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MODEL OF CONCRETE AT THE EARLY AGES

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Summary: *The mechanical behaviour of solidifying and further hardening concrete is influenced by the rapidly progressing hydration which makes the experimental work difficult and as a result there is few experimental data for concrete at such early ages. Since the Chen model of plasticity is very suitable for modelling of hardened concrete due to the different strengths considered in compression and tension, it was sought to make use of this model also for description of mechanical behaviour of hardening concrete. The material parameters considered in the modelling have to be dependent on the progressing hydration. The hydration is quantified with help of the degree of hydration. The performance of the modified Chen model of plasticity is described in an illustrative example.*

1. Introduction

Concrete, one of the most common buildings materials, is designed to serve a hard solid. Unlike in the case of steel structures where steel members are being assembled already as a material of the design properties, concrete in majority of cases is placed as liquid substance and then gradually it become the solid material of the designed properties.

Recently, the emphasis is put on the speed of construction. Therefore it is necessary to find how much construction process can be expedited. This often results in overloading of solidifying and hardening concrete. At this stage the structure of concrete is not fully developed. The effect of overloading, of course, depends on the specific application, as overloading can also result in proper consolidation of concrete, which is actually a positive result. On the other hand, the formwork is not infinitely stiff and so engineers try to reduce the hydrostatic pressure by the assumption that at some age the concrete can, at least partially, support itself. Then, if we know what is going on inside concrete, inside the formwork, one can increase the height of continuously poured lifts of concrete that means to expedite the construction process.

This paper describes an extension of Chen model of plasticity [Chen 1982] for use with concrete in the very early ages. Since the Chen model of plasticity is very suitable for modelling

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of hardened concrete due to the different strengths considered in compression and tension, it was sought to make use of this model also for description of mechanical behaviour of hardening concrete. The material parameters considered in the modelling have to be dependent on the progressing hydration. The hydration is quantified with help of the degree of hydration, [Štemberk & Tsubaki 2003]. Due to this modification the range of application of the Chen model can be extend to hardening concrete.

2. Method of solution

2.1. Evolution of microstructure of concrete

The solidifying and hardening state is dominated by the rapid progress of hydration, which must be taken into account without exception when dealing with solidifying or hardening concrete. The evolutionary function, as its name implies, is introduced in order to describe the evolutionary changes in the microstructure of solidifying and hardening concrete and therefore to control the mechanical behaviour of concrete in the modelling. The function of microstructure evolution, which expresses the effect of aging, was identified from experiments on the evolution of penetration resistance, pullout resistance and compressive strength of concrete at the ages up to the final setting time, as shown in the paper [Štemberk & Tsubaki 2003], and is given by

$$h(t_n) = a_5 \cdot \left(\frac{a_3 t_n^{a_2}}{a_1 + a_3 t_n^{a_2}} \right)^{a_4}, \qquad \begin{aligned} a_1 &= 10, \\ a_2 &= 9.164 - 7.2W/C, \\ a_3 &= 0.72, \\ a_4 &= 1, \\ a_5 &= 15, \end{aligned} \tag{1}$$

where t_n is a normalised time with respect to the final setting time, W/C is the water/cement ratio (in decimal) and a_1, a_2, a_3, a_4 and a_5 are empirical parameters.

Figure 1 shows the comparison between the compressive strength evolution expressed by the evolutionary function (1) (scaling parameters are given in Figure 1) and experimental data.

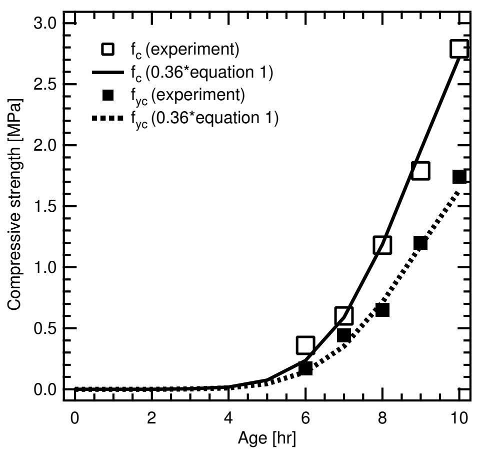

Figure 1: Compressive strength evolution

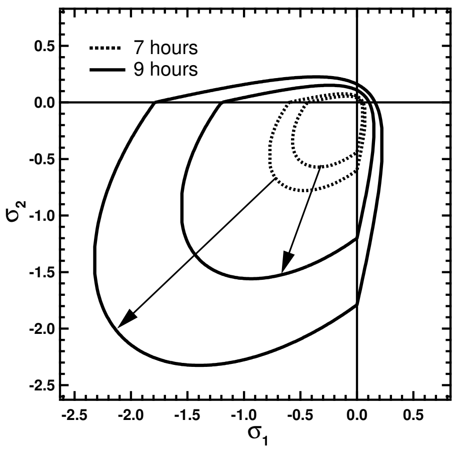

Figure 2: Evolution of initial yield and failure surfaces

For the multidimensional analysis of mechanical behaviour of concrete loaded at the early ages, the necessary input parameter is the time evolution of Poisson's ratio. The Poisson's ratio is dependent on the evolution of micro-structure. Construction of a simple function relating the Poisson's ratio and loading close to the end of the hardening stage can be found in [Štemberk & Kohoutková 2005].

