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TRANSFER MATRIX METHOD FOR THE HUMAN VOCAL FOLD MODELLING

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Summary: *The model of the human vocal fold is created with transfer matrix method. The model is made from 32 cylindric or conic elements for Czech vowel /u/. The calculation is carried out for harmonic and general periodic signals in the input of the vocal fold (position of the glottis). Time dependent acoustic pressure and acoustic speed in the output (position of the mouth) and natural frequencies of the fold are computed. The results are compared with computations with 3D FEM model.*

1. Introduction

The paper deals with use of transfer matrix method to determine dynamic characteristics of the human vocal fold. The advantage of this method is significant shortening of the computing time compared to computations with 3D finite element method.

Computing model of the real human vocal fold for Czech vowel /u/ was obtained with the help of direct transformation of MRI files into FEM models. Simplified model which is suitable for transfer matrix method was created on the basis of FEM model. Absorption of acoustic power at bounding surfaces and another kind of damping hasn't been under consideration.

2. Mathematical statement of the problem

2.1 Cylindric elements

For an acoustic duct - a tube with rigid wall - we can derive 1D Webster's wave equation for velocity potential:

$$\frac{\partial^2 \phi}{\partial x^2} - \frac{1}{c_0^2} \cdot \left(\frac{\partial^2 \phi}{\partial t^2} + \frac{r_S}{\rho} \cdot \frac{\partial \phi}{\partial t} \right) = 0 \qquad (1)$$

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where means

$\phi \equiv \phi(x,t)$	[m^2.s^{-1}]	velocity potential
ρ	[kg.m^{-3}]	medium density
x	[m]	longitudinal coordinate of acoustic duct
t	[s]	time
$S \equiv S(x)$	[m^2]	cross-sectional area of acoustic duct
c_0	[m.s^{-1}]	speed of sound
r_S	[kg.m^{-3}.s^{-1}]	specific acoustic resistance of the duct – to – length ratio

After solution (1) and substitution of boundary conditions we obtain relationship between input „1“ and output „2“ of acoustic duct (for acoust.pressure p [Pa] and ac.speed v [m.s^{-1}]).

$$\begin{bmatrix} p_2 \\ v_2 \end{bmatrix} = \underline{T_{2,1}} \cdot \begin{bmatrix} p_1 \\ v_1 \end{bmatrix} \tag{2}$$

where transfer matrix for the cylindric acoustic duct of length L [m] is

$$\underline{T_{2,1}} = \begin{bmatrix} \cosh(\gamma \cdot L) & -\dfrac{r_S + j \cdot \omega \cdot \rho}{\gamma} \cdot \sinh(\gamma \cdot L) \\ -\dfrac{\gamma}{r_S + j \cdot \omega \cdot \rho} \cdot \sinh(\gamma \cdot L) & \cosh(\gamma \cdot L) \end{bmatrix} \tag{3}$$

γ	[m^{-1}]	complex exponent
ω	[s^{-1}]	angular frequency of harmonic signal

When we neglect damping $(r_S = 0)$, we get

$$\underline{T_{2,1}} = \begin{bmatrix} \cos(k \cdot L) & -j \cdot z_0 \cdot \sin(k \cdot L) \\ -\dfrac{j}{z_0} \cdot \sin(k \cdot L) & \cos(k \cdot L) \end{bmatrix} \tag{4}$$

k [m^{-1}] wave number, $k = \dfrac{\omega}{c_0}$ (5)

z_0 [kg.m^{-2}.s^{-1}] wave resistance, $z_0 = c_0 \cdot \rho$ (6)

2.2 Conic elements

1D Webster’s wave equation for an acoustic duct with variable cross-section is

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{1}{S} \cdot \frac{\partial S}{\partial x} \cdot \frac{\partial \phi}{\partial x} - \frac{1}{c_0^2} \cdot \frac{\partial^2 \phi}{\partial t^2} = 0 \tag{7}$$

We can solve equation (7) with the help of new coordination ξ [m] (Fig.1) and after substitution of boundary conditions we obtain relationship between input and output of acoustic duct (2).

