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# MULTI-OBJECTIVE OPTIMIZATION APPROACH TO DESIGN OF RC FRAMES


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**Summary:** *This paper presents a discrete optimization of reinforced concrete structures based on an efficient combination of deterministic and stochastic optimization strategies. The deterministic optimization algorithm is used for the detailing of a reinforced concrete cross-section for a given combination of internal forces. The multi-objective stochastic optimization algorithm is then applied to the optimization of a whole structure in terms of basic structural characteristics like types of materials, dimensions of elements or profiles of steel bars.*


## 1. Introduction

An attempt to create an effective design procedure for reinforced concrete structures design goes through the history of Civil Engineering. We limit our attention to frame structures, which are the major part in this field as one of the basic building blocks of various construction systems.

It would be highly desirable to solve the whole design problem as one optimization task but the number of all possible solutions is too high for realistic frame structures. Therefore, it appears to be inevitable to split the process of structural design into two parts – the detailing of a reinforced concrete cross-section and the optimization of a whole structure in terms of basic structural characteristics like types of materials, dimensions of elements or profiles of steel bars.

The main goal of the first part is to fit an interaction diagram of a RC cross-section to a given combination of load cases. Efficient procedures for fast evaluation of internal forces for a general cross-section and stress-strain relationship were proposed in [8]. This task, for a given reinforcing bar diameter, thus reduces to a mere checking of admissible combinations of reinforcements.

The second part of a frame design focuses on the proportioning of building blocks. The goal is to find the best combination of discrete inputs that is, in an appropriate sense, optimal from the point of view of the total cost of the structure as well as maximum deflection of structural members.


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For the single objective case, our experience [5] shows that a modified version of the genetic algorithm based procedure called Augmented Simulated Annealing method is capable of solving this combinatorial task. In this contribution, the multi-objective approach is introduced to tackle several conflicting objectives. The Strength Pareto Approach algorithm [9] is then used for the determination of trade-off surfaces for selected criteria. For this purpose, two important objectives are selected and defined - the total price of a resulting structure and the maximum deflection of structural members. As results of the presented research, Pareto-optimal solutions can be plotted to demonstrate the non-linearity of this design problem and to show the applicability of this approach in Civil Engineering practice.

## 2. Design parameterization

As already mentioned above, we search for a frame structure simultaneously considering price of the structure and maximum deflection as the objectives of optimization. For simplicity, we limit our attention to 2D problems and elements with rectangular cross-sections. Hence, we consider frame structures located in the *xz* plane and our interest is restricted to internal forces acting in this plane: the bending moment $M_y$, the normal force $N_x$ and the shear force $V_z$.

