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INFLUENCE OF ELECTRIC MOTORS CHARACTERISTICS ON THE BEHAVIOUR OF DRIVEN SYSTEMS.

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Summary: *The more precise mathematical models of dynamic systems must contain also more exact description of sources of excitations. The paper deals with the analysis of electric motors characteristics and of interaction of motor with limited driving power and excited system. The moment characteristic* $M(\dot{\varphi},\ddot{\varphi},\alpha)$ *of the motor introduces new nonlinear effects into the whole system and causes more complicated responses.*

1. Introduction

Majority of authors in mechanical vibrations fasten their attention on the improvement of mathematical model of the studied oscillating system e.g. by introducing nonlinear springs, various types of nonlinear damping, by introducing more degrees of freedom etc., but the external excitation is supposed to be produced by an ideal unlimited source of energy with always prescribed frequency $f=\omega/2\pi$. They neglect also the feedback effect of vibrating system on the property of exciters. The real sources of excitations are non-ideal, they have limited power, limited inertia, damping properties and their frequency fluctuates according to the instantaneous state of oscillating system.

The feedback effect exists both in mechanical vibrators (where the frequency mainly varies) and in electrodynamics exciters (where the damping is influenced). The mechanical cam-spring exciter driven by electromotor with limited power is investigated in the presented paper.

2. Characteristics of electric motors

Characteristics of motors with limited power depend on the type of motors and can be modeled by different functions of torsion moment $M(\dot{\varphi},\ddot{\varphi},\alpha)$ [Nm], where $\dot{\varphi}$ [rad s^{-1}=s^{-1}] is instantaneous angular velocity, $\ddot{\varphi}$ [rad s^{-2}=s^{-2}] is angular acceleration and α is parameter informing about energy input into the motor.

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The power characteristic $P(\dot{\varphi},\ddot{\varphi},\alpha)$ can be directly derived from moment characteristic $M(\dot{\varphi},\ddot{\varphi},\alpha)$:

$$P(\dot{\varphi},\ddot{\varphi},\alpha)=\dot{\varphi}\; M(\dot{\varphi},\ddot{\varphi},\alpha). \qquad (1)$$

2.1. Stationary characteristics

If the moment of motor does not depend on angular accelerations, the moment characteristic is simpler $M(\dot{\varphi},\alpha)$. This form is often used at analyze of interaction of vibrating system with nonideal motor [1, 2, 3, 4]. These characteristics are also often replaced by the tangent at idle run of motor without load

$$M(\dot{\varphi},\alpha)=-M_0(\dot{\varphi}-\omega(\alpha)), \qquad (2)$$

where M_0 [Nm =s^{-1}] is the slope of characteristic at idle run with angular velocity $\dot{\varphi}=\omega(\alpha)$, α is the corresponding parameter of energy input.

The linear model (2) of characteristic is admissible for a small difference $(\dot{\varphi}-\omega(\alpha))$, that is for small loads. The real motor has nonlinear properties at greater differences $(\dot{\varphi}-\omega(\alpha))$, and the linear model (2) cannot describe the real behavior of motor at greater loads.

More exact characteristic of real motor can be modeled by using exponential function:

$$M(\dot{\varphi},\omega(\alpha))=M_1\left(1-e^{M_0(\dot{\varphi}-\omega(\alpha))}\right), \qquad (3)$$

where M_1 is maximum of torsion moment at very low revolutions $\dot{\varphi}\to 0$, M_0 is proportional to the slope of characteristic at idle running i.e. at $\dot{\varphi}=\omega(\alpha)$:

$$\left(\frac{dM/M_1}{d\dot{\varphi}}\right)_{\dot{\varphi}=\omega(\alpha)}=-M_0\,. \qquad (3a)$$

A simple example of the set of moment characteristics versus angular velocity $\dot{\varphi}$ according to equations (1) and (2) for $M_0=1$, $M_1=1$, and for several values of input parameter α corresponding to the free revolutions $\omega(\alpha)=3,4,...10$ is shown in Fig. 1.

Several motors, e.g. asynchronous, have no monotonic falling down characteristics, but these increase in the low revolutions range $\dot{\varphi}\in(0,\dot{\varphi}_{M\max})$, reaches maximum $M_{\max}$ at the velocity $\dot{\varphi}_{M\max}$ and then at $\dot{\varphi}>\dot{\varphi}_{M\max}$ the moments fall down. An example of such set of characteristics is described by

$$M(\dot{\varphi},\omega(\alpha))/M_1=(c_1+c_2\,\omega(\alpha)+c_3\dot{\varphi})[1-c_4\exp(\dot{\varphi}-\omega(\alpha))], \qquad (4)$$

where various forms of characteristics can be reached by suitable selection of parameters

c_1,c_2,c_3,c_4. For $c_1=0.5$, $c_2=0.05$, $c_3=0.1$, $c_4=1$ is a set of such characteristics shown in Fig. 2. Maximum moment rises with increasing parameter α (or $\omega(\alpha)$).