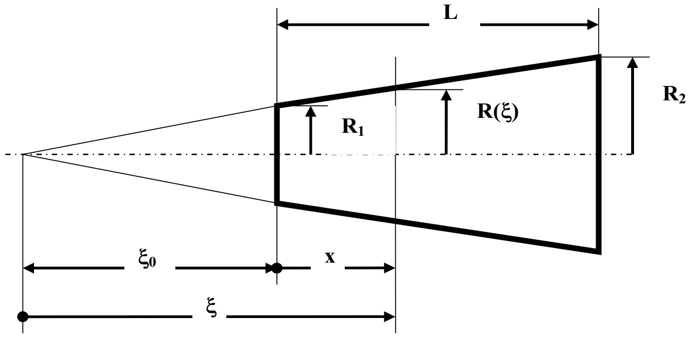

Fig.1 - Conic element

Transfer matrix for the conic acoustic duct of length L [m] is generally

$$\underline{T_{2,1}} = \begin{bmatrix} {}^{11}t_{2,1} & {}^{12}t_{2,1} \\ {}^{21}t_{2,1} & {}^{22}t_{2,1} \end{bmatrix} \tag{8}$$

and all it's complex components are dependent on k, z_0, L, ξ_0.

In case $R_1 = R_2 \Rightarrow \xi_0 \to \infty$ components of transfer matrix converts into

$${}^{11}t_{2,1}(\xi_0 \to \infty) = \cos(kL) \tag{9}$$

$${}^{12}t_{2,1}(\xi_0 \to \infty) = -j \cdot z_0 \cdot \sin(kL) \tag{10}$$

$${}^{21}t_{2,1}(\xi_0 \to \infty) = -\frac{j}{z_0} \cdot \sin(kL) \tag{11}$$

$${}^{22}t_{2,1}(\xi_0 \to \infty) = \cos(kL) \tag{12}$$

which are components of transfer matrix for the cylindric acoustic duct without damping.

2.3 Natural frequencies of the vocal fold

Natural frequencies of acoustic duct are obtained by substituion of boundary conditions. If the input (or output) is opened/closed, acoustic pressure/speed is equal to zero in that position.

We can divide vocal fold into system of N_E elements and then

$$\begin{bmatrix} p_{N_E+1} \\ v_{N_E+1} \end{bmatrix} = \underline{T_{N_E+1,1}} \cdot \begin{bmatrix} p_1 \\ v_1 \end{bmatrix} = \underline{T_{N_E+1,N_E}} \cdot \underline{T_{N_E,N_E-1}} \cdot \ldots \cdot \underline{T_{i+1,i}} \cdot \underline{T_{i,i-1}} \cdot \ldots \cdot \underline{T_{2,1}} \cdot \begin{bmatrix} p_1 \\ v_1 \end{bmatrix} \tag{13}$$

where transfer matrix between input and output is

$$\underline{T_{N_E+1,1}} = \begin{bmatrix} {}^{11}t_{N_E+1,1} & {}^{12}t_{N_E+1,1} \\ {}^{21}t_{N_E+1,1} & {}^{22}t_{N_E+1,1} \end{bmatrix} \qquad (14)$$

For boundary conditions „O-O“ (input opened, output opened) we get frequency equation from the term

$${}^{12}t_{N_E+1,1} = 0 \qquad (15)$$

Analogically for boundary conditions „C-C“ (input closed, output closed) we get

$${}^{21}t_{N_E+1,1} = 0 \qquad (16)$$

For boundary conditions „O-C“:

$${}^{22}t_{N_E+1,1} = 0 \qquad (17)$$

And for „C-O“:

$${}^{11}t_{N_E+1,1} = 0 \qquad (18)$$

By the numerical solution of frequency equation we obtain wave numbers k [m^{-1}] and then natural frequencies f [Hz].

3. Calculation results

3.1 Natural frequencies

There are first five natural frequencies on the pictures Fig.6-Fig.9. These quantities were computed for one cylindric element (Fig.2), one conic element (Fig.3), system of three cylinders (Fig.4) and system of two cones (Fig.5). The computation was done with the transfer matrix method (TMM) and finite element method (FEM) for four boundary conditions: input/output – opened/closed.