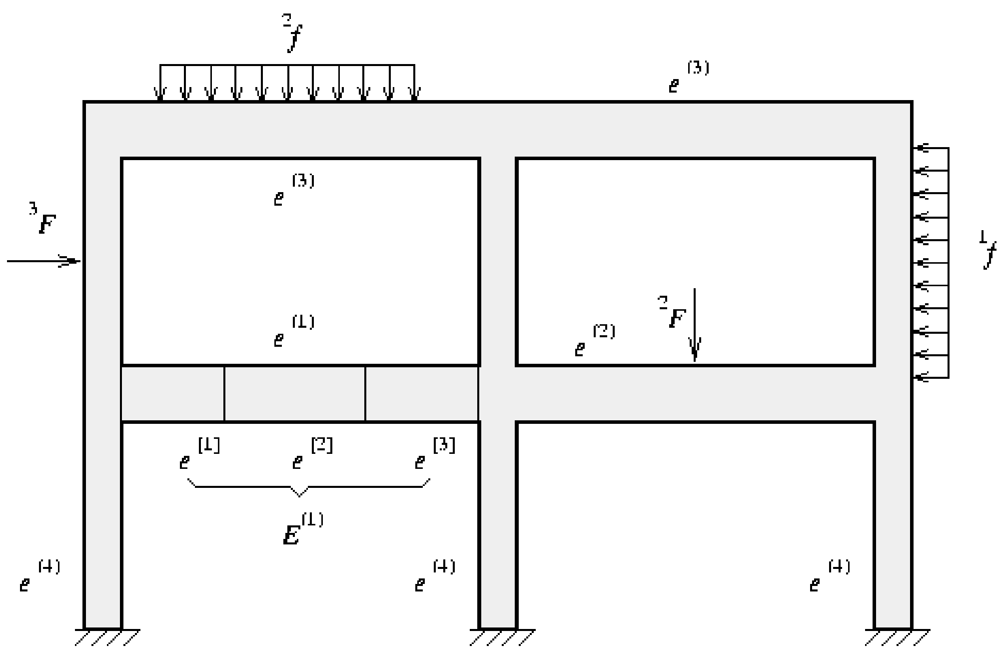



Fig. 1 An example of a frame structure

From the construction point of view as well as optimization itself it appears to be advantageous to decompose the whole structure into $n_d$ *design elements* (see Fig. 1). These user-defined blocks are parts of a structure which a-priori possess identical optimized parameters like dimensions of the cross-section, the area and the diameter of the bending reinforcement etc. In addition, we assume that the structure is discretized into $n_e$ *finite elements,* used for the determination of an internal forces distribution. In the sequel, we will denote a quantity $X$ related to the $i$-th finite element as $X^{[i]}$ while a quantity related to the $i$-th design element as $X^{(i)}$, i.e., values related to finite elements are indexed by square brackets, while quantities connected with design elements are denoted by round ones. Further, $e^{[i]}$ and

$e^{(i)}$ are used for the $i$-th finite and design elements, respectively. The symbol $E^{(i)}$ is reserved for the set of finite elements, related to the $i$-th design element, i.e.,

$$E^{(i)} = \left\{ e^{[j]} : e^{[j]} \cap e^{(i)} \neq \emptyset, j = 1, \ldots, n_e \right\}. \tag{1}$$

Furthermore, the analyzed structure is supposed to be loaded by $n_c$ user-supplied load cases. A quantity $X$ related to the $i$-th design element and the $c$-th load case is denoted as ${}^{c}X^{(i)}$.