2.2. Chen model of plasticity

Chen model of plasticity is a three-parameter model for concrete displaying isotropic hardening [Chen 1982]. This model expresses the elastoplastic behaviour of concrete. The typical behaviour of concrete is varying stress-strain characteristic under tension and compression. Two different, but similar, functions were proposed for each of the loading surfaces, in the compression-compression region and in the tension-tension or tension-compression regions. In our example, compression loading is considered hence only the equations for the compression-compression region are in the paper introduced.
The failure surface is assume in the compression-compression region

$$f_u^c(\sigma, h) = J_2 + \frac{A_u(h)}{3} I_1 - \tau_u^2(h) = 0. \tag{2}$$

The initial yield surface in the compression-compression region is given by

$$f_0^c(\sigma, h) = J_2 + \frac{A_0(h)}{3} I_1 - \tau_0^2(h) = 0, \tag{3}$$

where $A_0(h)$, $\tau_0(h)$, $A_u(h)$ and $\tau_u(h)$ are material constants which can be determined from simple tests. They are determined as functions of the ultimate stresses under uniaxial compression, $f_c(h)$, and under equal biaxial compression, $f_{bc}(h)$, and of the initial yield stresses under similar conditions, $f_{yc}(h)$ and $f_{ybc}(h)$. With increasing strength of concrete the loading surfaces are expanding. This is evidenced in Figure 2 where the dotted lines denote the initial yield surface and the failure surface at the age 7 hours and the solid lines denote the initial yield surface and the failure surface at the age 9 hours.

3. Application

To illustrate the applicability of the presented approach, a real structure was considered. The Border bridge is a part of the newly constructed D8 highway connecting Prague (Czech Republic) and Dresden (Germany). This composite bridge is about 500 metres long and overpasses a deep valley. The intermediate columns are circa 50 metres tall, which prohibits pumping concrete directly from the bottom of the valley to the bridge deck, which is designed as reinforced concrete slab. Therefore, the concrete needs to be transported to the location of placement across the already finished reinforced concrete deck. As falling behind the schedule is a very possible threat, especially in this case, when the construction site is located in a mountainous area, where it is a subject to unfavourable weather conditions, a tool for estimation of the earliest possible entrance to the newly concreted section of the deck is desirable, moreover, when the tool also provides some information on the possible damage caused by premature loading.

The material characteristics of concrete in the early ages are obtained from experiments. Simple techniques based on the uniaxial compression and penetration tests were proposed. For example, the experimental data gathered by Byfors yielded, from today's point of view, very useful data on the evolution of modulus of elasticity and compressive strength of solidifying and further hardening concrete, e.g. [Byfors 1980]. This experimental data were for investigation of the reinforced concrete deck of the composite bridge modelled the section of the concrete deck (see Figure 3) and analyse it at the ages from 6 to 24 hours under compression. Figure 3 shows the form of deformation of young concrete under the excessive compressive load.

The main objective was to describe the behaviour of the concrete deck under compressive load corresponding to the truck carrying fresh concrete. The data for loading force were obtained at a construction site. The force representing the truck tyre is defined as a 90 kN on one wheel axis, here the front axis was used. Therefore the pressure on the slab was derived as a force of 45 kN on the area under wheel, whose standard contact area is circa 20 x 30 cm.

Figure 4 shows results of the analysis. There are shown decreasing displacements with increasing age of the concrete under constant loading. The growing of strength at the ages from 6 to 12 hours is also obvious from this graph.

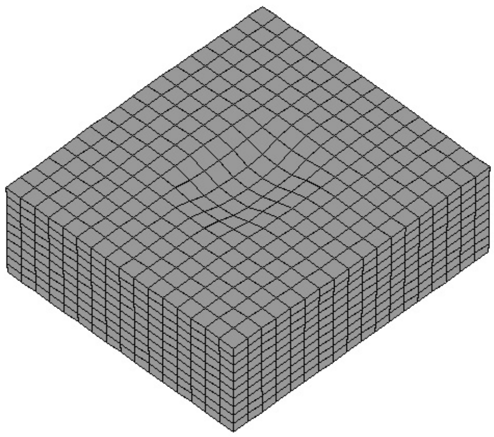

Figure 3: Section of analysed concrete deck - deformation

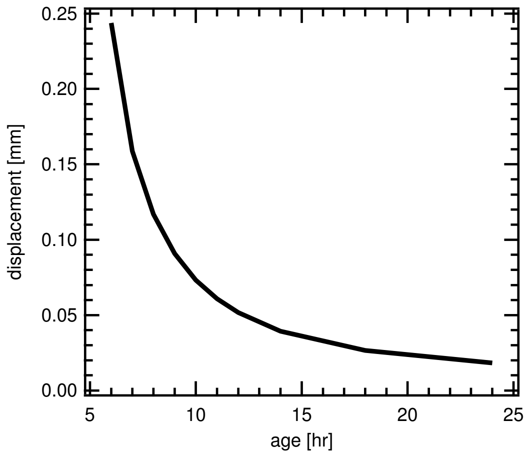

Figure 4: Vertical displacement at various ages

4. Conclusions

In this paper, modification of Chen model of plasticity was described, so it is possible to use this model also for solidifying and hardening concrete. Therefore, the parameters of Chen model are define as a function of microstructure evolution. The data for an example were obtained experimentally.

Results obtained from this modified Chen model of plasticity was compared with results of the experiments. A very good agreement of both results verifies the possibility of using Chen model of plasticity not only for hardened concrete but also for hardening concrete with yet evolving microstructure.

The applicability of the presented model is shown on the real structure and were the possibility of entrance to the newly concrete section was investigated.

The proposed model can be extended further so that the uncertainty is accounted for. An approach to fuzzification of the Chen model of plasticity described above was presented in [Kruis & Štemberk 2004]. A similar fuzzification procedure was presented in [Kruis & Štemberk

2005], where the varying settlement of a tall reinforced concrete wall during construction was quantified with respect to the uncertainty contained in the material description. The advantage of the fuzzy set approach lies in the form of the output values, which can serve immediately as the input parameters for a financial analysis of economical feasibility of the entire construction project.

5. Acknowledgment

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