Constant maximum of moment can be obtained for equation

$$M(\dot{\varphi},\omega(\alpha))/M_1=\frac{(c_1+c_2\,\omega(\alpha)+c_3\dot{\varphi})[1-c_4\exp(\dot{\varphi}-\omega(\alpha))]}{1+0.02\,\omega(\alpha)^2}\,. \qquad (5)$$

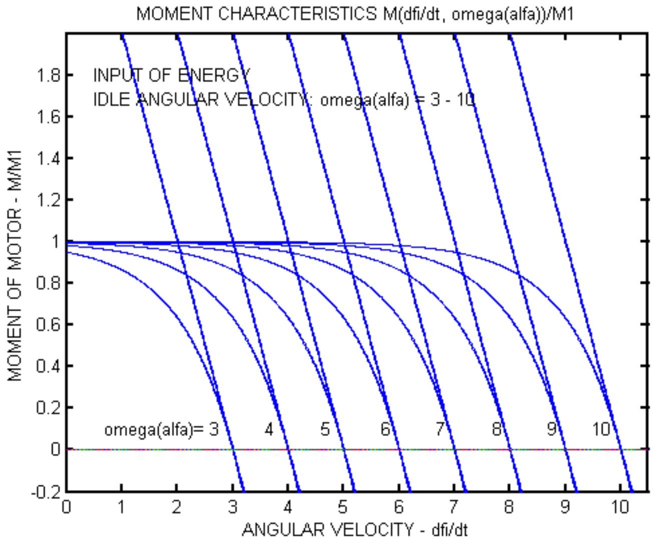

Fig. 1

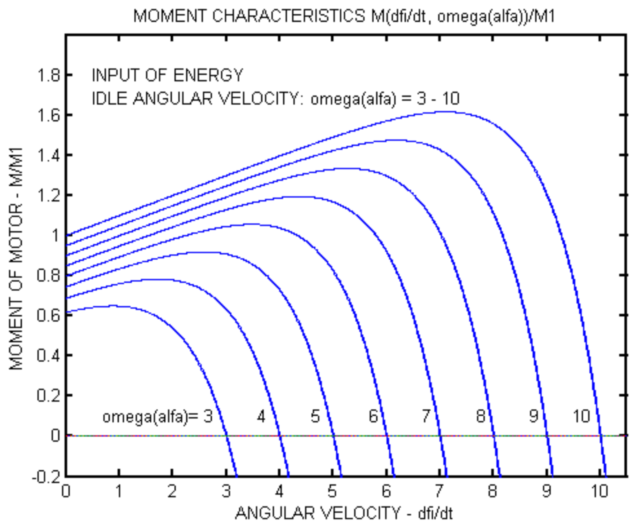

Fig. 2

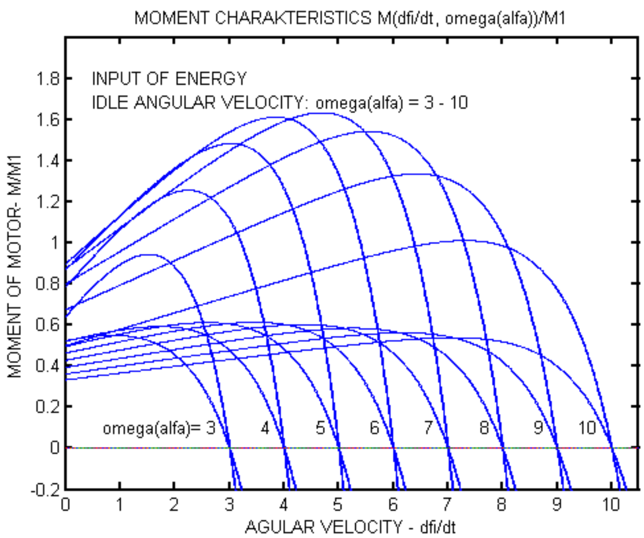

Fig. 3

Torsional moment of some motors decreases at too high values of α. This is the case for example of internal combustion engine at very open choke valve. Corresponding characteristics can be written in following form

$$M(\dot{\varphi},\omega(\alpha))/M_1=(0.3\omega(\alpha)+0.5\dot{\varphi})\,[1-c_4\exp(\dot{\varphi}-\omega(\alpha))](1-\omega(\alpha)/12) \tag{6}$$

and they are shown in Fig. 3 together with characteristics having constant maximum of torsion moment independent on value α (eq. 5).

