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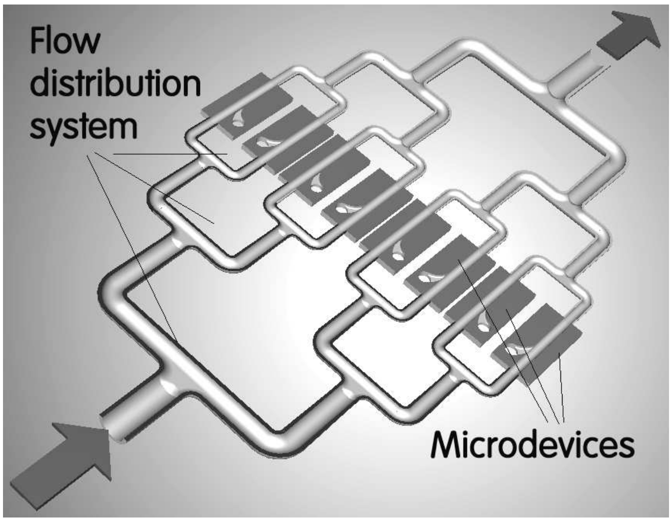
MULTI-LEVER FLOW BRANCHING

V. Tesař*

Summary: *Paper presents basic concepts governing distribution of a fluid flow into a large number of parallel devices. Derived laws for multi-level progressive division into branches are first validated by comparison with known data on mammalian respiratory and cardiovascular systems. Then the optimality criterion is sought for the specific conditions of microfluidics, with channels etched in a planar substrate.*

1. Introduction

The subject of the present paper became recently of particular importance due to the emergence of microfluidics - handling fluid flows in devices of submillimetre characteristic dimensions [1, 17]. When the objective are large-scale effects, the microdevices are not scaled up (which would mean losing the very advantages which the small size brings) but "*numbered up*": operated in large numbers in parallel. Figs. 1 and 2 show typical alternatives of distributing the total supply flow into the large number of paths through the microdevices. Usually, a reverse process of summing up the flows takes place on the exit side. Altogether, the flow division and branching may occupy more space than the actual fluid processing devices. Finding the optimum configuration is a task far from simple as it may perhaps seem to be. There may be also special reasons for using the flow distribution networks: the two systems of inlet branches in Fig. 3, bringing close together two different fluids, solve the task of fluid mixing, rather difficult in microfluidics where the typical low Reynolds numbers lead to absence of turbulence and micro-mechanical stirrers tend to be rather tricky. In some particularly sophisticated versions, especially on the downstream side, the in-



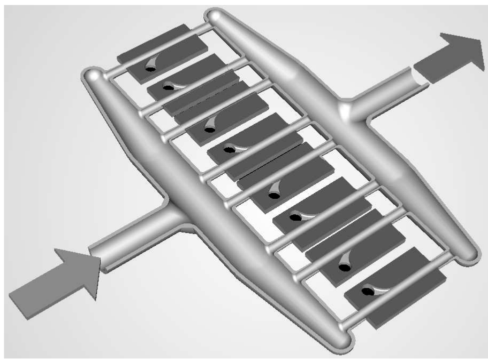
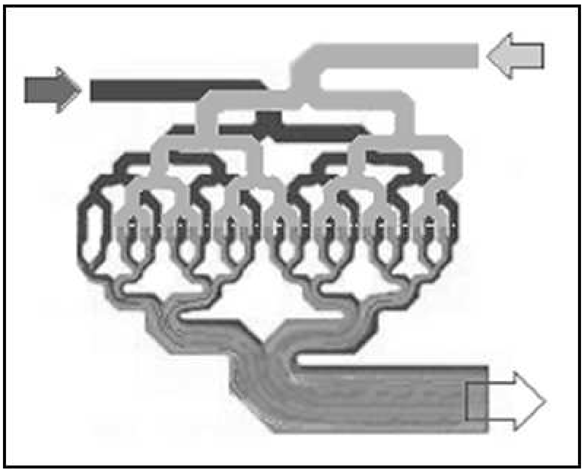


Fig. 1 (Left) A typical multi-level branched distribution system supplying fluid into a number of fluidic devices operated in parallel. The mirror arrangement of branches on the exit side performs the reverse task of flow summation to produce the desirable large output.