Natural frequencies of real human vocal fold (vowel /u/, Fig.6, Fig.7) were computed with TMM for cylindric elements, conic elements and with FEM. As we can see on Fig.10, TMM with conic elemenst gives fairly good results for boundary conditions „input (glottis) opened, output (mouth) opened“. The results for conditions „input closed, output opened“ are worse. The values obtained by computation with TMM and cylindrical elements are not satisfactory.

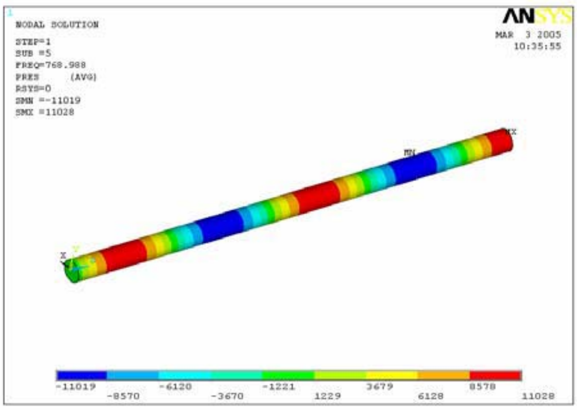

Fig.2 - cylindric element, 5.eigenvalue (FEM), boundary conditions „O-C“

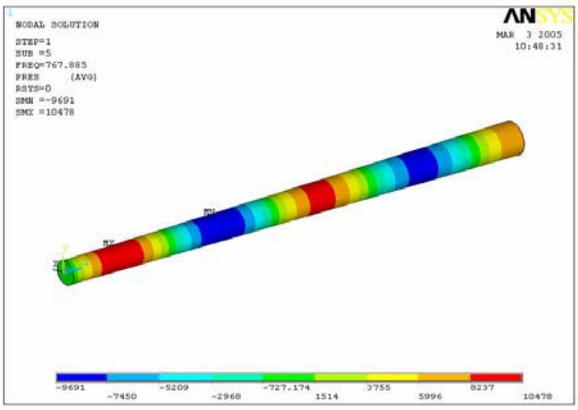

Fig.3 - conic element, 5.eigenvalue (FEM), boundary conditions „O-C“

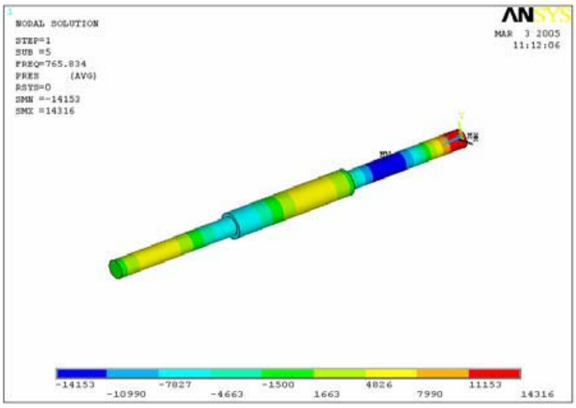

Fig.4 – 3 cylindric elements, 5.eigenvalue (FEM), boundary conditions „O-C“

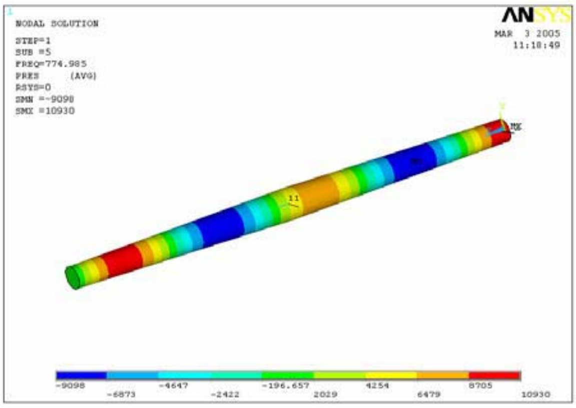

Fig.5 – 2 conic elements, 5.eigenvalue (FEM), boundary conditions „O-C“

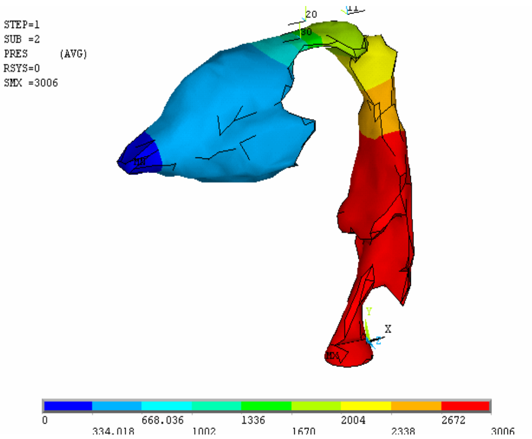

