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FLOW BEHAVIOUR OF VOČADLO-TYPE FLUIDS DURING BACK EXTRUSION

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Summary: *In food industry a back extrusion represents one of the cheapest and time-saving experimental methods how to determine rheological characteristics of the fluids studied. This method is based on plunging of a circular rod into an axisymmetrically located circular cup containing the experimental sample. In the past this method was successfully applied to power-law, Bingham and Herschel-Bulkley fluids. The crucial point for determination of the rheological parameters characterising the individual types of fluids consists in deriving velocity profiles in a concentric annulus formed by a plunger (rod) and container (cup), and a relation between pressure gradient and a volumetric flow rate. The aim of this contribution is to present semi-analytical forms describing a velocity profile for Vočadlo-type fluids including a location of the plug flow region, and pressure gradient - flow rate dependence.*

1. Introduction

At present standard rheometers provide sufficiently precise measurements characterising behaviour of non-Newtonian materials. In practice, this accuracy is not always necessary, and e.g. in food processing precise measurements are not always indispensable. Back extrusion (see Fig.1) represents a method providing relatively cheap and sufficient measurements of the rheological characteristics, see Steffe & Osorio (1987). This method is often used in food industry, e.g. for characterisation of tomato concentrate (Alviar & Reid, 1990), mustard slurry (Brusewitz & Yu, 1996), wheat porridge (Gujral & Sodhi, 2002), corn starch (Singh et al., 2002), caramel jam (Castro et al., 2000), rice (Sodhi et al., 2003), raspberry (Sousa et al., 2005), blackberry (Sousa et al., 2006), etc.

Its principle consists in penetrating of a circular plunger into an axisymmetrically placed circular container with a material studied. For a determination of rheological parameters appearing in the individual empirical rheological models, knowledge of a relation between pressure gradient P and volumetric flow rate q through an annulus formed by a plunger and a container is substantial. This relation is possible to derive from a relation for axial velocity profile of a material studied in an annulus. The present contribution aims at a derivation of both relations for the materials obeying the Vočadlo model.

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2. Application of the individual empirical constitutive models to back extrusion

Osorio & Steffe (1987) derived an analytical solution for a determination of consistency index *K* and flow behaviour index *n* in the 2P (two-parameter) power-law model

$$\tau = K\left|\dot{\gamma}\right|^{n-1} \cdot \dot{\gamma} \tag{1}$$

using a back-extrusion technique. The same authors (Osorio & Steffe, 1991) generalised their approach for the case of the 3P Herschel-Bulkley model

$$\tau = \tau_0 + K\left|\dot{\gamma}\right|^{n-1} \dot{\gamma} \quad . \tag{2}$$

This enables to take into account viscoplastic materials exhibiting a plug-flow region, nevertheless in this model a yield stress τ_0 represents a strict singular term.

The 3P Vočadlo (sometimes called Robertson-Stiff) model (Parzonka & Vočadlo, 1967; Robertson & Stiff, 1976) seems to be more user-friendly viscoplastic model involving a term with a yield stress in a more appropriate form

$$\tau = \left[K\left|\dot{\gamma}\right|^{\frac{n-1}{n}} + \left(\frac{\tau_0}{\left|\dot{\gamma}\right|}\right)^{\frac{1}{n}} \right]^n \dot{\gamma} \qquad \text{for} \quad \left|\tau\right| \geq \tau_0 \quad , \tag{3}$$

$$\dot{\gamma} = 0 \qquad \text{for} \quad \left|\tau\right| \leq \tau_0 \tag{4}$$

where *K* and *n* are consistency and flow behaviour indices, respectively; τ_0 stands for a yield stress.

3. Solution for the 3P Vočadlo model

The Vočadlo model rewritten in the form corresponding to the flow situation in a back extrusion (see Fig.1) is of the form

$$\tau_{rz} = \left[K^{\frac{1}{n}} \left|\frac{dv_z}{dr}\right|^{\frac{n-1}{n}} + \tau_0^{1/n} \left|\frac{dv_z}{dr}\right|^{-\frac{1}{n}} \right]^n \frac{dv_z}{dr} \qquad \text{for} \quad \left|\tau_{rz}\right| \geq \tau_0 \quad , \tag{5}$$

$$\frac{dv_z}{dr} = 0 \qquad \text{for} \quad \left|\tau_{rz}\right| \leq \tau_0 \tag{6}$$