Fig. 2 (Right) The branching may be replaced by a common manifold chamber open into all microdevices in parallel. Similar common collector vessel is again on the exit side. This layout may conserve space, but is obviously not favoured by natural selection in living beings: it is difficult to secure equal flowpath length needed for equal distribution of flows into all microdevices.

* Prof. Ing. Václav Tesař, CSc., Institute of Thermomechanics, Academy of Sciences of the Czech Republic, Dolejškova 5, 182 00 Prague 8, tel.: 420 26605 3282, fax: 420 28658 4695, e-mail: v.tesar@it.cas.cz



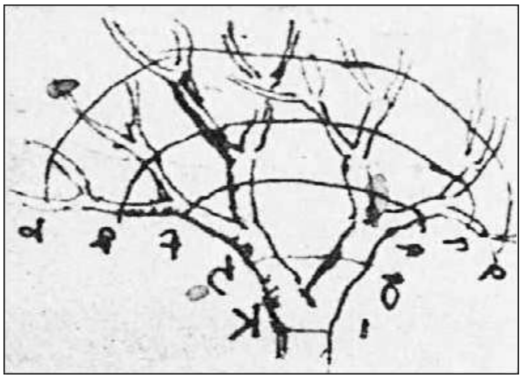
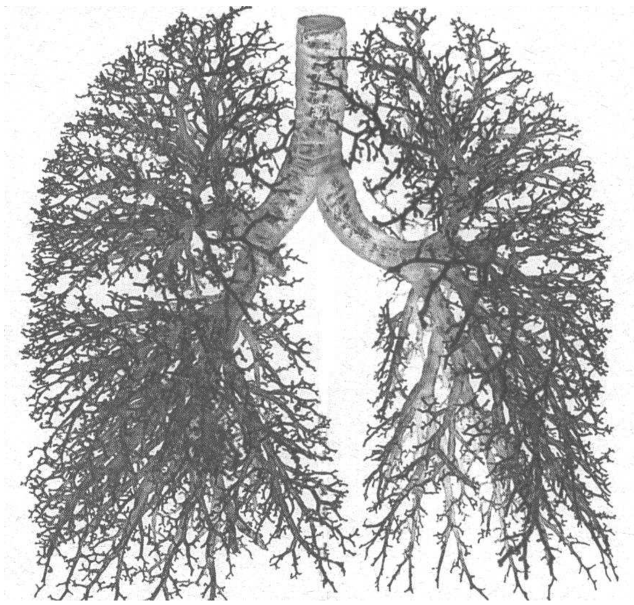


Fig. 3 (Left) Principle of one type of microfluidic mixers. Two independent inlet channel systems (in two parallel planes of a multi-layer arrangement) produce interleaved strips of the two mixed fluids - typical for most microfluidic (no turbulence) mixers. Strips are thin and this reduces the distance to be traversed further downstream by Fickian low Re diffusion which finally removes the concentration gradients.

Fig. 4 (Right) A sketch from Leonardo da Vinci’s notebook [2] depicting his theory of tree branching: the total cross section of branches at each branching level constant and equal to the cross section of the tree trunk.

dividual flows mutually interact, Fig. 17. At any rate, an effective design is increasingly important as microfluidic systems tend to be more and more complex. There are many variables and choice of optimality criteria is not obvious. Almost universally demanded basic requirement is to secure as much as possible identical conditions for all microdevices. This requires equal flowpath lengths, difficult to maintain in the alternatives shown in Figs. 2 and 5.