Fig.6 – vocal fold /u/, 1.eigenvalue (FEM), boundary conditions „C-O“

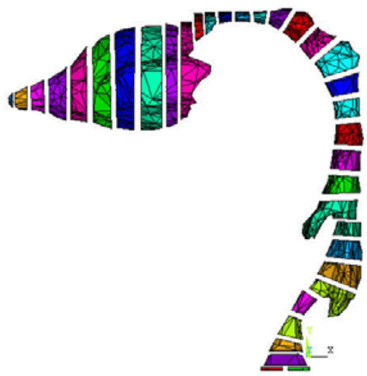

Fig.7 – dividing of vocal fold into 32 elements for using TMM

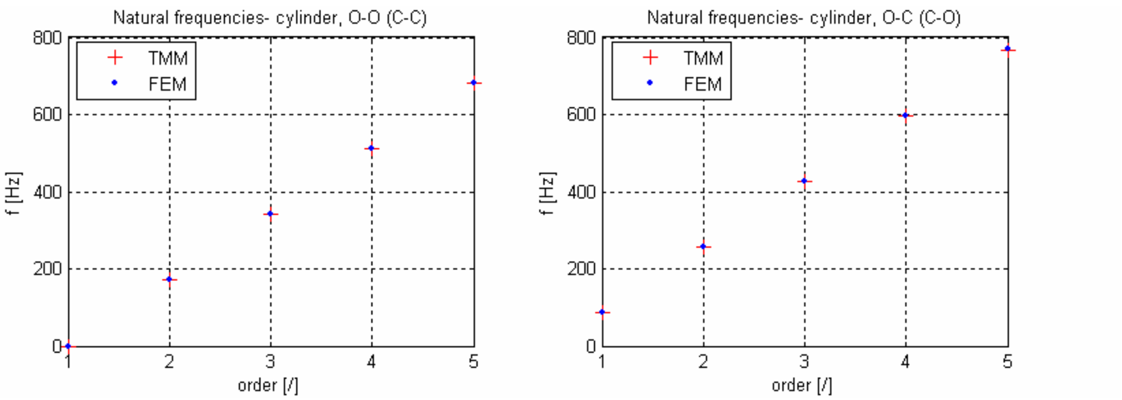

Fig.8 – Natural frequencies, cylindric element

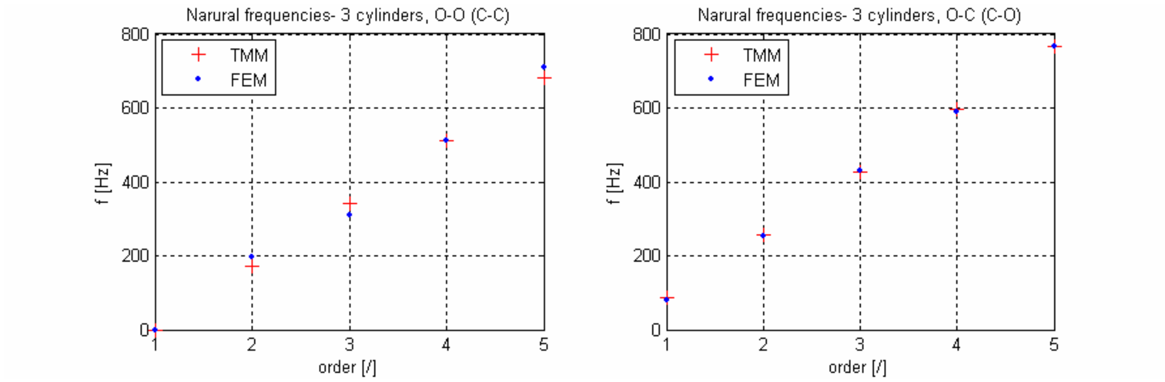

Fig.9 - Natural frequencies, system of 3 cylindric elements

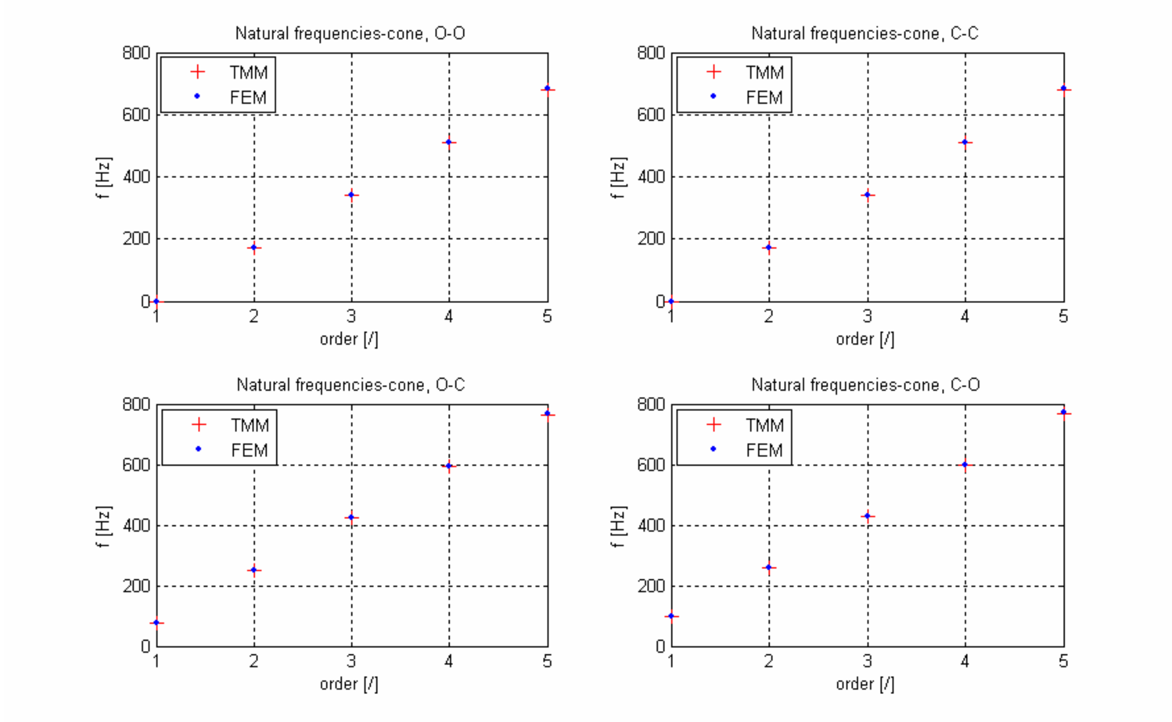

Fig.10 - Natural frequencies, conic element

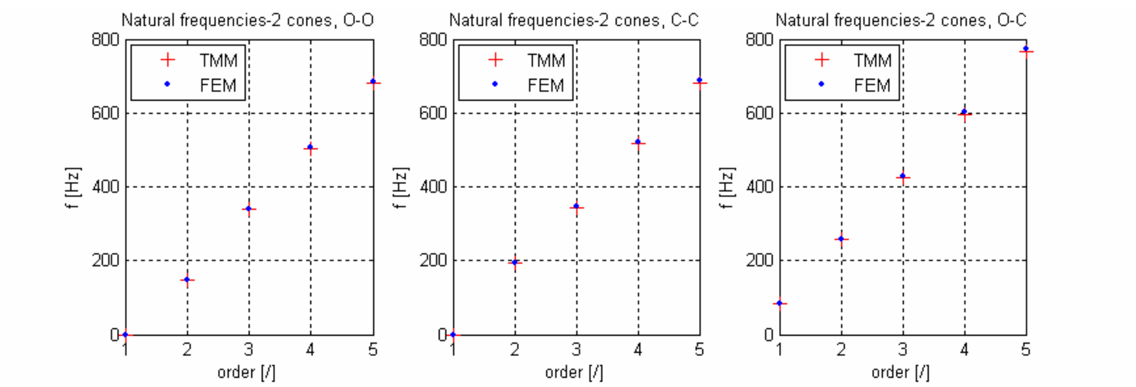

Fig.11 - Natural frequencies, system of 2 conic elements