2.1. Dynamic characteristics

Stationary characteristics described in previous chapter are often used for the solution of stationary or weakly non-stationary processes, where the variation of angular velocity $\frac{d(\dot{\varphi})}{dt}$ (or $\omega(\alpha(t))$) of motor revolutions is negligible against the velocity $\dot{\varphi}$. If this variation is considerable, the more exact mathematical expression has to be used:

$$M(\dot{\varphi},\ddot{\varphi},\omega(\alpha(t))),$$

where the input energy parameter $\alpha(t)$ is time variable. One possibility how to express the dependence on acceleration $\ddot{\varphi}$ is to add a function $M_1(\ddot{\varphi})$ to the stationary characteristic

$$M(\dot{\varphi},\ddot{\varphi},\omega(\alpha))=M(\dot{\varphi},\omega(\alpha))+M_T(\ddot{\varphi}). \tag{7}$$

The simplest dynamic characteristic is linear one

$$M(\dot{\varphi},\ddot{\varphi},\omega(\alpha))=M_0(\omega(\alpha)-\dot{\varphi})+b\ddot{\varphi}. \tag{8}$$

Influence of linear damping on the variation of moment *M* and angular velocity $\dot{\varphi}$ on the dynamic characteristics with coefficients of idle speeds $\omega(\alpha)=5-8\,\mathrm{s}^{-1}$, and $M_0=1\,\mathrm{kg\,m^2s^{-1}}$, $b=0.2\,\mathrm{kg\,m^2}$ for two amplitudes $a_\varphi=0.5;1\ [\mathrm{s}^{-1}]$ of velocity $\dot{\varphi}$ swinging around the average angular velocities $\dot{\varphi}_{av}=4-7\,\mathrm{s}^{-1}$

$$\dot{\varphi}=\dot{\varphi}_{av}+a_\varphi\cos\Omega t,\quad(\Omega=1\,\mathrm{s}^{-1}) \tag{9}$$

is shown in Fig. 4 in the form of elliptic hysteresis loops. Corresponding fluctuation of moment is given by (8):

$$M=M_0(\omega(\alpha)-\dot{\varphi}_{av}-a_\varphi\cos\Omega t)+0.2\,\Omega\,a_\varphi\sin\Omega t. \tag{8a}$$

The same fluctuation (9) of velocity $\dot{\varphi}$ of motor with static characteristics (4) produces the variation of moment *M*:

$$M=M_0(0.5+0.05\omega(\alpha)+0.1\dot{\varphi})\,[1+\exp(\dot{\varphi}-\omega(\alpha))]+0.2\ddot{\varphi}. \tag{10}$$

Result of numerical solution is shown in Fig. 5. The loops are no ellipses, but are rather distorted due to the curved static characteristics.

Dynamic characteristics are however affected also by the damping law $M_T(\ddot{\varphi})$. The dynamic loops in the case of nonlinear damping proportional to the third power of acceleration $\ddot{\varphi}^3$ are drawn in Fig. 6.