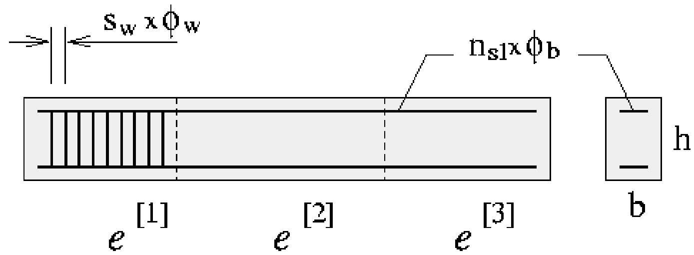


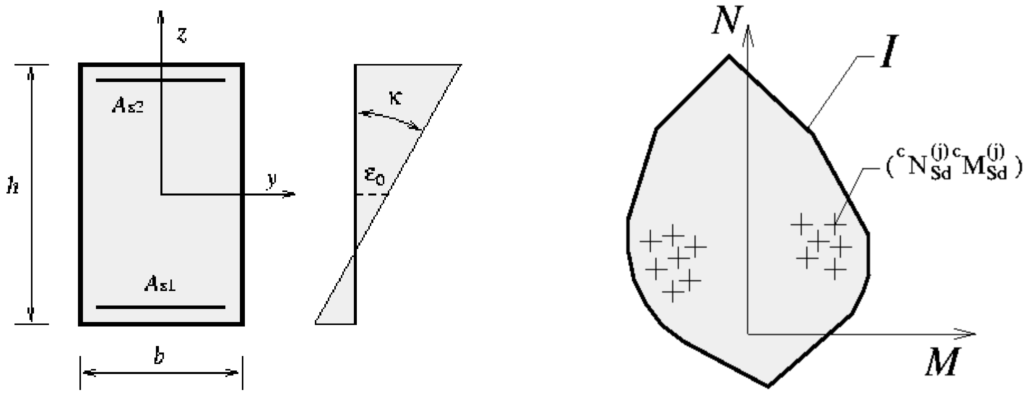

Fig. 2 An example of a design element

As described in the previous paragraph, the design element is used for the definition of basic optimization parameters (see Fig. 2). In our context, the design optimization parameters are the cross-section dimensions $b$ and $h$, the diameter of bending reinforcement $f_b$, the number of reinforcing bars located at the upper and the bottom surfaces of the design element denoted by $n_{s1}$ and $n_{s2}$ and, alternatively, the diameter of shear reinforcement $f_w$ and the spacing of stirrups $s_w$. We assume that the cross-sectional dimensions and stirrup spacing vary with a given discrete difference (e.g., 0.025 m), while $f_b$ and $f_w$ are selected from a given list of available dimensions.

## 3. Ultimate limit state

Generally speaking, the structural requirements imposed by a chosen design standard (e.g., EC2 [1] considered in this work) can be divided into two basic categories: load-bearing capacity and serviceability requirements. In the present work, the load-bearing capacity requirements, discussed in the present section, are directly incorporated into the reinforcement design. The serviceability requirements, on the other hand, are taken into account as the second optimization objective, see Section 4.2.

In our previous works [4] the optimization of cross-section reinforcement was carried out simultaneously with the determination of geometrical parameters of the structure. This approach, however, does not seem to be feasible for larger structures because it would result in a huge amount of optimized variables, rendering the whole problem unmanageable. Thus, we employ a conceptually simple procedure aimed at the reduction of the problem size based on powerful algorithms for fast evaluation of internal forces, that were developed in a work by R. Vondráček [7].




(a) (b)

Fig. 3 The cross-section scheme: (a) a plane of deformation, (b) an interaction diagram

First of all, we briefly list the basic ideas of the procedure of the evaluation of internal forces employed in this work and refer an interested reader to R. Vondráček's works [7], [8] for more detailed discussion. To that end, we assume that a given polygonal cross-section is subjected to a given linear distribution of the $e_x$ strain given by

$$e_x(z) = e_0 + zk\,, \tag{2}$$

where $e_0$ is the strain at the coordinate system origin and $k$ is the curvature in the $z$ direction (see Fig. 3a). Further, the response of a material is governed by a constitutive equation

$$s_x = s\,(e_x). \tag{3}$$

The internal forces $N_x$ and $M_y$ are then provided by the well-known relations

$$N_x = \iint_A s_x\,dA, M_y = \iint_A s_x z\,dA\,. \tag{4}$$

Converting the area integrals (4) into boundary integrals by the Gauss-Green formula together with the fact, that the cross-section is polygonal, yield after some manipulations

$$N_x = -\frac{1}{k^2}\sum_{i=0}^{n_p-1}\frac{1}{k_i}(ss(e^{(i+1)}) - ss(e^{(i)})), \tag{5}$$

$$M_y = -\frac{1}{k^3}\sum_{i=0}^{n_p-1}\frac{1}{k_i}[(z - e_0)ss(z) - 2ss(z) - 2sss(z)]_{e^{(i)}}^{e^{(i+1)}}, \tag{6}$$

where $n_p$ is the number of polygon segments, $k_i$ is the tangent of $i$-th polygon segment, $e^{(i)}$ is the value of the strain at the $i$-th polygon vertex and values $ss(\,.\,)$ and $sss(\,.\,)$ follow from recursions

$$sss(e) = \int_0^e ss(z)\,dz = \int_0^e [\int_0^z s(h)\,dh]\,dz = \int_0^e [\int_0^z [\int_0^h s(y)\,dy\,]\,dh]\,dz\,. \tag{7}$$

For detailed derivation and discussion of these relations together with the treatment of degenerate cases (i.e., $k \to 0$ or $k_i \to 0$) we again refer to the original works [7], [8].

