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FUZZY DYNAMIC STRUCTURAL ANALYSIS OF TWO-DIMENSIONAL FRAME WITH FUZZY COEFFICIENTS

P. Štemberk*, J. Kruis, Z. Bittnar****

Summary: *In this paper, an approach to dynamic analysis based on the fuzzy set theory is presented as a alternative to the classical stochastic dynamic analysis. The material parameters are considered as fuzzy quantities with a given distribution (fuzzy numbers). The dynamic analysis is then performed with help of the fuzzy arithmetic on α-cuts. The result is in the form of fuzzy numbers. In order to reduce computational time, the response surface function concept is employed. This approach is illustrated in an illustrative example of a 2D frame whose free vibration is analyzed.*

1. Introduction

Analysis of dynamically loaded structures requires, due to the uncertainties related to loading and material description, traditionally, stochastic analysis when the calculation is based on experimentally obtained statistical characteristics of some quantities. In case of earthquake, it is quite impossible to obtain such characteristics at the location where the analysis structure is situated. Then, some kind of interpolation needs to be assumed which already may cause errors in the calculation.

This paper shows a case of dynamic structural analysis based on the fuzzy set theory (Zadeh, 1965), which may serve as an alternative method the analyses based on the statistical approach. The material parameters are considered to be fuzzy quantities with a given distribution, i.e. fuzzy numbers with a desired shape of the membership function (Valliapan & Pham, 1993). The dynamic analysis is, then, performed with help of the fuzzy arithmetics on either the so-called α-cuts or computation-efficient (L,R) numbers (Kaufman & Gupta, 1985). In order to further improve the computational efficiency, inspired by (Akpan et al., 2001), the concept of a surface response function (Bucher et al., 1988; Rajashekhar & Ellingwood 1993) is utilized. This approach is demonstrated in an illustrative example of a 2D frame where the effect of uncertain material parameters transpires in corresponding distributions of natural modal shapes and natural frequencies of an analyzed two-dimensional frame. A methodology for a possible application to seismic design is also explained.

* Ing. Petr Štemberk, Ph.D., **Ing. Jaroslav Kruis, Ph.D a **Prof. Ing. Zdeněk Bittnar, DrSc.: Czech Technical University, Faculty of Civil Engineering, *Dept. of Concrete Structures and Bridges, **Dept. of Structural Mechanics; Thákurova 7; 160 29 Praha 6; tel.: +420.224 354 364, fax: +420.233 335 797; e-mail: stemberk@fsv.cvut.cz

2. Dynamic finite element analysis

The finite element method applied to dynamical problems of structures results into the form

$$\boldsymbol{M}\frac{d^2\boldsymbol{r}(t)}{dt^2} + \boldsymbol{C}\frac{d\boldsymbol{r}(t)}{dt} + \boldsymbol{K}\boldsymbol{r}(t) = \boldsymbol{f}(t), \tag{1}$$

where $\boldsymbol{M}$ denotes the mass matrix, $\boldsymbol{C}$ denotes the damping matrix, $\boldsymbol{K}$ denotes the stiffness matrix, $\boldsymbol{f}(t)$ denotes the load vector and $\boldsymbol{r}(t)$ is the vector of nodal displacemnts which are computed. t stands for time. Equation (1) represents a semidiscrete problem where the spatial coordinates are discretized while the time is still assumed to be continuous (Bathe, 1996; Bittnar & Šejnoha, 1992).

The analysis of natural frequencies (eigenvalues) and natural mode shapes (eigenvectors) of an undamped structure is based on the simplified relation

$$\left(\boldsymbol{K} - \omega_0^2\boldsymbol{M}\right)\boldsymbol{v} = \boldsymbol{0}. \tag{2}$$

The nonzero vector $\boldsymbol{v}$ is the eigenvector containing the natural mode shapes and ω_0 stands for the natural frequency. Equation (2) represents an generalized problem of eigenvalues. The most common method of solution of such problems is the subspace iteration (Bittnar & Šejnoha, 1992).