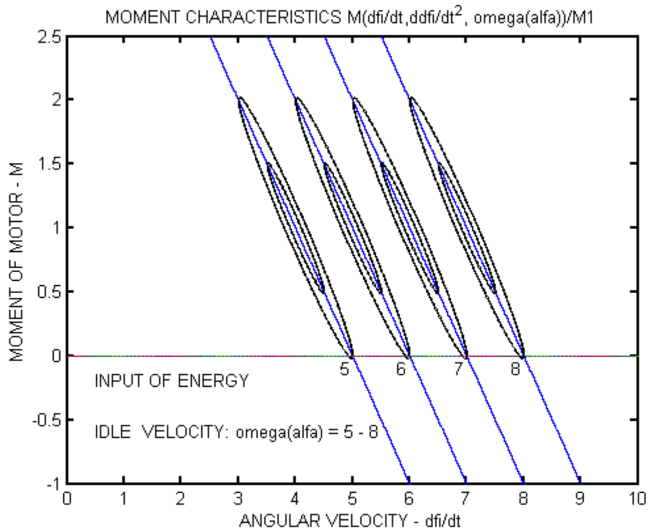

Fig. 4

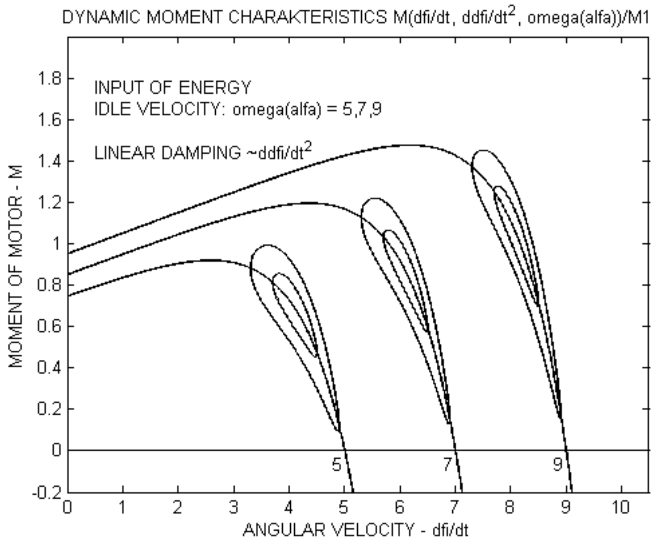

Fig. 5

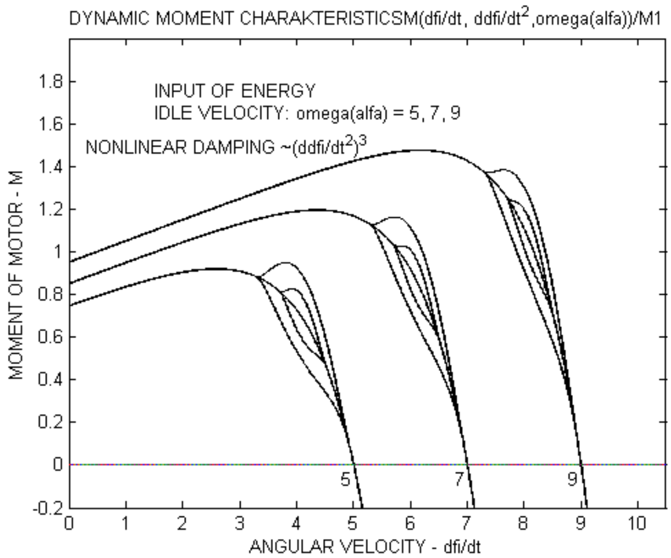

Fig. 6

Presented examples of dynamic characteristics are used as the first approximation of the real characteristics. The most exact description of dynamic properties of motors we get by considering the electromagnetic processes inside the electric motors.

4. Effect of electromagnetic circuit

The basic mathematical model of direct current electric motor was given e.g. in [5] at the presumption that the inductivity L, resistance R and magnetic flux ϕ vary linearly with the current. First of all, these physical magnitudes will be here considered as constants for simplicity.

The connection between the motor (1) and working machine (2) with a mechanical exciter (3) is supposed to be rigid.

Time variable voltage $U(t)$, which is the input into motor rotating with the angular velocity $\dot{\varphi}$, generates the current *i* according to the equation

$$L\,di/dt + Ri + \phi\dot{\varphi} = U(t). \tag{11}$$

This equation must be joined to the equation (12a) of revolution of rotor φ and as the case may be also to the equation of oscillating subsystem (12b):

$$I_m\ddot{\varphi} = \phi\, i - M_z(\dot{\varphi},t) - k_m r\sin\varphi\,(r\cos\varphi - y + y_0) \tag{12a}$$

$$m\ddot{y} + b\dot{y} + ky + \varepsilon f(y,\dot{y}) = k_m r(\cos\varphi - y + y_0). \tag{12b}$$

Electro-mechanic schematic structure of such system is in Fig. 7.

4.1. Statical moment characteristics (constant L, R, ϕ)

When the motor under the constant voltage $U = 40 \div 220V$ is loaded only by the constant torsion moment $M_z(\dot{\varphi},t) = M_z$ without oscillating component $(k_m = 0)$, then stationary angular velocity is

$$\dot{\varphi} = \omega = U_0/\phi - \left(R/\phi^2\right)M_z \quad \text{or} \quad M_z = \phi^2/R\,(U_0/\phi - \omega) \tag{13}$$

Moment characteristics are linear functions between ω and M_z, demonstrated in Fig. 8 by a set of straight lines with the slope

$$\frac{dM_z}{d\omega} = -\phi^2/R \tag{14}$$

and with idle angular velocity (at $M_z = 0$)

$$\omega(\alpha) = U_0/\phi. \tag{14a}$$

These characteristics are calculated for $R = 5\,\Omega$, $\phi = 0.7W_b$ (see [5]) and $U_0 = 40, 60, \ldots 220\,V$.

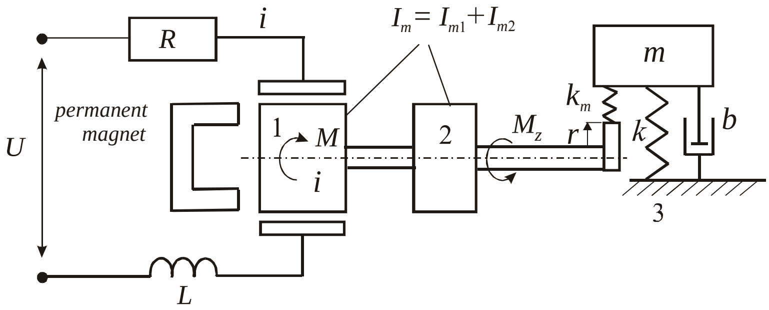

Fig. 7

4.2 Dynamical characteristics, moment versus revolutions.

Dynamic properties become evident at the time variation of some parameter of system, usually variation of input voltage U or loading moment M_z. In the further case let be