The branching task was solved by Nature in higher living organisms which require the flow division to supply nutrients for sustaining the cells, “*micro-devices*”, which are also



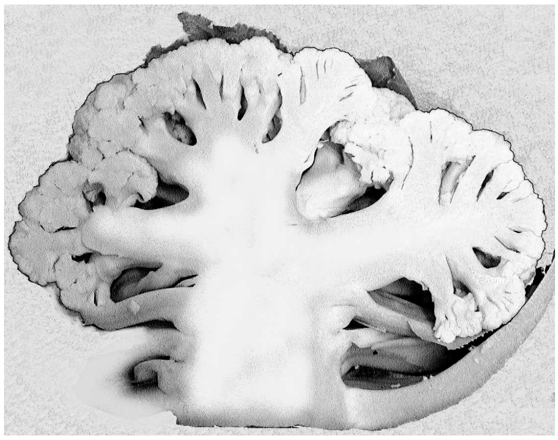


Fig. 5 (Left) Human respiratory system up to and including the $k=7$-th branching level (i.e. without the smallest-level branches, the inclusion of which – since they approximate a space-filling fractal - would make the branching design impossible to follow).

Fig. 6 (Right) Branched supply system of cauliflower. Even though at the final level the “microdevices” are effectively on a surface, the branching principle is used rather than the principle of the membrane Fig. 7.

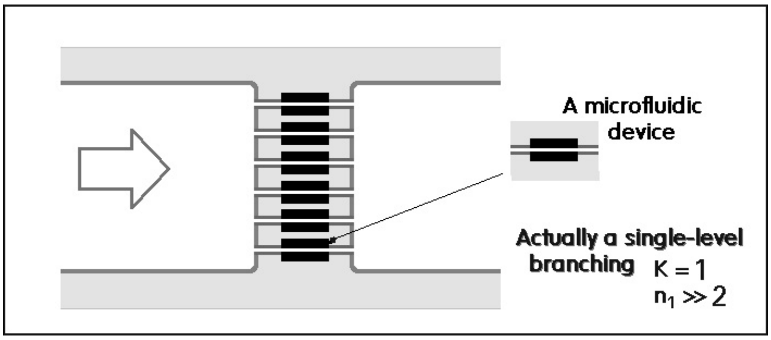


Fig. 5 The principle of a membrane, evolved from the common manifold principle Fig. 2 by decreasing the size and increasing number of micro-devices so that they occupy a surface perpendicular to the (local) flow direction. To save space, the membrane surface may be convoluted. The problem is how to achieve an equal distribution of flow into the devices, which must exhibit high hydraulic resistance to approach this goal.

"*numbered up*" rather than scaled up. Of many conceivable arrangements how to supply the cells, natural selection found and promoted as the most effective principle of the flow division the multi-level branching, with tube sizes progressively decreasing at each branching level. Mammalian respiratory (Fig. 5) and cardiovascular circulatory systems are an example. Also botanical organisms, plants, use the branching principle – Fig. 4, 6. The identity of conditions at the cell level is achieved by maintaining sameness of hydrodynamic conditions in daughter tubes at each branching level, where the small number of branches makes the task easier.

Designers of microfluidic systems can learn many useful lessons from the natural optimum solutions. It is, of course, necessary to keep in mind the different conditions in man-made branching systems:

a) The round tube cross sections are not used in microfluidics, where the micro-manufacturing techniques preferring planar designs. The space-filling tree of Fig. 6 cannot be directly simulated.

b) In plants – especially trees, the case studied already at about 1500 by Leonardo da Vinci [2], Fig. 4 – the branching laws are strongly influenced by the different underlying principle. The long plant cells pass through several subsequent branching levels, making inevitable the constancy of total cross sectionals.

c) Plants are not static. They grow continuously and the branching system must be arranged so as to cater for the necessary presence of budding new branches.

d) In pulmonary systems, the diffusion across the pipe walls becomes progressively important with the branching level as the size decreases. Optimality criteria valid for convective transport at large scales cease to be applicable at the diffusion-dominated levels.

e) The mammalian distribution trees operate in unsteady periodic flow regimes. In respiratory systems the flow direction is reversed at each half of the operating cycle. This double (forward and backward) use of the same branching tubes saves space (and tissue) but makes the hydrodynamics more complicated. Perhaps somewhat strangely, this flow reversation principle is not used in the cardiovascular circulatory system. Nevertheless the hydrodynamics is also complex there, due to the operation in pulsatile regime.

