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EXPERIMENTAL EVALUATION OF THE MAGNUS FORCE COEFFICIENT OF THE ROTATING SPHERICAL PARTICLE

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Summary: *The Magnus force coefficient was determined from comparison of theoretical and experimental trajectory of rotating spherical particle falling in calm water. Theoretical trajectories of the particle were calculated using the 2D numerical model of the rotating spherical particle moving in fluid and the proper value of the Magnus force coefficient was established from condition of the best fitting of the experimental trajectory by the calculated one. The mutual influence of the translational and rotational movements was described.*

1. Introduction

The translational movement with the simultaneous rotation of a solid body in fluid is important for many physical problems and engineering applications, for example, the particle saltation in an open channel with a rough bed. In the above mentioned cases, the lateral force, known as Magnus force, acts on the particle:

$$\vec{f}_M = C_M \Omega_p \rho \left[\vec{\omega} \times \vec{V}\right], \tag{1}$$

where Ω_p is the particle volume, ρ is the fluid density, $\vec{\omega}$ is the vector of the particle instantaneous angular velocity, $\vec{V}$ is the vector of the instantaneous translational velocity of the particle center of mass, C_M is the Magnus force coefficient. Usually it is supposed that the particle shape is spherical. Such particle movement in fluid is defined by two dimensionless parameters: the particle Reynolds number $\mathrm{Re}_p = \left|\vec{V}\right| d_p / \nu$ and the rotational particle Reynolds number $\mathrm{Re}_\omega = \left|\vec{\omega}\right| r_p^2 / \nu$ (where d_p and r_p is particle diameter and radius, respectively, ν is the kinematical viscosity of the fluid). Reynolds numbers Re_p and Re_ω are related to different parameters: the first of them to the particle diameter and the second one to the particle radius. This tradition was formed historically.

The theoretical analysis of the Magnus force was performed by Rubinov & Keller (1961) for $\mathrm{Re}_p \ll 1$ and $\mathrm{Re}_\omega \ll 1$. They deduced value of $C_M = 3/4$. Goldshtik & Sorokin (1968)

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theoretically derived that $C_M = 2$ for the range of Reynolds number $Re_p >> 1$ and $Re_\omega >> 1$. However, using of these values of C_M in the numerical models gives incorrect results because the real values of the coefficient C_M are more than order less of them.

Identification of the effect of Magnus force on a rotating sphere dates back to the 17th century, era of I. Newton. However, till the 20th century it was described only qualitatively. The first who estimated the Magnus force quantitatively was Maccoll (1928). He investigated the forces acting on a rotating wooden sphere in the air current. The sphere was rotated by an electric motor via a thin axle and the Magnus force was measured directly by detecting the force acting on the axle. However, such method is difficult to realize if Reynolds numbers are less then 10^4, because the force is too small to be detected. Maccoll found the Magnus force for $3.10^4 \le Re_p \le 10^5$ and $Re_\omega \le 3.10^5$. For the Reynolds number ratio $0.3 \le Re_p/Re_\omega \le 0.7$ the Magnus force coefficient can be roughly fitted by the formula:

$$C_M = 0.08\frac{Re_p}{Re_\omega}. \tag{2}$$

Results of Davies (1949) are valid for high Reynolds numbers: $Re_p=9.10^5$ and $Re_\omega \le 2.5.10^4$. Davies investigated golf balls in the air stream of constant velocity and from his results the formula for Magnus force coefficient depending on rotational Reynolds number only could be derived

$$C_M = \frac{5355}{Re_\omega}\left(1-\exp\left(8.25\cdot 10^{-5}\cdot Re_\omega\right)\right). \tag{3}$$

However, he noticed that for smooth balls Magnus force should be much less than that observed for dimpled golf balls.