Introducing the following dimensionless transformations (for notation see Figs.1,2 and rels.(3,4))

$$\xi = \frac{r}{R} \quad , \quad \varphi = \frac{v_z}{V} \quad , \quad T = \frac{2\tau_{rz}}{\left|P\right|R} \quad , \quad T_0 = \frac{2\tau_0}{\left|P\right|R} \quad , \quad \Lambda = \frac{\left|P\right|R}{2K}\left(\frac{R}{V}\right)^n \quad , \quad Q = \frac{q}{2\pi R^2 V} \tag{7}$$

the problem of flow within an annulus can be reformulated in the form

$$T = \frac{\lambda^2}{\xi} - \xi \quad , \tag{8}$$

$$\varphi(\kappa) = -1 \quad , \quad \varphi(1) = 0 \quad , \tag{9}$$

$$T = \left[\Lambda^{-s} \left| \frac{d\varphi}{d\xi} \right|^{1-s} + T_0^{-s} \left| \frac{d\varphi}{d\xi} \right|^{-s} \right]^n \frac{d\varphi}{d\xi} \qquad \text{for} \quad |T| \geq T_0 \,, \tag{10}$$

$$\frac{d\varphi}{d\xi} = 0 \qquad \text{for} \quad |T| \leq T_0 \tag{11}$$

where λ^2 is a dimensionless constant of integration, $s=1/n$.

If λ_i, λ_o denote the dimensionless boundary values of the plug flow region (see Fig.2), then from Eq.(8) it follows that

$$\lambda^2 = \lambda_i \lambda_o \quad , \tag{9}$$

$$\lambda_i = \lambda_o - T_0 \quad . \tag{10}$$

For simplification the following notation will be used in the further analysis

$$H(\xi) = \left| \xi - \frac{\lambda_i (\lambda_i + T_0)}{\xi} \right|^s \quad . \tag{11}$$

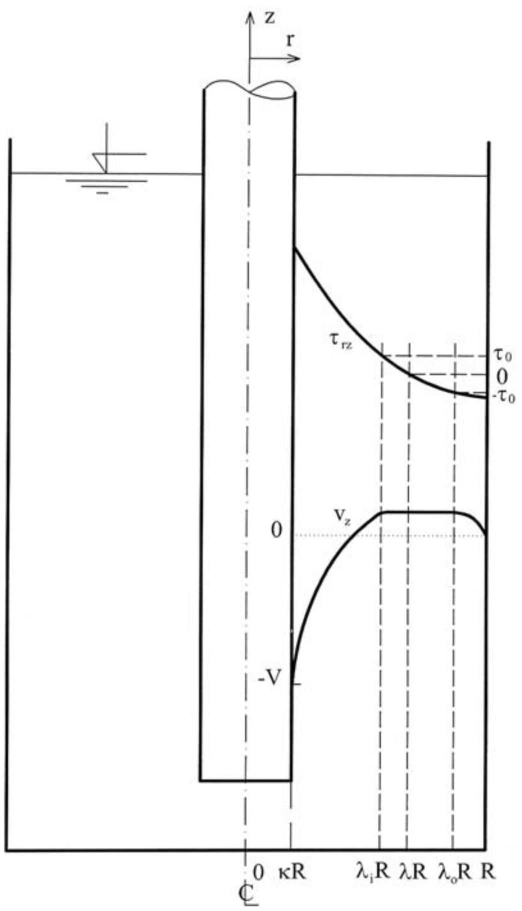

Fig.1 Definition sketch of a back extrusion.

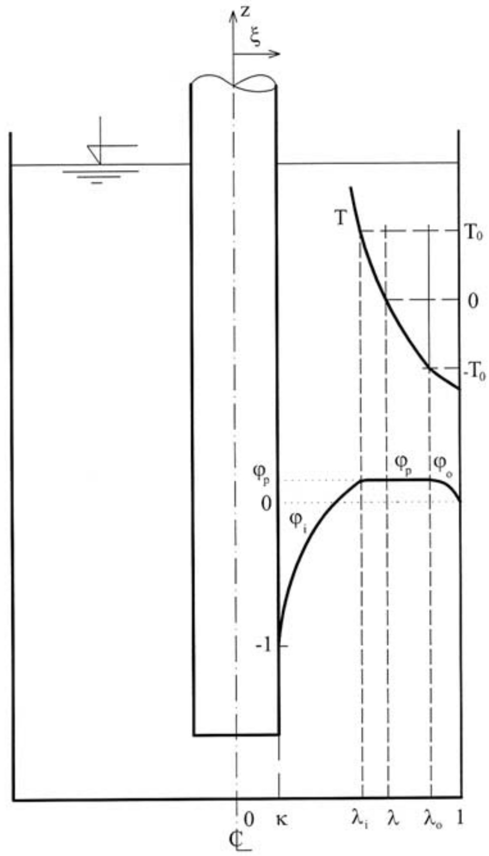

Fig.2 Definition sketch of a back extrusion after dimensionless transformations.