Despite the dissimilarities, it is useful to consider the mammalian arterial systems as a reference. With the number of terminal microdevices of the order of 10^{10} (= the number of capillaries in human cardiovascular system) and much deeper branching levels the natural solutions are more advanced than contemporary microfluidics. Investigated by a number of researchers, they provide useful background data to verify the derived relations.

2. Branching parameters:

In the typical case of a multi-level branching, Fig. 8, a pipe belongs to the branching level k if there are k branchings between it and the input "aorta" (level 0). The distribution network is composed of total K branchings in each flowpath between the aorta and the end level micro-devices - "capillaries" (level $k = K$). For simplicity of this introductory discussion, it is useful to consider a system of circular cross section pipes with negligible pressure drops associated with the bifurcation or junction processes – so that the pressure conditions are determined by the pressure drops across the pipes due to friction. Length of a typical branch at an intermediate level k is l_k, and its dia-

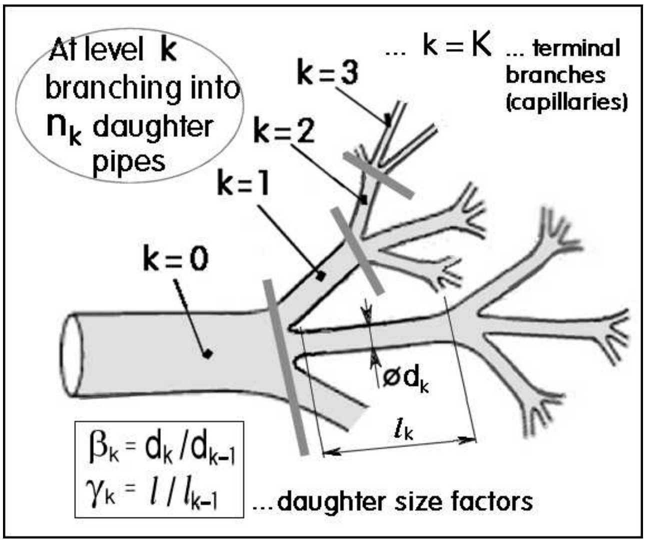


Fig. 8 Basic parameters of the multi-level branching: at the level k between the aorta ($k = 0$) and the capillary ($k = K$) there are N_k pipes. The branching factors n_k, β_k, γ_k ,… are the ratio of the distal to proximal quantity. Very often – but not necessarily – the system is self-similar with identical value of the parameters at each branching level.

meter is d_k. Geometry of the branching is characterised by the distal-to-proximal channel size factors: The geometric factors are diameter ratio (Fig. 8)

$$\beta_k = d_k / d_{k-1} \quad \text{... (1)}$$

and the length ratio

$$\gamma_k = l_k / l_{k-1} \quad \text{... (2)}$$

Because the essential requirement is the equal distribution of the flow into the branches, the discussion may be limited to ***equipartition*** branching, with all daughter branches at a level k identical. The kinematic parameter of the branching is the velocity ratio

$$u_k = w_k / w_{k-1} \quad \text{... (3)}$$

where velocity w_k is averaged over the cross section and, in pulsatile systems, over time. The ratio of pressure drops across the branches is

$$p_k = \Delta P_k / \Delta P_{k-1} \quad \text{... (4)}$$

and the ratio of the power dissipated by friction in the branches

$$\alpha_k = \dot{A}_k / \dot{A}_{k-1} \quad \text{... (5)}$$

The total number of branches at a level k (assuming a single inlet pipe, $n_0 = 1$) is

$$N_k = n_1 n_2 \ldots n_k = \prod_{j=1}^{j=k} n_j$$

where

$$n_k = N_k / N_{k-1} \quad \text{... (6)}$$

The most popular alternative in man-made systems (e.g. Fig. 1) are the ***bifurcation*** networks with $n_k = n = 2$ at all k levels. Fig. 8 presents the trifurcation case $n_k = n = 3$ and Fig. 13 the rather exceptional quadrifurcation $n_k = n = 4$.

To use the available space efficiently, the branching pattern resembles a space-filling fractal – of course not sharing its theoretical property $K \to \infty$ and its non-integer dimension. Like fractal objects, the networks are typically ***self-similar***, not only with $n_k = n$, but also the same values of the geometric factors $\beta_k = \beta$ and $\gamma_k = \gamma$ at all k levels.