3. Fuzzification of dynamic finite element analysis

The uncertainty, which is present in input parameters, can be tackled with help of the fuzzy set theory (Zadeh, 1965). In this theory, the uncertain quantities are defined in terms of fuzzy sets. Unlike in the classical set theory, here a membership of an element to a fuzzy set also assumes the values between 0 and 1, where 0 means "does not belong" and 1 means "definitely belongs" to a fuzzy set. Usually, the fuzzy sets represent vague verbal evaluation. In cases when a fuzzy set represents a numeral, it is called a fuzzy number.

Fuzzy numbers

The notion of a fuzzy number arises from the experience of the everyday life when many phenomena which can be quantified are not characterized in the terms of absolutely precise numbers. Fuzzy numbers are fuzzy sets which are defined on the set of real numbers. Their membership function assigns the degree of 1 to the central, also called nominal, modal or mean, value and lower degrees to other numbers which reflect their proximity to the central value according to the used membership function. The membership function should thus decrease from 1 to 0 on both sides of the central value. Such fuzzy sets are called fuzzy numbers.

Fuzzy arithmetic

A fuzzy arithmetic operation depends on the definition of a fuzzy number. In the cases when fuzzy numbers are defined by a set of α-cuts, the problem of fuzzy arithmetic is reduced to the well-known arithmetic operations on intervals, which are applied to each α-cut. Implicitely, this means a sequence of binary combinations on each α-cut in order to obtain the minimum and the maximum value for each α-cut. The finite element method converts a problem into a system of linear equations, in this case a system of fuzzy linear equations, which comprises an extensive number of arithmetic operations. This fact makes the formulation in the above terms merely unsolvable due to the number of all necessary binary operations.

To eliminate the drawback of the α-cut formulation, new techniques for solving fuzzy linear equation systems have been developed, e.g. (Bulckley, 1991). However, these techniques are not easily applicable to robust problems, such as the fuzzy dynamic finite element analysis. Therefore, another technique for reducing the large number of binary combinations, originally developed for other problems, e.g. statistical analysis, was exploited.

Response surface function

Fuzzy analyses, as well as stochastic analyses, suffer from non-occurrence of analytical solutions in the case of non-deterministic input data. This situation can be remedied by the following. Let $\tilde{\boldsymbol{x}} \in \tilde{\boldsymbol{X}}$ je denote the vector of input data from the space of input data $\tilde{\boldsymbol{X}}$ and $\tilde{\boldsymbol{y}} \in \tilde{\boldsymbol{Y}}$ denote the vector of output data from the space of output data $\tilde{\boldsymbol{Y}}$. Both, stochastic and fuzzy, analyses require the knowledge of response which can be written

$$\tilde{\boldsymbol{y}} = \mathcal{F}(\tilde{\boldsymbol{x}}), \tag{3}$$

where $\mathcal{F}$ denotes the response of a system (structure) to the input data collected in the vector $\tilde{\boldsymbol{x}}$. This represents a mapping from the space $\tilde{\boldsymbol{X}}$ to the space $\tilde{\boldsymbol{Y}}$. The non-occurence of analytical solution requires application of a suitable numerical method which discretizes the problem and solves it numerically. The space $\tilde{\boldsymbol{X}}$ is discretized by an *n*-dimensional space, $\boldsymbol{X}$, and similarly the space $\tilde{\boldsymbol{Y}}$ by an *m*-dimensional space, $\boldsymbol{Y}$. A stochastic analysis based on simulation methods generates thousands or millions of samples of input data (the vectors $\boldsymbol{x}$) and then the deterministic computation follows. The fuzzy analysis based on α-cuts requires computation of all combinations of input data which also leads to thousands or millions of samples. Both approaches yield the response of a system based on a huge amount of output data (thousands or millions of the vectors $\boldsymbol{y}$) obtained from many executions of standard (deterministic or crisp) solutions.

