measured:

$$U_3(t) = \frac{b}{a} U_2(t), \tag{16}$$

the functions of correlation $R_{u2u3}(\tau)$ and $R_{u3u2}(\tau)$ in (15) assume the form of

$$\begin{aligned} R_{u3}(\tau) &= E[U_3(t) \cdot U_3(t+\tau)] = \left(\frac{b}{a}\right)^2 R_{u2}(\tau) \\ R_{u2u3}(\tau) &= E[U_2(t) \cdot U_3(t+\tau)] = \frac{b}{a} E[U_2 \cdot U_2(t+\tau)] = \frac{b}{a} R_{u2}(\tau) \\ R_{u3u2}(\tau) &= \frac{b}{a} R_{u2}(\tau) \end{aligned} \tag{17}$$

Finally,

$$\begin{aligned} R_{x3}(t) &= \left(\frac{1}{a+b}\right)^2 \cdot R_{u2}(\tau) = \frac{b}{a(a+b)^2} R_{u2}(t) - \frac{b}{a(a+b)^2} R_{u2}(\tau) \\ &+ \frac{b^2}{a^2(a+b)^2} R_{u2}(\tau) = \frac{(a-b)^2}{a^2(a+b)^2} R_{u2}(\tau) \end{aligned} \tag{18}$$

Process variance σx_3^2 is determined by comparing time t for autocorrelation function to zero.

$$\sigma x_3^2 = R_{x3}(0) \tag{19}$$

If, however, we wish to obtain the characteristics of the process in the frequency domain, spectrum density is determined from Chinczyn relationship

$$S_{x3}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{1\infty} R_{x3}(\tau) \cdot \exp(-i\omega\tau)\, d\tau \,, \tag{20}$$

and, finally,

$$S_{x3}(\omega) = \frac{(a-b)^2}{2\pi\, a^2 (a+b)^2} \int_{-\infty}^{+\infty} R_{x3}(\tau) \exp(-i\,\omega\tau)\, d\tau \tag{21}$$

Conclusions

The analysis presented showed that, it is possible to determine the correlation function of rotary vibration from simple analytical formulae

Depending on what is needed, the results of calculation can be presented in transient or frequency domain.

Literature

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