Once we are able to evaluate internal forces for a given plane of deformation determined by $k$ and $e_0$, the boundary of the ***interaction diagram I*** (see Fig. 3b) for a given cross-section can be simply constructed by evaluating the values of the bending moment $M_y$ and the normal force $N_x$ for a given set of extremal deformation planes. Then, the cross-section can sustain the given normal force $N_{Sd}$ and the bending moment $M_{Sd}$ ***iff***

$$(N_{Sd}, M_{Sd}) \in I. \tag{8}$$

In the design procedure, we assume that we are provided with the dimensions of a cross-section $b$ and $h$ and the diameter of the longitudinal reinforcing bars $f_b$. Next, the Codes of Practice provide us with the minimum and maximum values of reinforcement areas $A_{s1} + A_{s2}$, which can be easily converted to a minimum/maximum number of reinforcing bars $n_{s,min}$ and $n_{s,max}$. Then, one can find the minimum reinforcement area such that the condition (8) holds for all elements and load cases, i.e.

$$({}^{c}N_{Sd}^{[j]}, {}^{c}M_{Sd}^{[j]}) \in I,\, j \in E^{(i)}, i = 1, \ldots, n_d, c = 1, \ldots, n_c. \tag{9}$$

Although the proposed procedure is extremely simple, it performs satisfactorily thanks to the very efficient implementation of internal forces evaluation. Furthermore, it effectively eliminates infeasible solutions and thus substantially decreases the dimensionality of the problem.

## 4. Objective functions

Having defined (and appropriately reduced) the domain of all admissible structures, the most suitable solution from this set is to be selected. For this purpose we need to measure the quality of each structure. As mentioned previously, we have selected both the total price of a structure and the maximum deflection as the objectives to be optimized. Note that some deflection limit usually serves as constraint during an optimization process while here is an objective.

### 4.1 Design economy

The total price of the structure follows from the expression

$$f(\mathbf{X}) = V_c P_c + W_s P_s + A_c P_{Ac}, \tag{10}$$

where $\mathbf{X}$ stands for the vector of design variables, $V_c$ is the volume of concrete, $W_s$ is the weight of steel and $A_c$ is the area of concrete connected with form-work; $P_c$, $P_s$ are the prices of concrete per unit volume and steel per kilogram, $P_{Ac}$ is the price of form-work per square meter, which is added to simulate construction costs[1].

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[1] See the paper [6] for more than seventy references dealing with cost optimization of reinforced concrete structures.

### 4.2 Serviceability limit state

As the second objective of the optimization, the maximum deflection of the analyzed structure will be considered. In the current implementation, the maximum sagging of the ***i***-th design element due to the ***c***-th load case is determined on the basis of a simple numerical integration algorithm. To this end, suppose for simplicity that the internal forces distribution for the given load case and design element is known from the elastic analysis. For the given values of the bending moment $M_y$ and the normal force $N_x$[2], the parameters of the deformation plane $e_0$ and $k$, recall Fig. 3a, can be efficiently found by the Newton-Raphson algorithm [7]. Then, under the assumptions of small deformations and small initial curvature, the deflection curve follows from the familiar relation,

$$\frac{d^2 w(x)}{dx^2} = -k\,(M_y(x)), \tag{11}$$

which yields, after integrating Eq. (11) twice,

$$w(x) = -\int_0^x [\int_0^z k\,(M_y(h))\,dh\,]\,dz \; + C_1 x + C_2 \tag{12}$$

with integration constants $C_1$ and $C_2$ determined from boundary conditions for a given design element. Also the analyzed element can be split into several equidistant parts with the length $\Delta x$ and Eqs. (11) and (12) can be replaced by their discretized counterparts. The maximum deflection of the design element is then straightforwardly determined as the extremal value found for all load cases.

## 5. Multi-objective optimization algorithm

The Strength Pareto Evolutionary Algorithm (SPEA), firstly introduced by Zitzler and Thiele [10] in 1999, was selected as the multi-objective optimizer in the present study. The key ideas of this algorithm can be summarized as [10]: storing non-dominated solutions externally in a second, continuously updated population, fitness assignment with respect to the number of external non-dominated points that dominate it, preserving population diversity using the Pareto dominance relationship and incorporating a clustering procedure for the reduction of the non-dominated set. Moreover, all these features are actually independent of the form of crossover and mutation operators. Therefore, it is possible to use operators developed for the single-objective optimization problem [4] without any changes. Last, but certainly not least, advantage of this algorithm is its conceptual simplicity and freely available C++ source code. An interested reader is referred to the article [10] and the Ph.D. thesis [9] for more detailed description of the algorithm as well as extensive numerical investigation of its performance.