In order to reduce the necessary number of computation runs, the concept of a response surface function has been used many times. The basic idea of the response function is to approximate the operator $\mathcal{F}$ by a suitable function which should be as simple as possible. The function for the *k*-th output parameter can be written in the form

$$f^{(k)}(x) = a^{(k)} + \sum_{i=1}^{n} b_i^{(k)} x_i + \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}^{(k)} x_i x_j, \tag{4}$$

where the superscript identifies an output parameter and *n* denotes the dimension of the space of the input data, $\boldsymbol{X}$. The unknown coefficients are obtained from the least square method in the following way. Let the set of input parameters contain *s* samples. Each sample is located in the vector $\boldsymbol{x}^{[i]}$, where the superscript identifies a sample. The standard computation gives output data, which are collected in the vectors $\boldsymbol{y}^{[i]}$. The coefficients of the response function minimize the following expession

$$F^{(k)}(a^{(k)}, b_i^{(k)}, c_{ij}^{(k)}) = \sum_{i=1}^{s} \left(f^{(k)}(\boldsymbol{x}^{[i]}) - y_k^{[i]} \right)^2. \tag{5}$$

In many cases, it is not necessary to use the quadratic terms. Considering only the linear terms simplifies further computations.

4. Numerical example

As an example, the natural frequency analysis of a two-dimensional frame with four floors made of reinforced concrete is considered. The overall height of the frame is 16 meters and the width is 5 + 5 meters. The dimensions of beams and columns are identical (0.5 x 0.5 m). It is assumed that the building was erected in four consecutive lifts. Each lift consists of placing concrete in three columns and in the beam which connects the upper ends of the columns. Therefore, it is further assumed that there are only four types of concrete whose composition can possibly differ. The influencing material parameters are the modulus of elasticity, E and the density, ρ. E a ρ are fuzzy input parameters with nominal values of 30 GPa and 2500 kg/m^3, respectively, which can change by $\pm$10% and with a linear membership function (triangular fuzzy numbers).

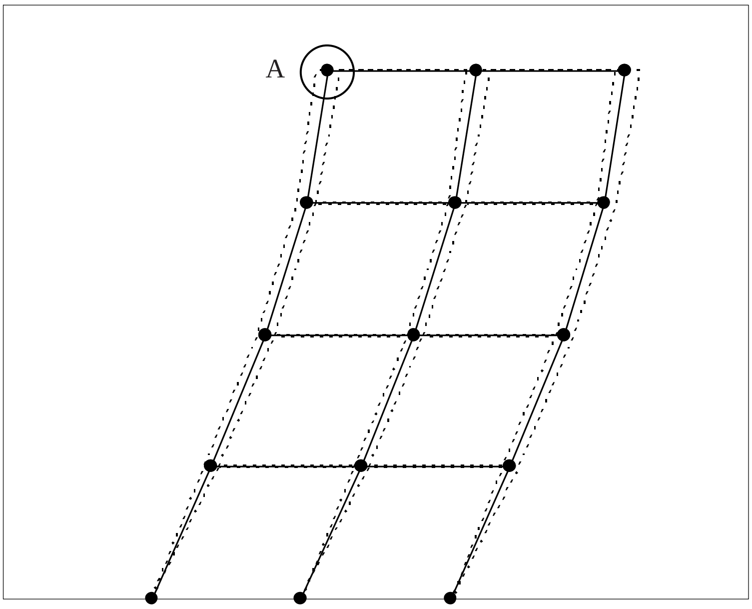

Fig. 1 Mode shape 1

For our illustrative purposes, we need 125 response surfaces functions to describe the first five natural vibration modes, i.e. a response surface function to express each natural frequency and the horizontal and vertical displacements in each joint (three joints on each of the four floors) for each natural mode shape. In order to obtain sufficient input and output data for calculation of the coefficients of the response surface functions it was decided to take three values (minimum, modal value, maximum) for each material parameter, E and ρ, that means $3^{2\times4}$ (=6561) independent runs of the dynamic finite element analysis. The specific form of Equation (4) in this example was

$$\begin{aligned} f^{(k)}(x) = {} & b_1^{(k)}E_1 + b_2^{(k)}E_2 + b_3^{(k)}E_3 + b_4^{(k)}E_4 + \\ & + b_5^{(k)}\rho_1 + + b_6^{(k)}\rho_2 + b_7^{(k)}\rho_3 + b_8^{(k)}\rho_4 + b_9^{(k)}. \end{aligned} \tag{6}$$