In the self-similar cases, the expression for the number of all branches at the level k simplifies to

$$N_k = n^k \quad \dots (7)$$

3. Analysis – pulmonary and cardiovascular natural branchings

The goal is to identify the dependence of the four parameters β_k ; γ_k ; u_k ; p_k on the branching number n_k. Some of the relations between them, which restrict the degrees of freedom, are reasonably valid while others are rather speculative:

A) The first, generally most plausible assumption is a straightforward consequence of Castelli's version (constant volume flow rate) of the mass conservation law. Under the usual simplifying conditions of incompressibility $v = \mathrm{const}$ and assuming zero capacitance (no fluid accumulation) in the network, there is the same proximal and distal total volume flow rate at all branching levels

$$N_k \dot{V}_k = N_k \pi d_k^2 w_k / 4 = N_{k-1} \dot{V}_{k-1} = N_{k-1} \pi d_{k-1}^2 w_{k-1} / 4$$

so that from eqs. (1), (3), and (6) $\quad n_k \beta_k^2 u_k = 1 \quad \dots (8)$

B) Accepting the simplification of the node between connected branches behaving as a constant pressure space, the pressure drops are only due to frictional hydraulic losses in the pipes. Another assumption (somewhat more forced) of the losses ΔP being governed by the Hagen-Poiseuille law may be then accepted, with the resultant linear dependence on the volume flow rate (where R – resistance):

$$\Delta P_k = R_k \dot{V}_k$$

In fact, most pipes in typical branching networks are usually not long enough for the flow being fully developed, as the Hagen-Poiseuille law assumes. Nevertheless, this may be perhaps at least a suitable starting point.

$$R_k = \frac{128\,\nu}{\pi\, v\, d_k^4}\, l_k$$

$$\dot{V}_k = \pi d_k^2 w_k / 4$$

- which means $\Delta P_k \sim l_k w_k / d_k^2$

or $\quad p_k \beta_k^2 = \gamma_k u_k \quad \dots (9)$

It should be noted that assuming the Hagen-Poiseuille law validity leads to the expression for the power $\dot{A}$ [W] dissipated in a particular pipe branch as:

$$\dot{A}_k = \Delta P_k \dot{V}_k = R_k \dot{V}_k^2$$

$$\dot{A}_k \sim l_k w_k^2$$

$$\alpha_k = \gamma_k u_k^2 \quad \dots (10)$$

C) ***The isokinetic alternative***

The most popular version of engineering branching layouts, the (da Vinci's) ***isokinetic*** branching $u = 1$, reduces eq. (8) to

$$\beta_k = n_k^{-1/2} \quad \dots (11)$$

In particular, for the bifurcating ($n = 2$) self-similar ($u_k = u = 1$, $n_k = n$) isokinetic case, there is for all branchings:

$$\beta = n^{-1/2} = 0.707,\ \gamma = n^{-1/3} = 0.7937,\ u=1,\ p = n^{2/3} = 1.587$$

However, data for naturally developed fluid flow branching networks do not support this alternative to be the best solution. In particular, in cardiovascular systems the mean velocity w_K of blood flow in the capillaries ($k = K$) is much slower than in the aorta. In human arterial system the ratio of capillary/aorta velocities is

$$w_K / w_0 \approx 10^{-3}$$

....

C) A different alternative for the third assumption was introduced by Bengtsson & Edén [8]. It states that arterial systems are built with constant power dissipated per unit pipe wall area. This is a reasonable requirement for a biological system, securing an equal stress distribution among all its constitutive cells, keeping a well adapted general equilibrium in the body.

> It is interesting to consider order-of-magnitude numerical values for human arterial system - mainly according to [8]. The basal metabolism of roughly 100 W corresponds to specific power of the order 1 W /kg. Typical cell size is 10.10^{-6} m and density 10^3 kg/m^3, so that the dissipated power per typical human body cell is 1 pW. Average speed of flow through the capillaries is of the order of $w_K = 10^{-3}$ m/s and the diameter of the capillaries roughly $d_K = 20.10^{-6}$ m, the pressure drop across a capillary is approximately 3 kN/m^2, so that the dissipated power is of the order of 10 W per capillary. The length of a capillary being of the order of 10^{-3} m, this power is dissipated over the mantle area ~ 10^3 m^2, where there are roughly 10^{13} cells; thus the power dissipated - the mechanical power load on each cell – is also of the order of 1 pW.