Using a conical pendulum technique, Barkla & Auchterlonie (1971) estimated the lift force of a sphere rotating in the air in the range of moderate Reynolds numbers: $1500 \le Re_p \le 3000$ and $800 \le Re_\omega \le 8000$. Based on their results the Magnus force coefficient can be determined as:

$$C_M = 0.034 \pm 0.008. \tag{4}$$

In the range of moderate Reynolds numbers Tsuji et al. (1985) observed trajectories of the sphere, which impinged on an inclined plate submerged in water and bounced. The trajectories were compared with the calculated ones. The experiments were made in the range $550 \le Re_p \le 1600$ and $Re_\omega \le 560$ and the coefficient C_M could be evaluated as:

$$C_M = 0.15 \pm 0.04. \tag{5}$$

Oesterle B. & Dinh Bui (1998) measured lift force acting on a rotating sphere moving in viscous fluid with the constant linear and angular velocities for the intermediate Reynolds numbers: $10 \le Re_p \le 140$ and $5 \le Re_\omega \le 420$. They examined the trajectory of the sphere moving upwards in a liquid at rest. The sphere was equipped with two very thin cylindrical axles. The motion was induced by means of two suspension threads, which were coiled on the axles, yielding a rotational velocity. Their investigation yields the following expression for Magnus force coefficient:

$$C_M = \frac{1}{12}\frac{Re_p}{Re_\omega}\left(1-\exp\left(-0.1\cdot Re_\omega^{0.4}\cdot Re_p^{0.3}\right)\right)+\frac{3}{4}\exp\left(-0.1\cdot Re_\omega^{0.4}\cdot Re_p^{0.3}\right). \tag{6}$$

This brief review shows that there is still a lack of information in the particle Reynolds number range $3\,000 \le Re_p \le 30\,000$, which corresponds, for instance, to the saltation process. In the present study the Reynolds numbers fall in the range: $300 \le Re_p \le 40\,000$ and $200 \le Re_\omega \le 40\,000$. Experiments were carried out with rubber balls moving and rotating in calm water. The analysis of the experimental data is done with the aim to find the real value of C_M. The main difficulty of this analysis is that it isn't possible to get solely the Magnus force. Other forces, such as drag force and drag rotation moment of forces, act also on the ball.

The mutual influence of the translational and rotational particle movements was observed, which causes increasing of the drag force and the drag rotational moment. The formulas, which take into account this influences were suggested and the Magnus force coefficient value was computed. Its values were found more than order less than the theoretically determined values.

2. Experimental procedure

The experiments were carried out in the rectangular glass vessel 300 mm long, 200 mm wide and 800 mm high. The water depth was kept on the level 730 mm. The rubber spherical balls with diameter from 13 to 37 mm were used as particle model. The hairlines were drawn on the balls along two perimeters of the ball with the angle of 90^0 between them to make possible to visualise particle rotation. Each measured particle was speeded up in the special chute what ensured the required particle rotation and translational velocity in the given plane. Different levels of the initial height of the particles in the chute were used to provide different values of the initial translational and angular velocities of the individual particle. Immediately after the particle entry to the water the value of the translational Reynolds number was $Re_p < 40\,000$ and the value of the rotational Reynolds number was $Re_\omega < 40\,000$.

The balls density was chosen close to that of the water, what makes possible to use the standard video system recording images at rate 50 frames per second for visualization. For each experiment 150-200 images were obtained. From the images geometric and kinematical properties of the particle motion were found. However, only the frames outside the unsteady entrance region were used for analysis of the experimental data. The first experimental points were eliminated in order to reject the non-steady process of entry in water, involving air bubbles influence and surface perturbations.

The pilot analysis of the experimental particle trajectories was done. Its radius of curvature was defined and the centripetal force was calculated. It has shown that the value of the Magnus force coefficient is about one tenth of the theoretical evaluations.

3. The numerical simulation

The system of equations describing the particle motion is

$$\frac{d\overline{V}}{dt} = \frac{1}{\rho_p}\left(\vec{F}_d + \vec{F}_g + \vec{F}_B + \vec{F}_M + \vec{F}_m\right), \tag{7}$$

$$J\frac{d\vec{\omega}}{dt} = \vec{M}\,, \tag{8}$$

where ρ_p is the particle density, J is the particle momentum of inertia.