$$M_z(\dot{\varphi},t) = M_{z0} + M_1 \cos \omega_1 t \, ,$$

where ω_1 is a frequency independent on revolution $\dot{\varphi}$. Due to the variable load at constant input voltage U, the angular velocity $\dot{\varphi}$ and current i vary according to equations (11), 12a) (at k_m=0). Result of numerical solution for $L = 0.05H$, $I_m = 0.01$ kgm^2 and loading moment $M_{z0} = 2$ Nm, $M_1 = 2$ Nm, $\omega_1 = 5\,\mathrm{s}^{-1}$ is presented by the elliptic hysteresis loops on the left side of Fig. 8. The loops area, corresponding to the lost energy, increases at rising velocity of loading variation, i.e. at higher frequency ω_1. This is shown on the right side of the same Figure, where the dynamic trajectories for twice higher frequency $\omega_1 = 10\,\mathrm{s}^{-1}$ are drawn.

4.3. Nonlinear properties of magneto-electric circuit.

Graphs in Fig. 4 were computed at assumption of constant values of L, R, ϕ. In the real motor, these physical quantities change theirs values in dependence on current i. According to [5] let us use linear form:

$$L = L_0 + L_1 i$$
$$R = R_0 + R_1 i \tag{15}$$
$$\phi = \phi_0 + \phi_1 i \, .$$

As the resistance R changes very moderately for used values of i (in comparison with changes of ϕ and L), the resistance R will be considered as constant $(R_1 = 0)$.

Equations of motion of rotor loaded by moment $M_z(t)$ are then

$$(L_0 + L_1 i) di/dt + R_0 i + (\phi_0 + \phi_1 i)\omega = U(t) \tag{16}$$
$$(\phi_0 + \phi_1 i)i - M_z(t) = I_m \, d\omega/dt \, .$$

Stationary state at $M_z(t) = M =$const. and $U(t) = U =$const. means $di/dt = 0$ and $d\omega/dt = 0$. Equations (16) then simplify to

$$(R + \phi_1 \omega)i = U - \phi_0 \omega \tag{16a}$$
$$(\phi_0 + \phi_1 i)i = M$$