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[2]Note that for the notational simplicity, indices *c* and *i* are omitted in the present section.

## 6. Examples and results

We demonstrate the aforementioned design procedure on the benchmark problems, already considered in a conference paper [2]. In particular, two different statically determined structures are examined.

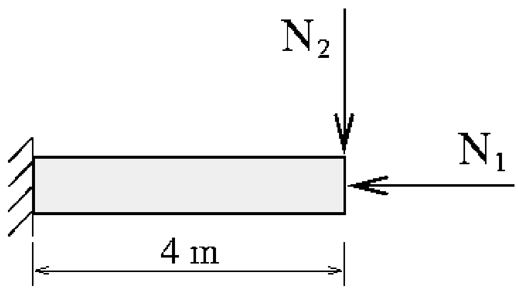


Fig. 4 First example - a cantilever beam

### 6.1 A cantilever beam

Firstly, a cantilever beam, see Fig. 4, with the 4.0 meter span was studied. A concrete model with cylindrical ultimate strength equal to 20 MPa (Class C 16/20) was considered with steel model with the 410 MPa yield stress (Class V 10 425). The cantilever was loaded with two loading cases: (${}^1N_1$=1800 kN, ${}^1N_2$=100 kN) and (${}^2N_1$=300 kN, ${}^2N_2$=100 kN). The theoretical cover of steel reinforcement was set to 0.05 m and the supposed diameter of shear reinforcement was 0.06 m. In the design procedure, the beam width was restricted to $b \in \{0.3, 0.35, 0.4, \text{and } 0.45\}$ m while the heights $h \in \{0.4, 0.5, \text{and } 0.6\}$ m were considered. The longitudinal reinforcement profiles were selected from the list $f_b \in \{10, 12, 14, 16, 18, 20, 22, 25, 28, 32, \text{and } 36\}$ mm. The individual unit prices appearing in Eq. (33) were considered $P_c$ = 2,500 CZK/m$^3$, $P_s$ = 25 CZK/kg and $P_{Ac}$ = 1,250 CZK/m$^2$, respectively[3]. Finally, the integration step $\Delta x$ = 0.25 m was considered for the deflection analysis.

Results are shown in Fig. 5 and Fig. 6 by the visualization methodology presented in the author's thesis [3]. The principle is that all Pareto-optimal solutions can be sorted in the terms of individual functions and also their **x** values can be sorted/drawn in this order. Such picture can give us "sensitivity" information on the variables' influence on the objective function and, therefore, to significantly help the designer to choose the proper solution.

It can be seen that there are 39 non-dominated solutions, which are characterized by the maximal value of the height of the beam $h$ and by non-monotonously increasing amount of steel, see Fig. 5(d). It is also important, that solutions are not created by the small steel profiles which are probably not able to sustain applied internal forces.

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[3]The symbol CZK stands for Czech Crowns.