In (7) the terms of the right-hand side denote the following forces per unit volume:

the drag force

$$\vec{F}_d = -\frac{3}{4}\rho\frac{C_d}{d_p}\left|\vec{V}\right|\vec{V}\,, \tag{9}$$

the submerged gravitational force

$$\vec{F}_g = (\rho_p - \rho)\vec{g}\,, \tag{10}$$

the Basset history force

$$\vec{F}_B = -9\frac{\rho}{d_p}\left(\frac{\nu}{\pi}\right)^{\frac{1}{2}}\int_0^t \frac{d\vec{V}}{d\tau}\frac{d\tau}{(t-\tau)^{1/2}}\,, \tag{11}$$

the Magnus force

$$\vec{F}_M = C_M\rho\left[\vec{\omega}\times\vec{V}\right], \tag{12}$$

the added mass force

$$\vec{F}_m = -\rho C_m\frac{d\vec{V}}{dt}\,. \tag{13}$$

The moment of the force acting on the rotating particle in the fluid is:

$$\vec{M} = -C_\omega\frac{\rho}{2}\vec{\omega}\left|\omega\right|r_p^5\,, \tag{14}$$

where $\vec{g}$ is the gravitational acceleration, $C_m = 0.5$ is the added mass coefficient, C_d, C_M and C_ω are the drag force coefficient, Magnus force coefficient and the drag rotation coefficient, respectively.

The aim of the present study is to evaluate the values of the Magnus force coefficient C_M. The values of the coefficients C_d and C_ω will be also derived with the purpose to take into account the mutual influence of the translational and rotational motion of the particle.

The value of the drag force coefficient for the particle translational motion in fluid without rotation (Nino & Garcia, 1994) is given as function of the particle Reynolds number:

$$C_{d0} = \frac{24}{\mathrm{Re}_p}(1 + 0.15(\mathrm{Re}_p)^{\frac{1}{2}} + 0.017\,\mathrm{Re}_p) - \frac{0.208}{1 + 10^4\,\mathrm{Re}_p{}^{-0.5}}\,. \tag{15}$$

and the value of the rotation drag coefficient for the particle rotating in fluid around its center of mass without translation movement (Sawatzki, 1970) is used.

4. Results

The above mentioned system of equations was solved numerically using the 2D model of the particle movement in fluid. The experimental and calculated trajectories and the particle kinematical parameters as functions of time were plotted. The values of the drag force coefficient C_d, Magnus force coefficient C_M and the drag rotation coefficient C_ω were found by the method of the best fitting of experimental data.

4.1. The drag rotation coefficient

The drag rotation coefficient $C_{\omega 0}$, obtained by Sawatzki (1970) for the sphere rotating around its centre of mass in fluid without translational movement, was used in the numerical model. The typical plots of the particle angular velocity ω versus time are shown in Fig. 1 for experimental and numerical data. The calculated values of the particle angular velocity are greater than the experimental ones. It means that the translation motion of the particle increases in reality the drag rotation moment. This fact must be taken into account in the numerical model.

To increase the value of the drag rotation coefficient C_ω the slight calibration factor for it was introduced. The best result was obtained in the case:

$$C_\omega = C_{\omega 0}\left(1 + 0.0044\sqrt{\mathrm{Re}_p}\right). \tag{16}$$

The firm line in Fig. 1 corresponds to the expression (16). It was verified for all experiments and for each one the agreement was satisfactory.