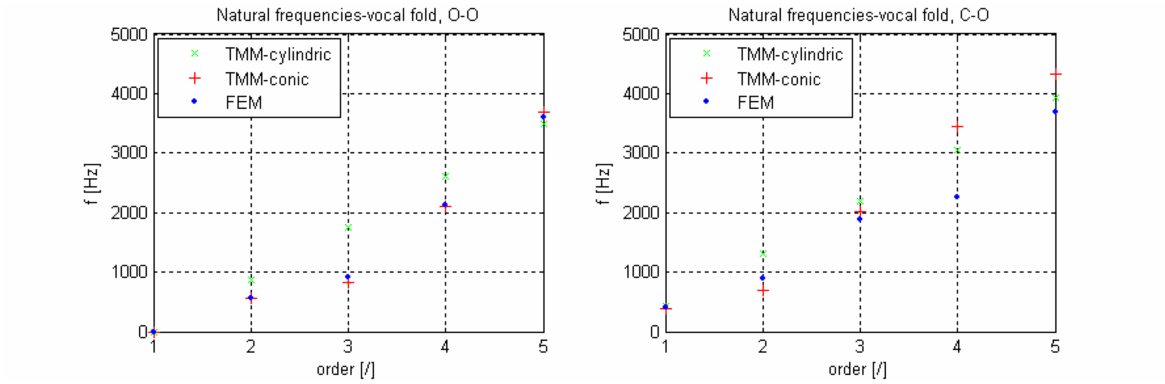

Fig.12 - Natural frequencies, vocal fold – Czech vowel /u/

3.2 Time dependent quantities in the output of vocal fold

Time dependent acoustic pressure, acoustic speed and volume flow in the output of the vocal fold were computed. At first harmonic signal with amplitude of speed 1 ms^{-1} and frequency 100 Hz was apply to input. We can see that TMM with cylindric (Fig.13) and conic (Fig.14) elements gives different results.