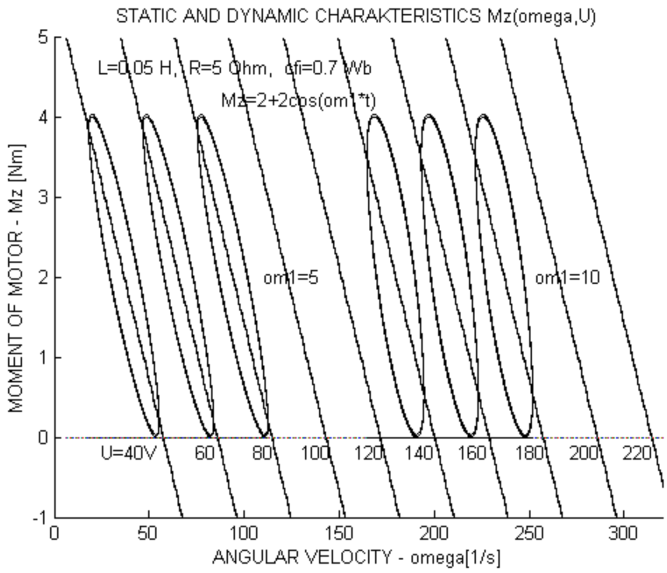

Fig. 8

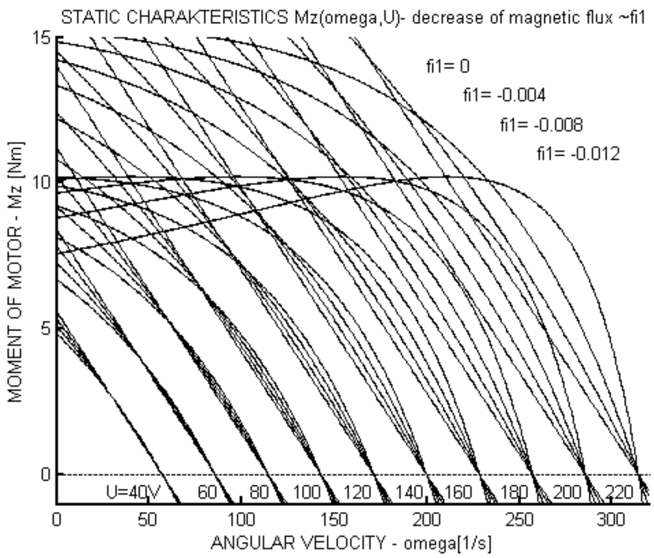

Fig. 9

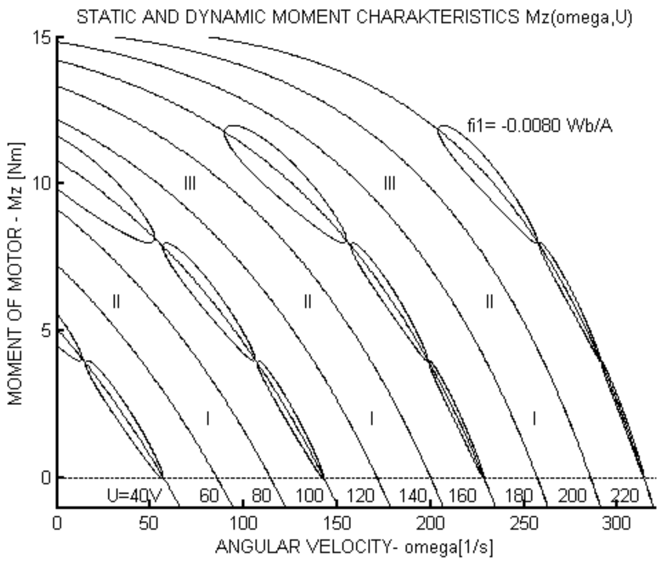

Fig. 10

Elimination of current i gives relation between M and ω, that is stationary characteristic of electro-mechanic system of motor:

$$M = \left(\frac{U-\phi_0\,\omega}{R+\phi_1\,\omega}\right)^2 \phi_1 + \left(\frac{U-\phi_0\,\omega}{R+\phi_1\,\omega}\right)\phi_0 = (U-\phi_0\,\omega)(U\,\phi_1 + R\phi_0)/(R+\phi_1\,\omega)^2. \qquad (17)$$

Idle angular velocity $\omega(\alpha)$ (free-load run) are given by the point of intersection of characteristic with horizontal axis (M=0):

$$(U-\phi_0\,\omega(\alpha))=0, \quad \omega(\alpha)=U/\phi_0. \qquad (18)$$

Initial moment at start $(\omega=0)$ is

$$M(0)=U(U\,\phi_1+R\phi_0)/R^2. \qquad (19)$$

This moment is linearly proportional to the voltage U at $\phi_1=0$ and becomes lower at non-zero values $\phi_1<0$.

Idle angular velocity $\omega(\alpha)$ does not change at variation of ϕ_1, but this value influences the slope of characteristic at idle speed $\omega(\alpha)$:

$$(dM/d\omega)_{\omega=\omega(\alpha)} = (\phi_0\,\phi_1\,\omega(\alpha)-\phi_0 R-2U\,\phi_1)/(R+\phi_1\,\omega(\alpha))^3. \qquad (20)$$

Influence of decrease of magnetic flux ϕ_1 on the moment stationary characteristics is presented in Fig. 9, calculated according (17) for values $R=5\Omega$, $\phi_0=0.7W_b$, $U=40,60,80,...220V$, and for $\phi_1=0;-0.004;-0.008;-0.012$ W_b/A. It is evident, that the dependence of magnetic flux on the current considerably changes moment characteristics, the motor has limited power, and near the idle velocity it became harder source of energy.

Dynamic characteristics of motor as electro-magnetic-mechanic system can be obtain by solution of differential equations (16). Using parameters: $L_0=0.05H, R_0=5\Omega$, $R_1=0,\ \phi_0=0.7W_b,\ \phi_1=-0.008W_b/A,\ I_m=0.01\,\mathrm{kgm}^2$, decrease of inductivity let be $L_1=-0.0005\,H/A$. The variation of loading moment is

$$M_z = M_{z0} + M_1\cos\omega_1 t, \qquad (21)$$

where M_1=2Nm, $\omega_1=2.5$ rad/s, and central value $M_{z0}=2$; 6 and 10 Nm. The supply voltages at which loadings moments act are U=40; 100; 160; 220 V. Variation of moments M_z and angular velocities $\omega=\dot{\varphi}$ is illustrated in Fig. 10 by closed loops in the form of slightly deformed ellipses in three levels according to the central values $M_{z0}=2(I), 6(II), 10(III)$. The width of these ellipses decreases with increasing voltage U, and increases with the central value M_{z0} of loading moment.

The dynamic characteristics of real motor with limited power strongly influence the behavior of various technical drive systems.

5. Electromechanical drives

Electric motors are used for driving of lot of working machines types, the loading moments of which have very different forms: continuous, oscillating, stationary, non-stationary, with impacts, etc.