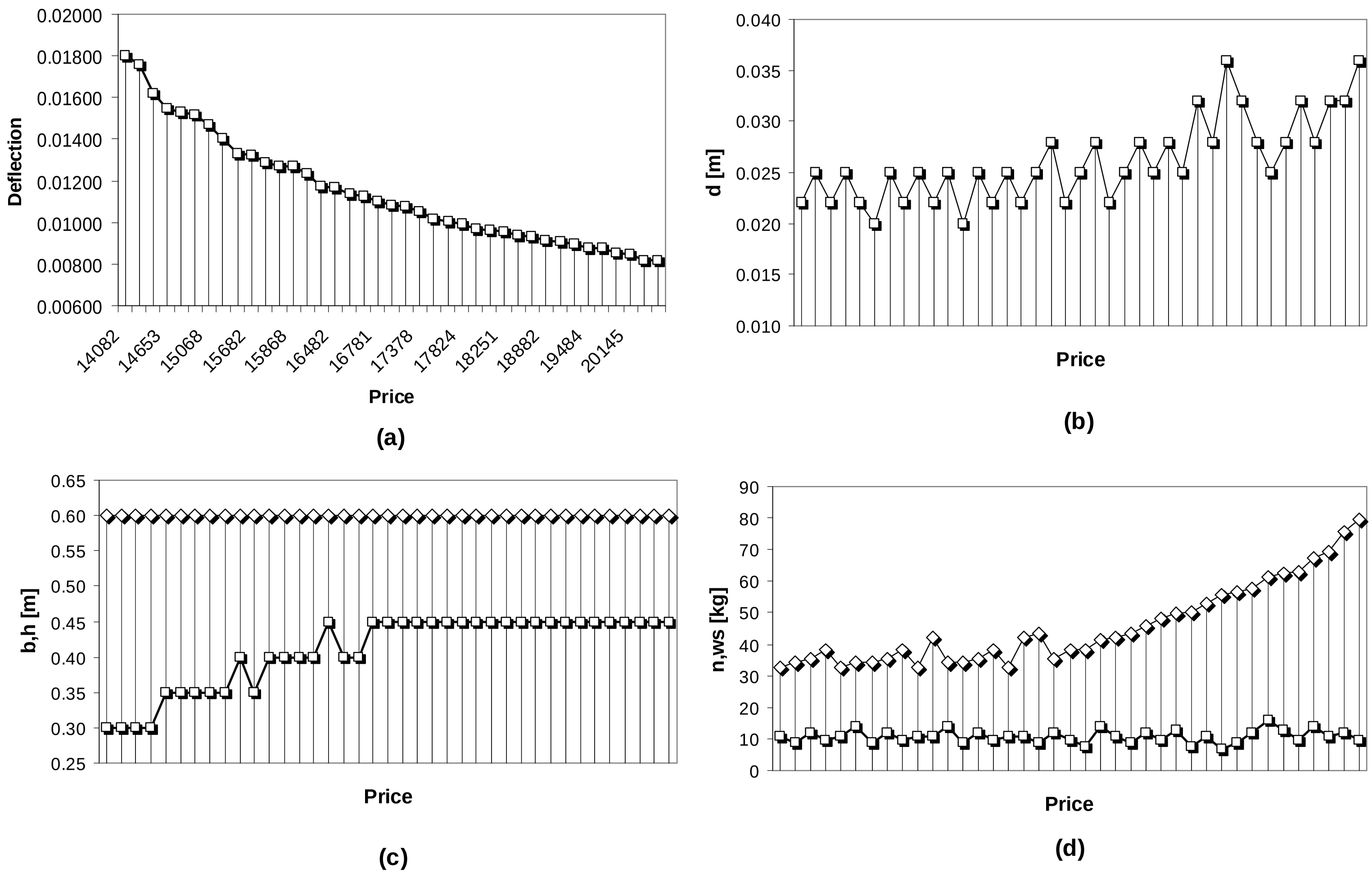


Fig. 5 Results for the cantilever beam example: (a) Pareto-front and Pareto sets - (b) steel profiles *d*, (c) the width *b* and the height *h*, (d) the number of steel bars *n* and the amount of steel $w_s$