The first four mode shapes are shown in Figs. 1 to 5 where the dotted lines represent all possible envelopes of response, in other words, the minimum and maximum values, which correspond to the values obtained for α-cuts α=0). The finite element model of this frame discretized each frame section (beam and column) by five beam elements, however, only the joint displacements are shown. This is why the fifth mode shape is not shown since there was no significant difference between the fourth and fifth mode shapes. In Fig. 2, a section of the frame is enlarged and the vertical displacements are 1000 times increased compared to the horizontal displacements so that one can see the distribution of possible displacements of the frame. The distribution of the first five natural frequencies is shown in Fig 6. It was observed that the response function gives very good results for dominant

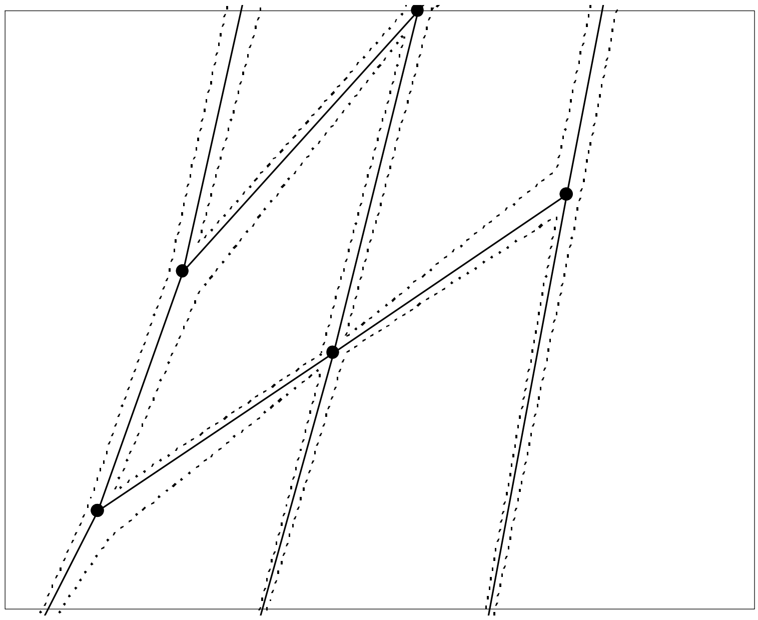

Fig. 2 Enlarged section of mode shape 1

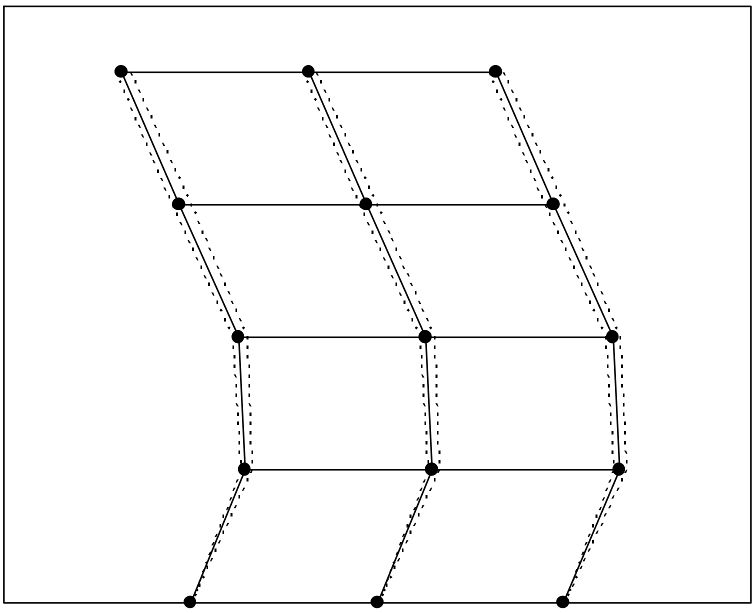

Fig. 3 Mode shape 2

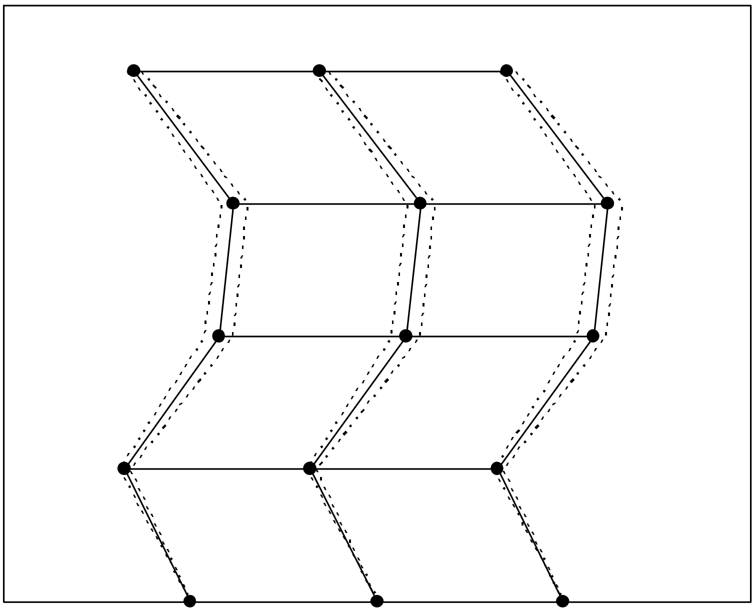

Fig. 4 Mode shape 3

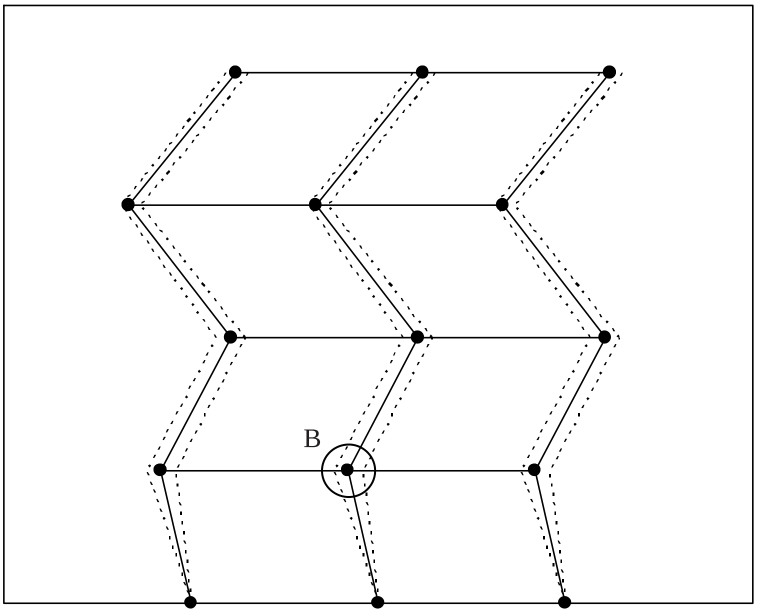

Fig. 5 Mode shape 4

displacements (at point A, which is the top left joint) in lower natural mode shapes, which are important for seismic design. For vertical displacements, which do not play an important role in seismic design (at point B, which is the intermediate joint of the first floor), the response function could not fit the proper shape of the membership function, which is evidenced in Fig. 7.