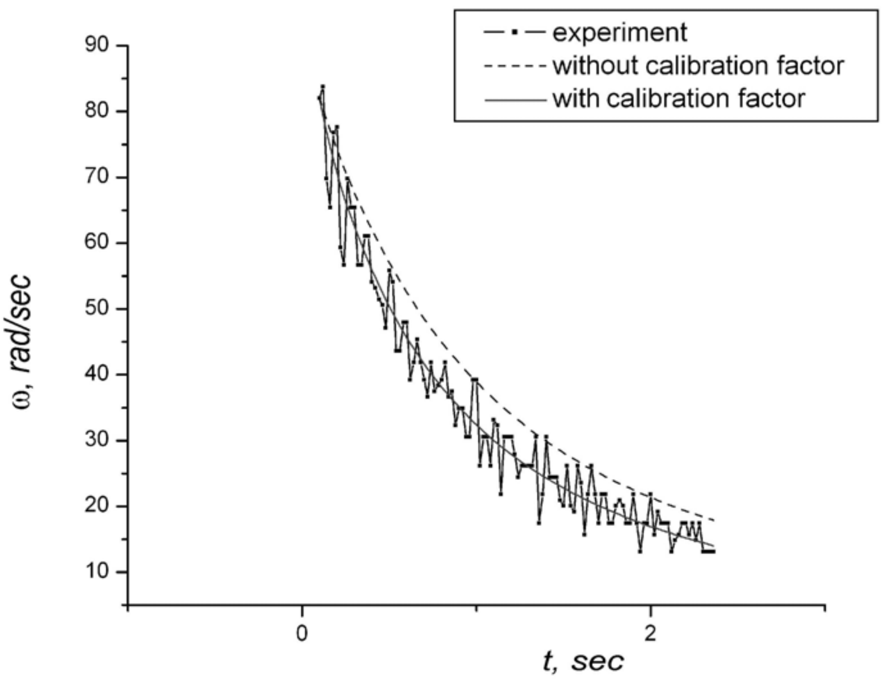

Fig. 1. The particle angular velocity versus time (D = 36.2 *mm*, ρ_p =1017 $kg\ m^{-3}$)

4.2. The drag force coefficient

To evaluate the drag force coefficient the expression (15) for the particle translational motion without rotation in fluid was used as the first approximation for numerical simulation. The experimental and calculated components and magnitude of the translational velocity vector as the functions of time are shown in the Fig. 2, Fig. 3 and Fig. 4. The co-ordinate axis x is horizontal and the co-ordinate axis y is vertical. It is obvious that the calculated absolute values of the translational velocity and of its components are greater than the experimental ones. It means that the rotational motion of the particle increases in reality the drag force what should be taken into account in the numerical model.

To obtain the proper value of coefficient C_d, the slight calibration factor for it should be introduced. The best result was obtained in the case:

$$C_d = C_{d0}\left(1 + 0.065 \cdot \mathrm{Re}_{\omega}^{0.3}\right). \tag{17}$$

The firm lines in the graphs in Fig. 2, 3, 4 correspond to the expression (17). This formula was verified for all experiments and for each of them the agreement was satisfactory.