The constant power loading (power $\dot{A}$ per unit mantle area $\pi d^2 \, l/4$) in both proximal and distal branches at the k-th level meets the condition

$$4 N_k \dot{A}_k / (N_k \pi d_k l_k) = 4 \dot{A}_{k-1} / (\pi d_{k-1} l_{k-1})$$

which, in view of eq. (10) leads to

$$u_k^2 = \beta_k \qquad \text{...(12)}$$

Inserting this relation into eqs. (8) and (9) results in

$$u_k = n_k^{-1/5}$$
$$\beta_k = n_k^{-2/5} \qquad \text{...(13)}$$

and the relation between the two remaining undetermined factors

$$\frac{p_k}{\gamma_k} = n_k^{3/5} \qquad \text{...(14)}$$

D) The last condition to be met in biological organisms follows from the assumption of the branching network being volume filling. For the assumption $d_k < l_k$, necessary for validity of the Hagen-Poiseuille law,

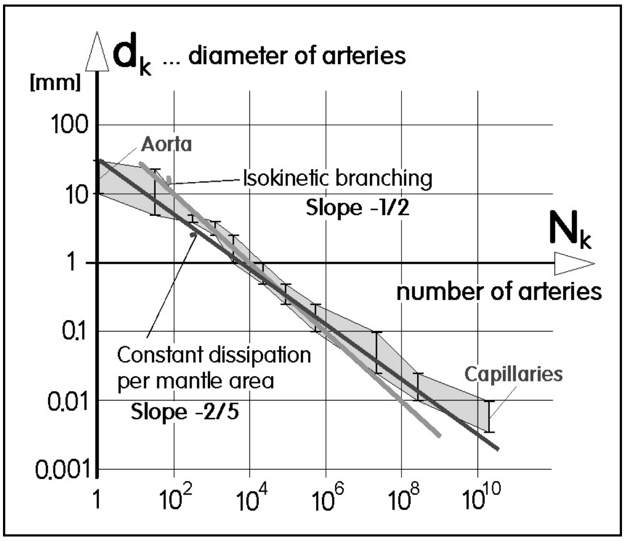
$$N_k l_k^3 = N_{k-1} l^3_{k-1}$$
$$\gamma_k = n_k^{-1/3} \qquad \text{...(15)}$$
$$p_k = n_k^{4/15}$$

Indeed, eqs.(13) and (15) mean $\beta_k/\gamma_k = n_k^{-1/15}$, the arteries become more elongated as the branching level increases, leading to better agreement with $d_k < l_k$,. If the tubes were not elongated, the condition D would become $N_k \, d_k^2 \, l_k = N_{k-1} d_{k-1}^2 l_{k-1}$, resulting, due to (13), in

$$\gamma_k = n_k^{-1/5} \qquad \text{...(16)}$$

4. Comparison with data

The branching laws derived to meet the above condition may be compared with data available for mammalian cardiovascular [7] and pulmonary [5, 6] networks. In the self-similar branchings, the dimensionless eq.(13), $\beta_k = d_k / d_{k-1} = n_k^{-2/5} = (N_k/N_{k-1})^{-2/5}$ is satisfied with $d_k \sim N_k^{-2/5}$ and $d_{k-1} \sim N_{k-1}^{-2/5}$ at each level k. This value of the exponent $-2/5$ is shown to hold both in Fig. 9 for the human arteries and in Fig. 10 for mammalian lungs - in the latter case, of course, only in the convection-dominated region of interest here, at small branching levels $k < 9$.