Important kinds of systems are vibrating systems, where many typical phenomena occur. There are instabilities, jumps, bifurcations, beats, subharmonic oscillations etc. The nonlinear properties of the mechanical oscillating system are mainly considered as the consequence of the presence of the internal nonlinear elements (spring or nonlinear damping) in the system. But the nonlinearity can be caused also by the non-ideal properties of the source of exciting forces. The behavior of the machine forced by the external harmonic force is very often studied on the simplified dynamical model, the exciting frequency ω of which is supposed to be constant, independent on the current state of vibrating system.

The energy sources of the real physical structures have always limited power and therefore, at the more exact studies of the behavior of vibrating systems, the properties of the sources of exciting energy as electromechanical subsystem have to be taken into consideration.

As on example let us investigate the system consisting of electromotor with characteristic described by equation (17) driving the linear oscillating system with one degree of freedom. The shaft of the motor is provided by a crank or cam mechanisms (eccentricity r) connected by spring (stiffness k_m) with the vibrating system (Fig. 7). Differential equations describe the properties of electromagnetic subsystem, torsion motion of rotor and vibrations of linear oscillating system:

$$\begin{aligned} & di/dt = \left[U(t) - Ri - (\phi_0 + \phi_1 i)\,\dot{\varphi}\right]/(L_0 + L_1 i) \\ & d^2\varphi/dt^2 = \left[(\phi_0 + i\phi_1)\,i - M_{z0} + k_m r \sin\varphi\,(y + y_0 - r\cos\varphi)\right]/I_m \\ & m\,d^2y/d^2t + b\,dy/dt + ky = -k_m\,(y + y_0 - r\cos\varphi). \end{aligned} \tag{22}$$

For generalization of the mathematical model, it is very convenient to introduce dimensionless parameters instead of 18 dimensional variables and constants:

$$\begin{aligned} & i[A],\ t[\mathrm{s}],\ U[V = \mathrm{kgm^2s^{-3}}A^{-1}],\ R[\Omega = \mathrm{kgm^2s^{-3}}A^{-2}],\ \phi_0[W_b = \mathrm{kgm^2s^{-2}}A^{-1}], \\ & \phi_1[W_b = \mathrm{kgm^2s^{-2}}A^{-2}],\ \varphi[1],\ L_0[H = \mathrm{kgm^2s^{-2}}A^{-2}],\ L_1[H/A = \mathrm{kgm^2s^{-2}}A^{-3}], \\ & M_z[\mathrm{Nm} = \mathrm{kgm^2s^{-2}}],\ K_m[\mathrm{N/m} = \mathrm{kgs^{-2}}],\ r[\mathrm{m}],\ y[\mathrm{m}],\ y_0[\mathrm{m}],\ I_m[\mathrm{kgm^2}], \\ & m[\mathrm{kg}],\ b[k\mathrm{gs^{-1}}],\ k[k\mathrm{gs^{-2}}]. \end{aligned} \tag{23}$$

It is sufficient to use only fewer variables (in our case only 18-4=14 dimensionless variables) for describing the motion of the whole electro-magneto-mechanical system. They are:

$$\begin{aligned} & y/r = x,\ y_0/r = x_0,\ t\sqrt{k/m} = \tau,\ b/\sqrt{k/m} = \beta,\ k_m/k = \kappa,\ \varphi = \varphi,\ I_m/mr^2 = \Theta, \\ & \frac{M_z}{kr^2} = \mu,\ iR\sqrt{m/k}/\phi_0 = I,\ U\sqrt{m/k}/\phi_0 = u,\ L_0\sqrt{k/m}/R = \Lambda_0,\ kL_1\phi_0/(R^2m) = \Lambda_1, \\ & \phi_1\sqrt{k/m}/R = \psi_1,\ \frac{Rr^2\sqrt{km}}{\phi_0^2} = \rho. \end{aligned} \tag{24}$$

The equations (22) can be then transformed into more simple form

$$
\begin{aligned}
&(\Lambda_0+\Lambda_1 I)\,dI/d\tau = u(\tau) - I - (1+\psi_1 I)\frac{d\varphi}{d\tau}\\
&\Theta\,\frac{d^2\varphi}{d\tau^2} = \frac{1}{\rho}\,[1+\psi_1 I]\,I - \mu + \kappa \sin\varphi\,(x + x_0 - \cos\varphi) \qquad (25)\\
&d^2x/d\tau^2 + \beta\,dx/d\tau + x = \kappa(\cos\varphi - x - x_0).
\end{aligned}
$$