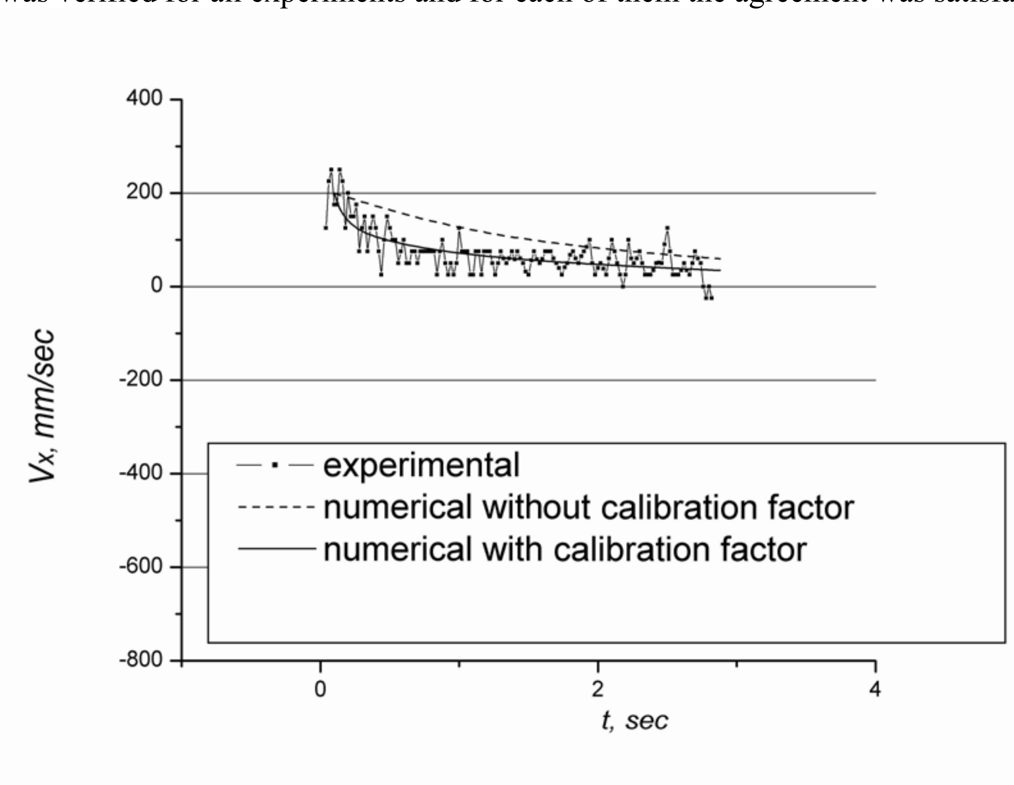

Fig. 2. The horizontal component V_x of the translational velocity versus time (D = 36.2 *mm,* ρ_p =1017 $kg\ m^{-3}$)

4.3. The Magnus force coefficient

The typical experimental particle center trajectory and the calculated one are shown in Fig. 5, where the firm line corresponds to the theoretical value of the Magnus force coefficient C_M = 2 according to Goldshtik & Sorokin (1968). In this case the trajectory of the particle center quickly twists into a helix due to the great centripetal force. It means that this value of C_M is much greater than the real one.

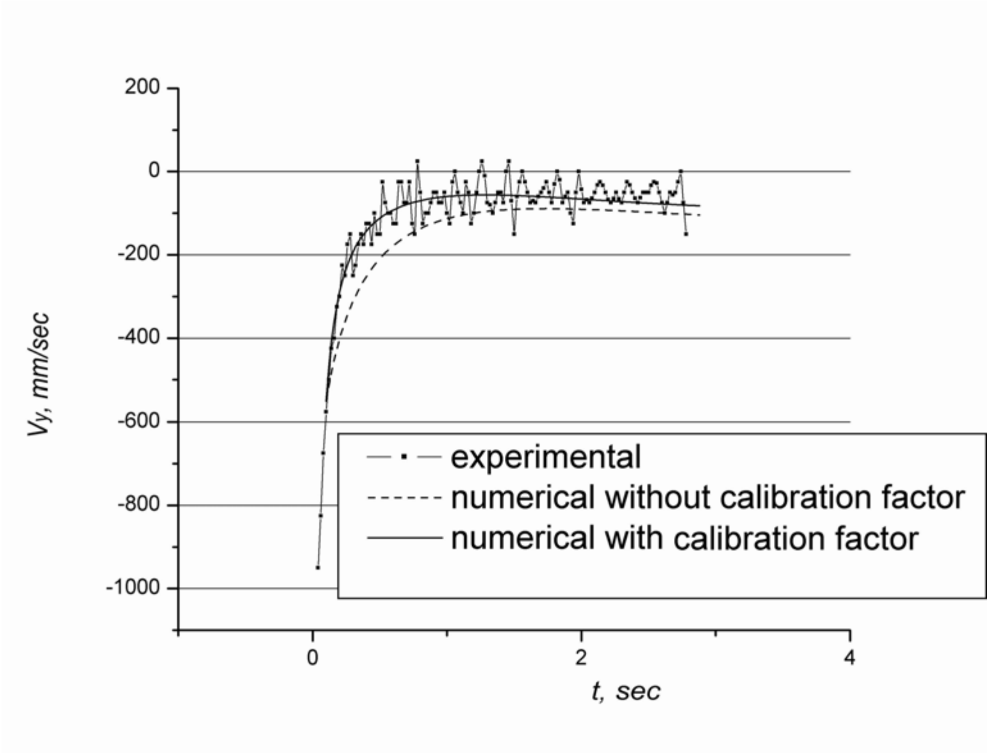

Fig. 3. The vertical component V_y of the translational velocity versus time (D = 36.2 *mm*, ρ_p =1017 $kg\ m^{-3}$)