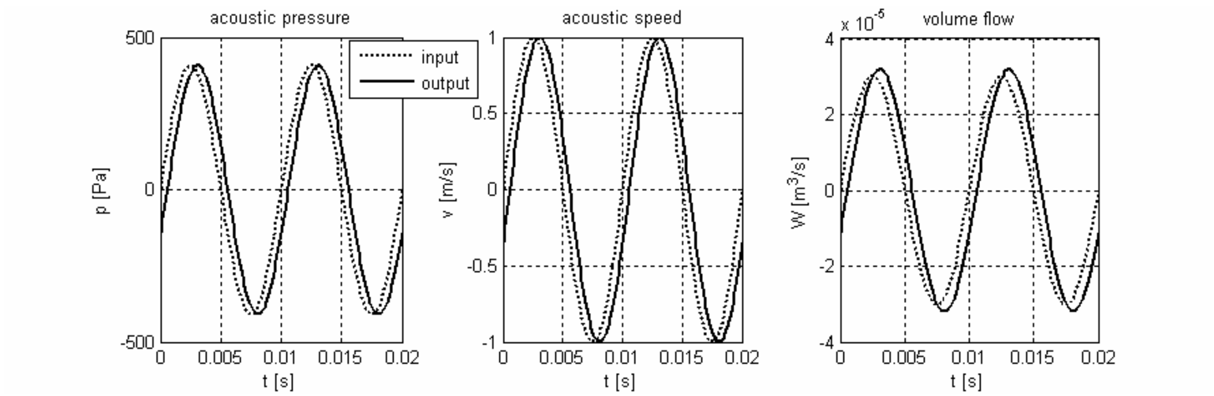

Fig.13 – Time dependance, harmonic signal, system of 32 cylindric elements

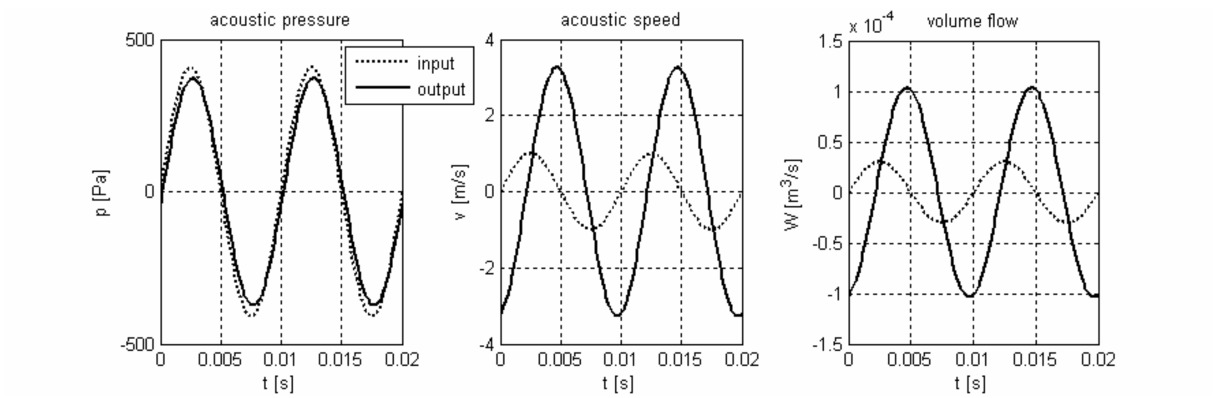

Fig.14 – Time dependance, harmonic signal, system of 32 conic elements

Second general periodic signal (decomposed by Fourier transform into 100 harmonic components) was apply to input. It is evident that computation with cylindric elements (Fig.15) changes only a phase for all harmonic components, but doesn't change an amplitude. Time dependent quantities in the output for the same periodic signal in the input, the same geometry of vocal fold, but using conic elements are on the Fig.16.