An example of application of these equations is shown in Fig. 11, where the dimensionless time (τ) history of oscillation displacement $(x = y/r)$, angular velocity (omega $= d\varphi/dt$), the total angle of revolution (φ) and current $(I = iR\sqrt{m/k}\,/\phi_0)$ in the motor winding are drawn. The input voltage was slowly increasing

$$u = U\sqrt{m/k}\,/\phi_0 = 5 + 0.01\tau.$$

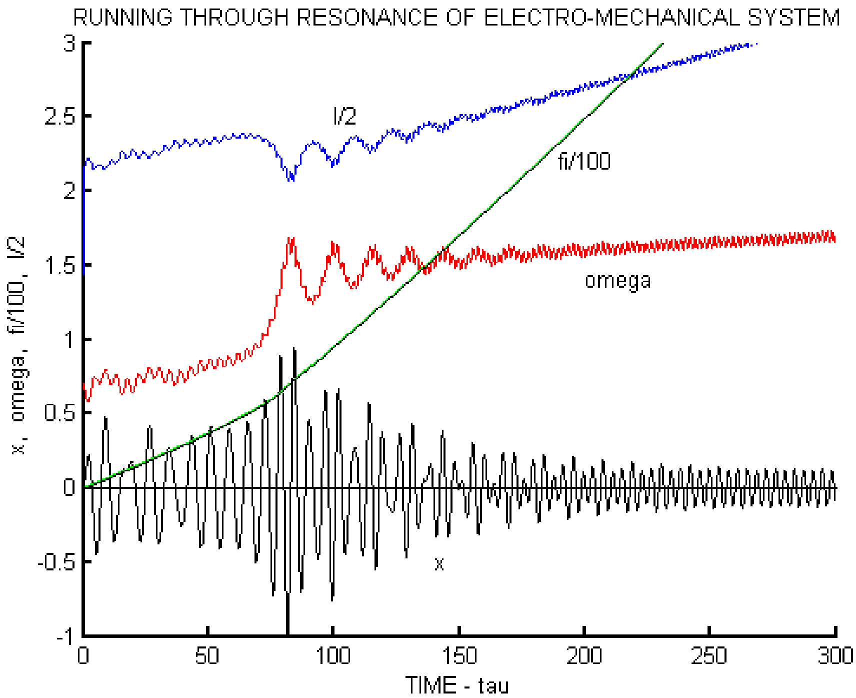

Fig. 11

Motor was loaded through the spring-cam exciter by oscillating linear system with parameters m =1000 kg, k =1000 kgs^{-2}, $\beta = b/\sqrt{km} = 0.05$, $\kappa = k_m/k = 0.2$. (see Fig. 7).

From diagrams in Fig. 11 it is evident, that oscillation with frequency ω of displacement x of mechanical subsystem causes the oscillation both of the angular velocity $\dot\varphi = \omega$ and of current I with the twice higher frequency 2ω. When the angular frequency ω reaches value of eigenfrequency $\sqrt{k/m} = 1$, a jump to the higher revolutions occurs and the displacement x

oscillates in beats. This beats copy themselves also into angular frequency ω and in current *I*. The total angle of revolution $\varphi = \int wd\tau$ is given by a monotonous increasing curve $\varphi/100$.

Conclusion

The interaction between the source of energy with limited power - an electromotor - and the working vibrating machine is studied.

Several types of simple mathematical models of static and dynamic characteristics of electro-motors are proposed.

More exact characteristics are obtained by considering the properties of electro-magnetic circuit. Corresponding characteristics have to be described by differential equations both in dimensional quantities and in non-dimensional form, very suitable for general analysis. An example of dynamic behavior of such a system passing through resonance zone is attached.

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References

[1] Půst, L.: Stability and transient phenomena in the nonlinear systems, Proc.: *Ninth World Congress on the Theory of Machines and Mechanisms,* Politecnico of Milano, Italy, 1995, Vol. 2, pp. 1489-1493.

[2] Guz, A.N., Markuš, Š., Půst, L.: Dinamika těl vzajimodějstvujuščich so sredoj, Kiew, Naukova Dumka, 1991.

[3] Brasil, R.M.L., Balthazar, J.M.: Nonlinear oscillations of a portal frame structure excited by a non-ideal motors, Proc. *2nd Int. Conf. Control of Oscillations and Chaos*, St. Peterburg, 2000, Vol. 2, pp. 275-278.

[4] Půst, L.: Vliv zdroje buzení na kmitání soustavy s jedním stupněm volnosti, Sb. Computational Mechanics 2004, Nečtiny ZČU Plzeň, D.II,. str. 421-428.

[5] Kratochvíl C., Procházka F., Pulkrábek J.: Porovnání dynamických vlastností elektromechanických pohonových soustav se stejnosměrnými motory velkého a malého výkonu, Proc. Computational Mechanics 2004, ZČU Plzeň, Nečtiny, D.I.,str. 231-238.

[6] Půst L.: Dynamická vazba mezi kmitající soustavou s rázy a zdrojem buzení. Proc. Interakce a zpětné vazby, ÚT-AVČR, Praha, 2004, str. 125-134.