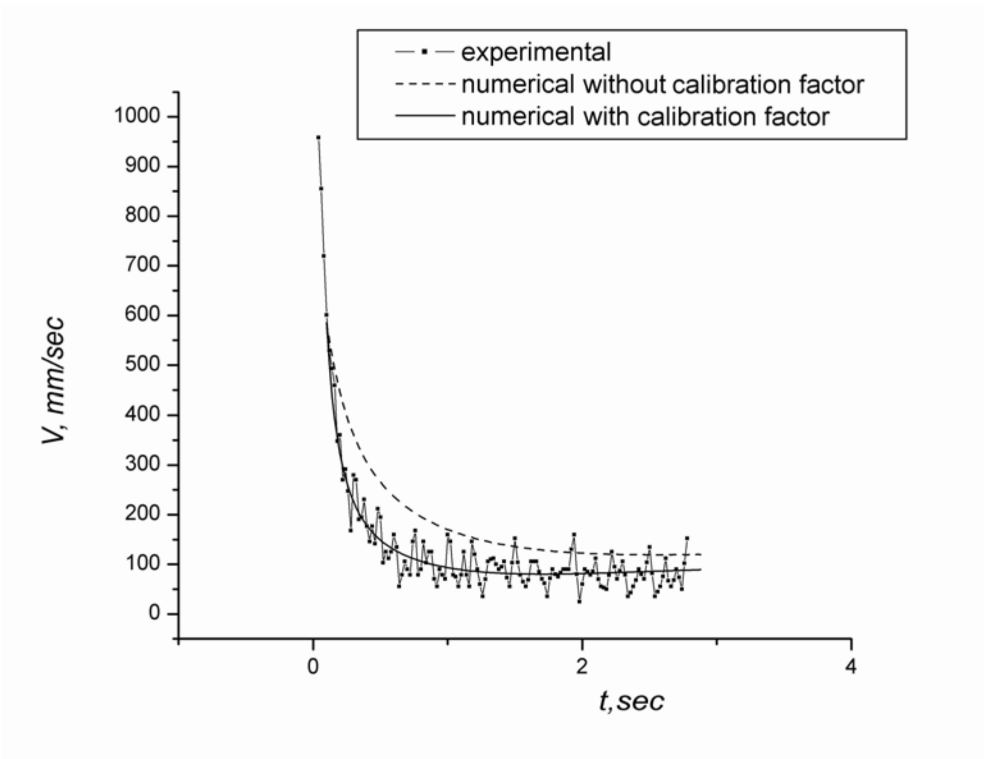

Fig. 4. Magnitude of the translational velocity V versus time t (D = 36.2 *mm*, ρ_p =1017 $kg\ m^{-3}$)