The solution of the above stated problem provides the following expressions for the inner, plug-flow region and outer velocity profiles

$$\frac{d\varphi_i}{d\xi} = \Lambda^s \left[\left(\frac{\lambda^2}{\xi} - \xi \right)^s - T_0^s \right] \qquad \text{for } \kappa \le \xi < \lambda_i \text{ (where } \frac{d\varphi}{d\xi} > 0 \text{)} \quad , \tag{12}$$

$$\frac{d\varphi_p}{d\xi} = 0 \qquad \text{for} \quad \lambda_i \le \xi \le \lambda_o \quad , \tag{13}$$

$$\frac{d\varphi_o}{d\xi} = -\Lambda^s \left[\left(\xi - \frac{\lambda^2}{\xi} \right)^s - T_0^s \right] \qquad \text{for} \quad \lambda_o < \xi \le 1 \quad \text{(where } \frac{d\varphi}{d\xi} < 0 \text{)} \quad . \tag{14}$$

From the condition of continuity of the velocity profile

$$\varphi_i(\lambda_i) = \varphi_o(\lambda_o) \tag{15}$$

it follows that λ_i is a solution of the equation

$$\int_{\kappa}^{\lambda_i} \Lambda^s H(\xi) d\xi + \int_{\lambda_i+T_0}^{1} \Lambda^s H(\xi) d\xi - (2\lambda_i + T_0 - \kappa - 1)\Lambda^s T_0^s - 1 = 0 \quad . \tag{16}$$

If we compare a volumetric flow rate q through an annulus as given by rel.(7) and visually in Fig.1, we get

$$2\pi R^2 V Q = \pi(\kappa R)^2 V \quad . \tag{17}$$

From here it follows that

$$Q = \kappa^2 / 2 \quad . \tag{18}$$

As the determination of dimensionless flow rate Q is basically similar to that derived in Malik & Shenoy (1991) for power-law fluids, in the following we only introduce the final result

$$\begin{aligned} Q = & -\frac{1}{2}\left(\frac{1-s}{3+s}\lambda^2 - \kappa^2\right) - \left[\frac{1+\kappa^3-\lambda_i^3-\lambda_o^3}{6} - \frac{1-s}{2(3+s)}\lambda^2(1+\kappa-\lambda_i-\lambda_o)\right]\Lambda^s T_0^s + \\ & + \frac{\Lambda^s}{2(3+s)}\left[(1-\lambda^2)^{1+s} - \lambda_o^{1-s}(\lambda_o^2-\lambda^2)^{1+s} + \lambda_i^{1-s}(\lambda^2-\lambda_i^2)^{1+s} - \kappa^{1-s}(\lambda^2-\kappa^2)^{1+s}\right] \end{aligned} \tag{19}$$

Comparing rels.(18,19) we obtain

$$\begin{aligned} & -\frac{1-s}{2(3+s)}\lambda^2 - \left[\frac{1+\kappa^3-\lambda_i^3-\lambda_o^3}{6} - \frac{1-s}{2(3+s)}\lambda^2(1+\kappa-\lambda_i-\lambda_o)\right]\Lambda^s T_0^s + \\ & + \frac{\Lambda^s}{2(3+s)}\left[(1-\lambda^2)^{1+s} - \lambda_o^{1-s}(\lambda_o^2-\lambda^2)^{1+s} + \lambda_i^{1-s}(\lambda^2-\lambda_i^2)^{1+s} - \kappa^{1-s}(\lambda^2-\kappa^2)^{1+s}\right] = 0 \end{aligned} \tag{20}$$

Combining these semi-analytical results presented above with the experimental data provided by a compression testing machine such as those manufactured by Instron Corp., it is possible to determine the rheological parameters τ_0, K, n appearing in the Vočadlo model, rel.(3).

4. Conclusion

The 3P Vočadlo model in its form eliminates a singularity appearing e.g. in the 3P Herschel-Bulkley model. 'Smoothness' of the Vočadlo model results in better application to the numerical procedures as e.g. a semi-analytical one in back-extrusion characterisation of rheological behaviour of various food materials.

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