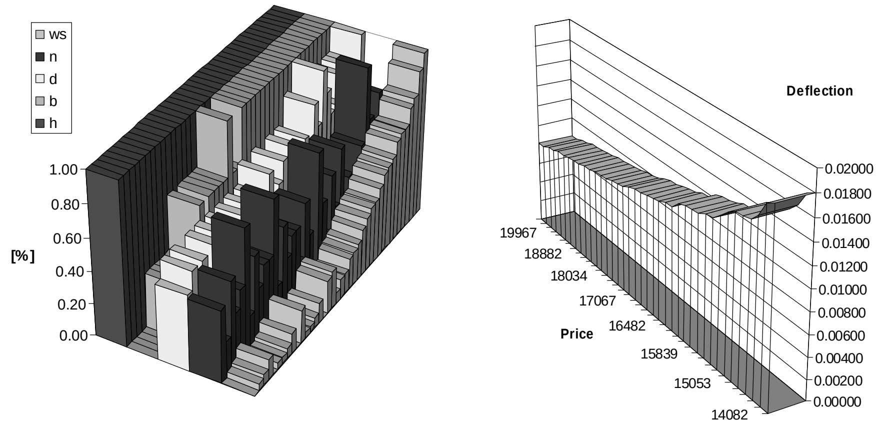


Fig. 6 Results for the cantilever beam example depicted in 3D

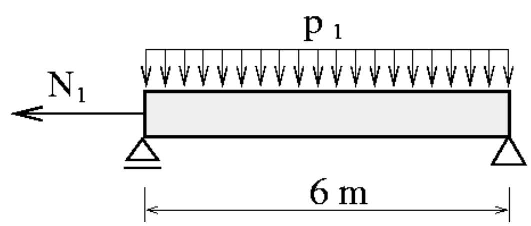


Fig. 7 Second example - a simply supported beam

### 6.2 A simply supported beam

The second example studied was a simply supported beam, see Fig. 7. The span was considered 6 m. The concrete and the steel model were the same as in the previous example, as well as a reinforcement cover, a shear reinforcement profile, geometrical parameters $\boldsymbol{b}$, $\boldsymbol{h}$ and $f_b$. The beam was loaded with three loading cases: (${}^{1}p_1 = 62.5$ kN/m, ${}^{1}N_1$ = -240 kN), (${}^{2}p_1 = 62.5$ kN/m, ${}^{2}N_1 = -1440$ kN) and (${}^{3}p_1 = 62.5$ kN/m, ${}^{3}N_1 = 480$ kN).

At this example, we simulated the scenario of a growing price of steel. The question placed here is: “What will happen if a price of steel grows for 20%?”. Therefore, the Case 1 is characterized by unit prices $P_c$ = 2,500 CZK/m$^3$, $P_s$ = 25 CZK/kg and $P_{Ac}$ = 1,250 CZK/m$^2$ and the Case 2 by the same values for $P_c$ and $P_{Ac}$, but the value of $P_s$ is set to 30 CZK/kg.