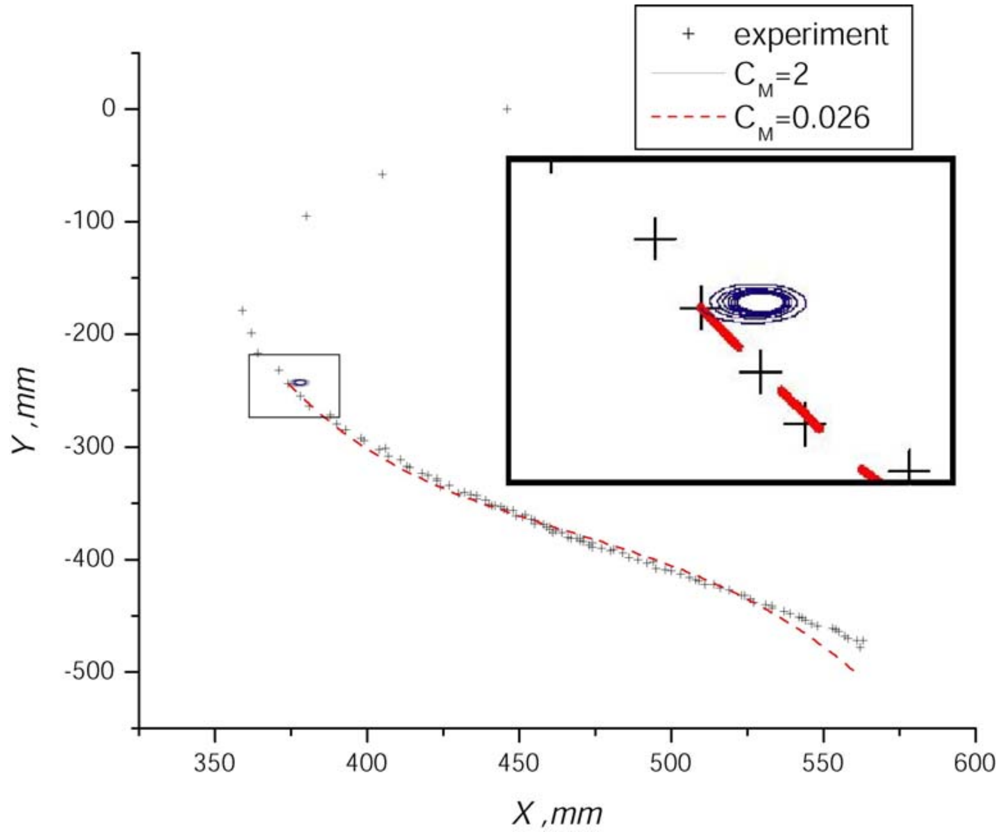

Fig. 5. The trajectories of the particle (D = 36.2 *mm*, ρ_p =1017 *kg* m^{-3})

The real Magnus force coefficient C_M was selected to fit experimental data of the particle trajectories and was chosen to be a constant value for each experimental run. It fell within the range $0.023 \leq C_M \leq 0.048$ for individual experiments with different balls and different initial conditions. The Magnus force coefficient C_M deviation can be explained by its dependence on both translational and rotational particle Reynolds numbers.

5. Conclusions

The experiments were carried out aiming to determine the Magnus force coefficient of the spherical particle. Drag force coefficient and drag rotation coefficient were determined for the case of simultaneous rotational and translational motion for Reynolds numbers in the range $300 \leq \mathrm{Re_p} \leq 40\ 000$ and $200 \leq \mathrm{Re_\omega} \leq 40\ 000$. The numerical simulation was used to evaluate the values of these coefficients.

The mutual influence of the translational and rotational particle movements was studied. It was found that the drag rotation coefficient C_ω depends on both translational and rotational Reynolds numbers. The drag rotation coefficient for the purely rotational movement $C_{\omega 0}$ determined by Sawatzki (1970) should be multiplied by the correction factor $C_\omega = C_{\omega 0}\left(1 + 0.0044\sqrt{\mathrm{Re}_p}\right)$ depending on particle translational Reynolds number $\mathrm{Re_p}$.

The drag coefficient C_d is dependent on both translational and rotational Reynolds numbers, too. Drag force increases as angular velocity increases. Also the drag coefficient for the purely translational movement C_{d0} should be multiplied by the correction factor $C_d = C_{d0}\left(1 + 0.065 \cdot \mathrm{Re}_\omega^{0.3}\right)$ depending on particle rotational Reynolds number Re_ω.