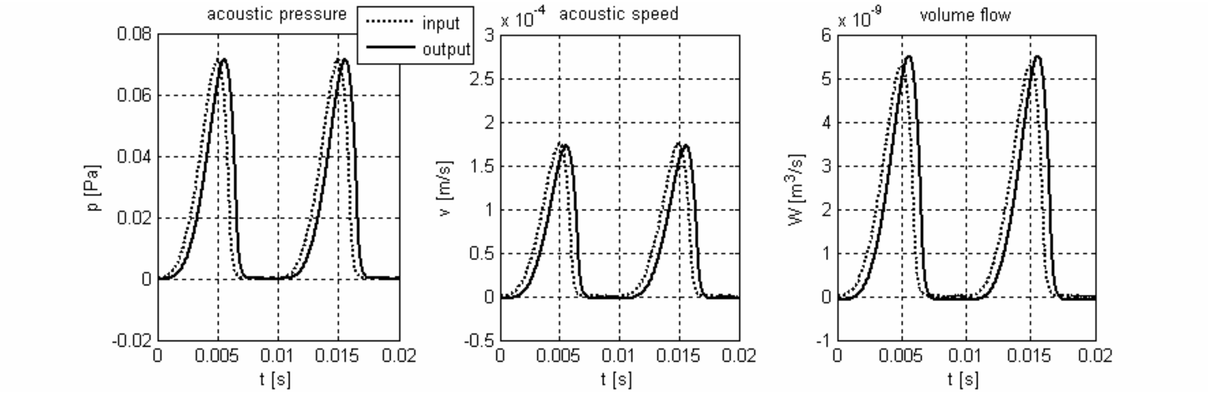

Fig.15 – Time dependance, periodic signal, system of 32 cylindric elements

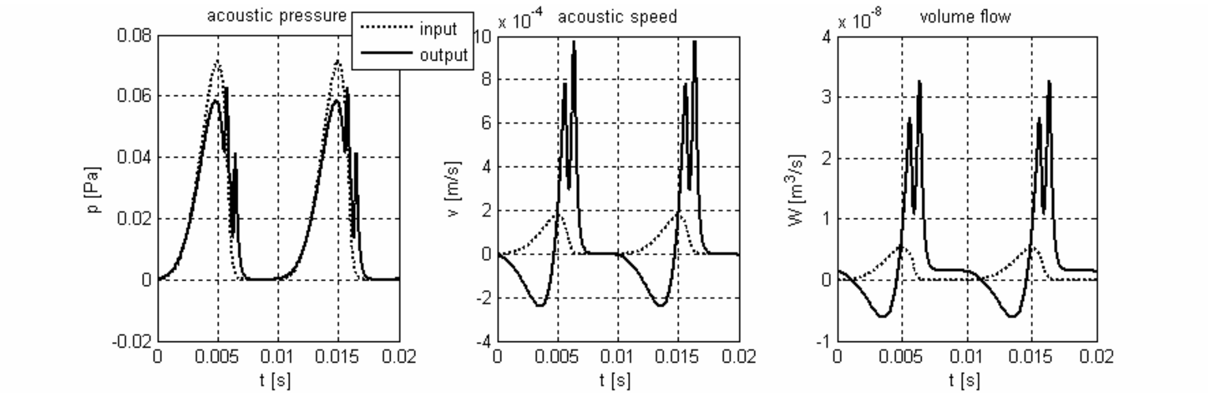

Fig.16 – Time dependance, periodic signal, system of 32 conic elements

4. Conclusion

The paper deals with use of transfer matrix method to determine natural frequencies of vocal fold and time dependent quantities in the output of vocal fold.

Natural frequencies computed for small number of elements accord with results obtained by FEM. Natural frequencies for vocal fold as a system of 32 elements don't accord so well. TMM with conic elements gives better results than TMM with cylindric elements (Fig.12).

TMM with cylindric elements gives bad results of time dependent quantities in the output, because it changes only phase for all harmonic components, but doesn't change an amplitude.

Absorption of acoustic power at bounding surfaces and another kind of damping is not under consideration of this paper.

5. Acknowledgement

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6. References

Titze, I.R. & Mapes, S. & Story, B. (1994) Acoustics of the tenor high voice . *J. Acoust. Soc. Am.* 95 (2), pp. 1133 – 1142.

O`Leidhin, E. & Murphy, P. (2003) Preliminary glottal source modeling for pathologic voices. 3rd International Workshop MAVEBA 2003, Firenze, Italy, pp. 237 – 240.

Dedouch, K. & Horáček, J. & Vampola, T. & Vokřál, J. (2003) Velofaryngeal insufficiency studied using finite element models of male vocal tract with experimental verification.

3rd International Workshop MAVEBA 2003, Firenze, Italy, pp. 229 – 232.

Dedouch, K. & Horáček, J. & Vampola, T. & Švec, J. & Kršek, P. & Havlík, R. (2002) Acoustic modal analysis of male vocal tract for Czech vowels. Interaction and feedbacks 2002, pp. 13 – 20.

Novák, A. (1996) Phoniatrics and pedaudiology. Voice disorders – principles of voice physiology, diagnostics, treatment, reeduction and rehabilitation. Unitisk, s.r.o., Prague.

Radolf, V. & Vampola, T. (2004) Analýza vokálního traktu člověka pomocí metody přenosových matic, Interaction and feedbacks 2004, pp. 135-142

Merhaut, J., (1971) Teoretické základy elektroakustiky, Academia, Praha