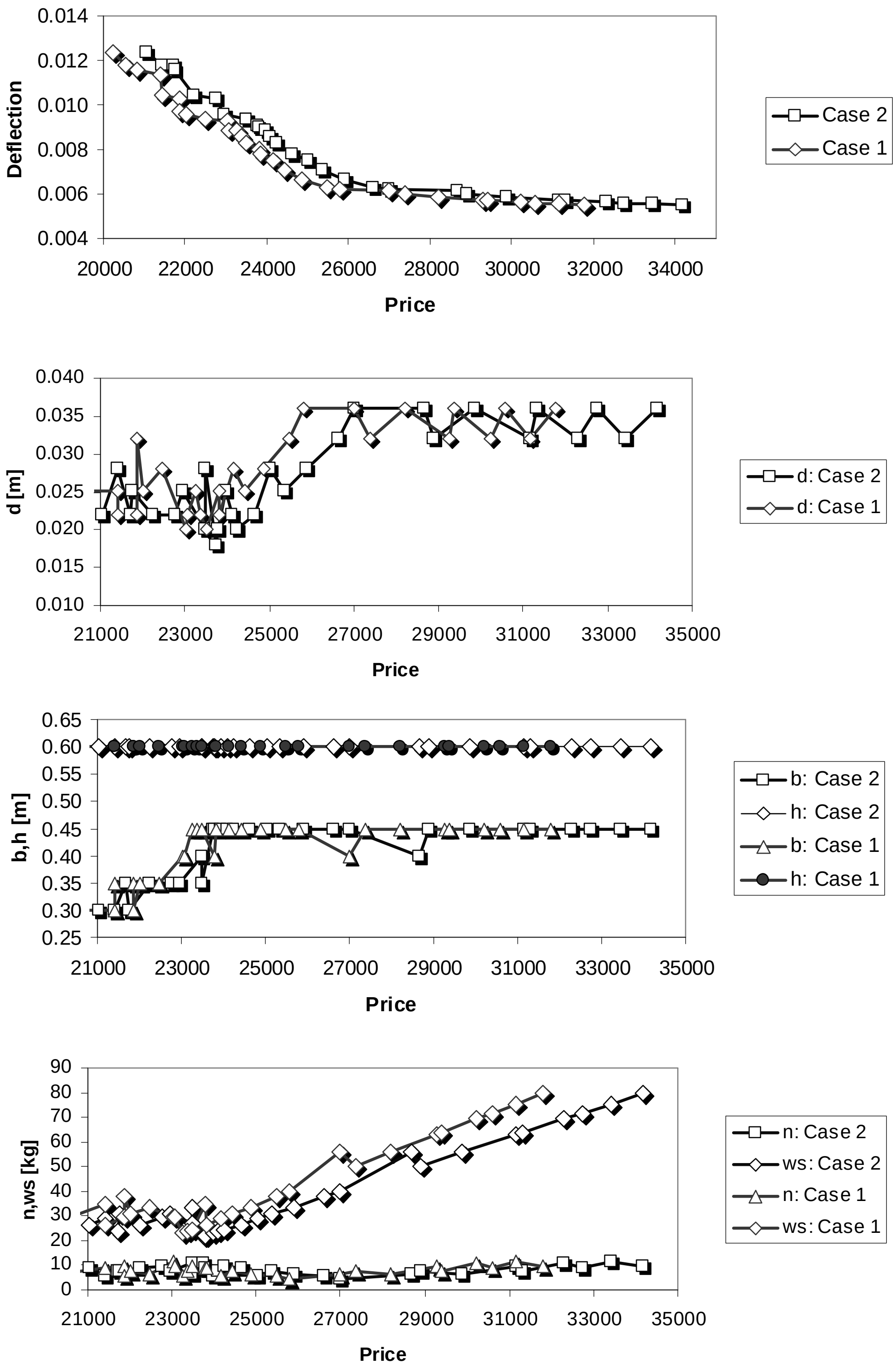


Fig. 8 Results for the simply supported beam example: (a) Pareto-fronts and Pareto sets - (b) steel profiles *d*, (c) the widths *b* and the heights *h*, (d) the number of steel bars *n* and the amount of steel $w_s$

Results are shown again in Fig. 8. The Case 1 is created by 30 non-dominated solutions and the Case 2 by 29 and both cases are characterized by the maximal value of the height of the beam *h*. On the first sight, the growth of the steel price shifts the Pareto-front of the more expensive Case 2 to the right, see Fig. 8(a). But still, there are some designs, where both cases meet each other. The next interesting point is the decrease of the amount of steel, as can be visible in the Fig. 8(d). And finally, by inspecting both Pareto-sets it comes that the last 15 solution are the same - they differ only in the price. Thus, such optimal designs can be seen as stable (or at least less sensitive) with respect to perturbation of steel price and hence more “robust” from the practical point of view.

## 7. Conclusions

Nowadays engineering tasks are different types of designs and, especially, the structural design is very frequent one. And the design of a structure is internally an optimization problem. Moreover, the real design task is always multi-objective. Unfortunately, till nowadays it is often solved as a single-objective problem by combining different, usually conflicting, objectives into only one. This is so inappropriate intervention into the process of finding an optimal solution that the multi-objective methodology presented in this paper seems to be rather a necessity than a choice. As an addition, multi-objective algorithms enable solution for constrains in a more natural way as another objectives.

As an illustrative example, the design of RC frames is introduced. Although the number of all possible solutions for this particular design in a detail is manifold, it was shown that the computational cost can be minimized and the optimized design can be shifted closely to a practical use. To show its applicability, typical examples are solved and the Pareto-fronts in terms of the total price of a structure against its deflection are depicted. The results of the SPEA algorithm revealed that there are 39, 30 and 29 non-dominated solutions for the cantilever and simply supported beam problems, respectively. The trade-off surfaces for both problems appear in Fig. 5(a) and Fig. 8(a). It is clearly visible that even for these rather elementary design tasks, both Pareto-optimal fronts are non-convex and non-smooth due to discrete nature of the optimization problem. This fact justifies the choice of the selected optimization strategy and suggests its applicability to more complex structural design problems.

## Acknowledgements

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