The real value of Magnus force coefficient is much less than that predicted theoretically by Goldshtik & Sorokin (1968). The experimentally determined value of C_M is in good agreement with results of Maccoll (1928) and Barkla & Auchterlonie (1971), who measured lift force for adjacent ranges of particle Reynolds number Re_p. Maccoll (1928) presented his results for the Reynolds number range $3.10^4 \leq \mathrm{Re}_p \leq 10^5$ and $\mathrm{Re}_\omega \leq 3.10^5$; C_M fell in the range $0.027 \leq C_M \leq 0.053$. According to Barkla & Auchterlonie (1971) results the Magnus force coefficient could be evaluated as $C_M = 0.034 \pm 0.008$ for the Reynolds numbers range $1500 \leq \mathrm{Re}_p \leq 3000$ and $800 \leq \mathrm{Re}_\omega \leq 8000$. In our experimental study the Magnus force coefficient fell in the range $0.023 \leq C_M \leq 0.048$ for the Reynolds numbers range $300 \leq \mathrm{Re}_p \leq 40\,000$ and $200 \leq \mathrm{Re}_\omega \leq 40\,000$. As follows from the literature review the Magnus force coefficient is in general a monotonically decreasing function of the translational particle Reynolds number. It is assumed that C_M depends both on the translational and the rotational Reynolds numbers.

6. Acknowledgements

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7. Notation

C_d - drag force coefficient;

C_{d0} - drag force coefficient for the particle moving in fluid without rotation;

$C_m = 0.5$ - added mass coefficient;

C_M - Magnus force coefficient;

C_ω - drag rotation coefficient;

$C_{\omega 0}$ - drag rotation coefficient for the particle rotating in calm fluid;

d_p - particle diameter;

$\vec{F}_B$ - Basset history force per unit volume;

$\vec{F}_d$ - drag force per unit volume;

$\vec{F}_g$ - submerged gravitational force per unit volume;

$\vec{F}_m$ - added mass force per unit volume;

$\vec{F}_M$ - Magnus force per unit volume;

$\vec{f}_p$ - Magnus force;

$\vec{g}$ - gravitational acceleration;

J - particle momentum of inertia;

$\vec{M}$ - moment of the force acting on the rotating particle in fluid;

r_p - particle radius;

$\mathrm{Re}_p = Vd/\nu$ - particle Reynolds number;

$\mathrm{Re}_\omega = \omega r^2/\nu$ - rotational particle Reynolds number;

$\vec{V}$ - vector of the translational particle velocity;

t - time;

ν - kinematical fluid viscosity;

ρ - fluid density;

ρ_p - particle density;

$\vec{\omega}$ - particle angular velocity.

Ω_p - particle volume.

8. References

Barkla H. M. & Auchterlonie L. J. (1971) The Magnus or Robins effect on rotating spheres. *J. Fluid Mech., 47,* pp. 437-447.

Davies J. M. (1949) The aerodynamics of golf balls. *J. Appl. Phys., Vol. 20,* pp. 821-828.

Goldshtik M. A. & Sorokin V. N. (1968) On motion of particle in vortex chamber. *J. Appl. Math. Eng. Phys., 6,* pp. 149-152 (in Russian).

Maccoll J. W. (1928) Aerodynamics of a spinning sphere. *J. Roy. Aero. Soc., Vol. 32,* pp. 777-798.

Nino Y., Garcia M., 1994. Gravel saltation. 2. Modeling, *Water Resour. Res.,* 30(6), 1915-24.

Oesterle B. & Dinh Bui T.(1998) Experiments on the lift of a spinning sphere in the range of intermediate Reynolds numbers. *Exp. Fluids. 25,* pp. 16-22.

Rubinov S. I. & Keller J. B. (1961) The transverse force on a spinning sphere moving in a viscous fluid, *J. Fluid Mech., 11,* pp. 447-459.

Sawatzki O. (1970) Das Stromungsfeld um eine rotierende Kugel. *Acta Mechanica,* 9, pp. 159-214.

Tsuji Y. Morikawa Y. & Mizuno O. (1985) Experimental measurements of the Magnus force on a rotating sphere at low Reynolds numbers. *ASME J. Fluid Eng., 107.* pp. 484-488.