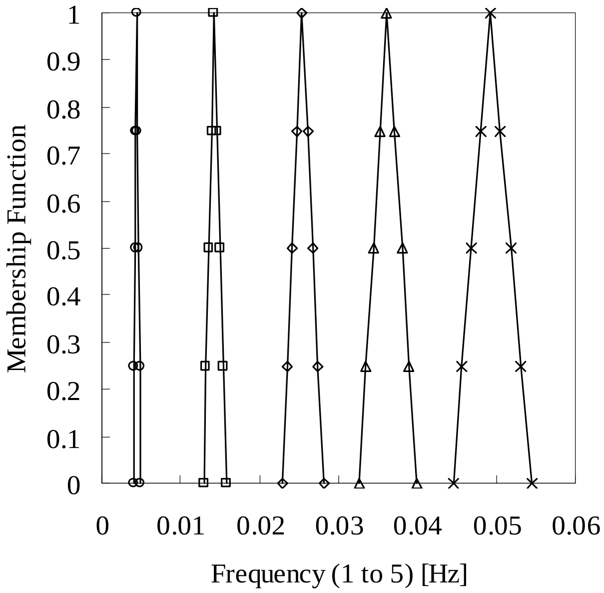

Fig. 6 Distribution of natural frequencies

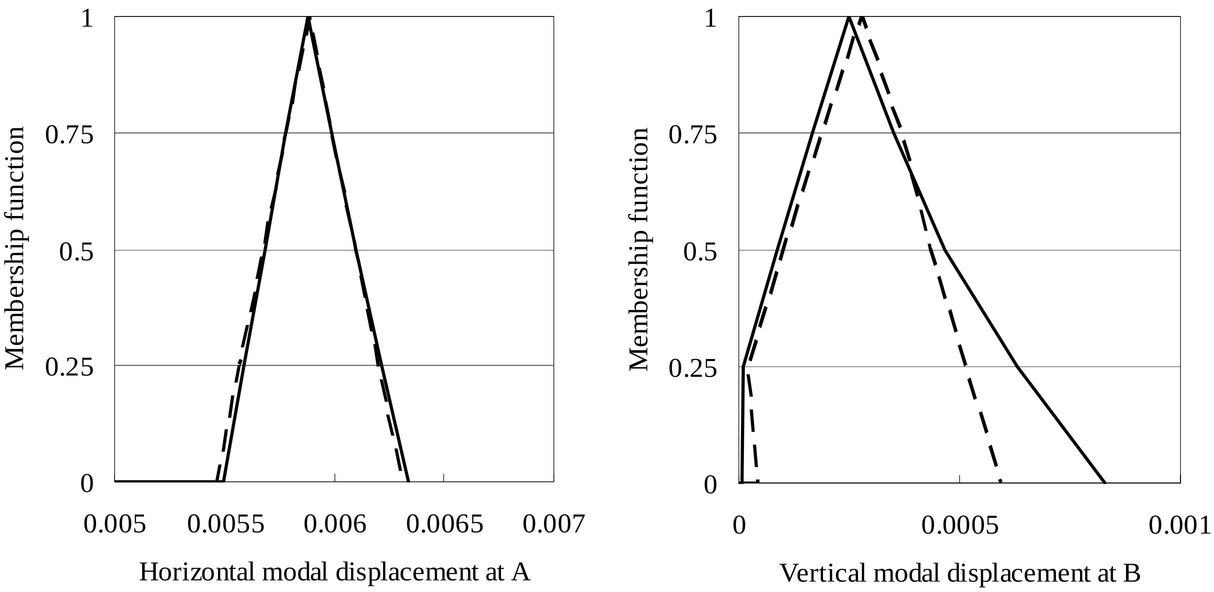

Fig. 7 Distribution of two displacements

5. Possible application

In the design of earthquake resistant structures, it is essential not to neglect any uncertainty as it may lead to an erroneous conclusion due to the dynamic simulation which may amplify such uncertainty beyond all limits. For those reasons it seems reasonable to express uncertain numerical data in terms of fuzzy numbers and use them as such in analyses to cover all possible solutions.

In the previous section, an approach to natural vibration analysis was shown which provides input data for further analyses considering, e.g. earthquake induced vibrations. The spectral-analysis based methods require only maximum values obtained for each natural mode for evaluation of excited vibration. Therefore, it is desirable to verify whether these values can be satisfactorily expressed by surface response functions which were obtained only by binary combinations of material parameters with three values (minimum, modal value, maximum). The resulting surface response function was also obtained for five values, corresponding to the α-cut values with α equal to 0, 0.5 and 1, however, that already meant $5^{2\times 4}$ (=390625) independent runs of the dynamic finite element analysis. The improvement was negligible, moreover, compared with the computational effort it proved truly unnecessary.

6. Conclusions

In this paper, it was shown that the fuzzy dynamic finite element method can be satisfactorily supplemented with the surface response function concept which considerably increases the computational efficiency. It was shown that input and output data collected through the binary combinations of only three values (minimum, modal value, maximum) for all varying material parameters yields surface response functions with the errors up to 5 % from the true results for dominant, and hence imporant.

7. Acknowledgement

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