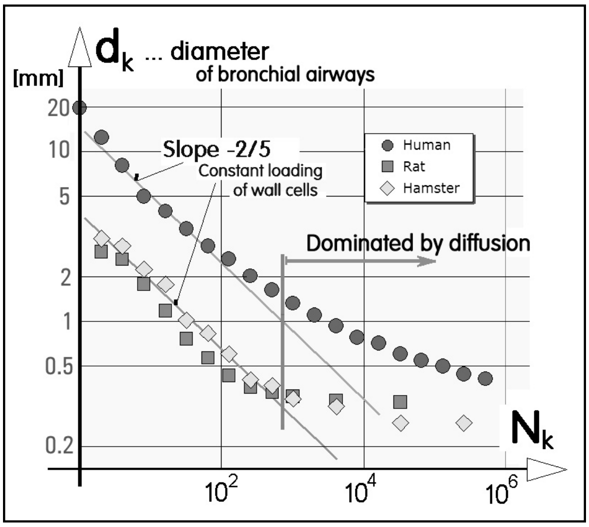


Fig. 9 (Left) Dependence of the diameter d_k of human arteries on the branching level k, which is characterised by the number of branches N_k. The slope of constant dissipated power loading of the arteries wall fits the data of Schnek [7] (the shaded area indicates typical uncertainty) visibly better than the isokinetic branching law.

Fig. 10 (Right) Measured [5, 6] diameters d_k of mammalian bronchial tubes at various branching levels k (characterised by N_k). Above the small size dominated by the diffusion across the walls, the slopes correspond well to the dependence derived for the constant dissipation per tube mantle area.

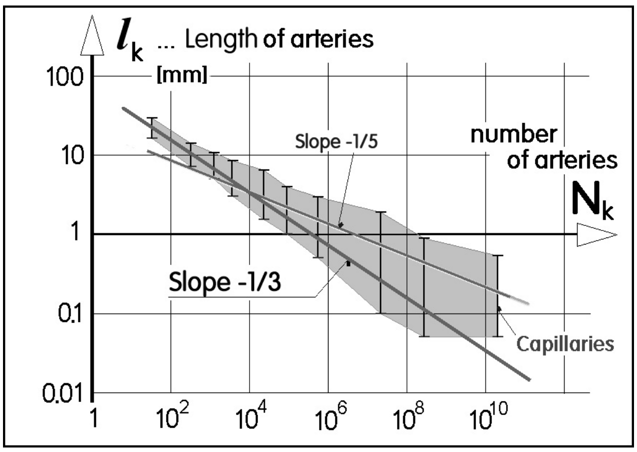


Fig. 11 Measured lengths l_k of human arteries at a branching level k, again characterised by the number of branches N_k [7]. compared with the slopes corresponding to volume filling by long tubes, slope $-1/3$ eq.(15), and by tubes of larger diameter-to-length ratio, slope $-1/5$ eq. (16).

Similarly, the distribution of human arterial lengths in Fig. 11 supports reasonably the space filling law $\chi_k = n_k^{-1/3}$ for narrow long tubes, eq. (15). The scatter, however, does not completely exclude validity of short-tube law eq. (16) – except at small k, where filling the space by short tubes is out of question anyway.

5. Optimality criteria dictated by manufacturing technology

Microfluidic flow distribution networks have to meet criteria similar to A, B, C, and D - but dictated by technological rather than biological factors. The contemporary planar manufacturing methods lead to different geometric conditions. Typically, the channels are etched to a constant depth h everywhere (Fig. 12). In the geometric branching factor β the diameter ratio is replaced by the ratio of channel widths b (Fig. 12)

$$\beta_k = b_k / b_{k-l} \quad \text{... (17)}$$

The criteria A, B, and D, adapted accordingly to these geometric constraints are then:

A) $n\,\beta\,u = 1$ ... (18)

B) $p = \gamma\,u \qquad \alpha = p\,\beta\,u = \gamma\,\beta\,u^2$ ... (19)

D) Plane filling property $n\,\beta\,\gamma = 1$ ... (20)

The power loading of channel surfaces, the basis of the criterion C for biological organisms, is hardly of importance there and has to be replaced by a different constraint.

Practically more important is the overall ***pressure drop*** across the branching. Alternatively, this criterion may be formulated as desirability of small power dissipated by friction.

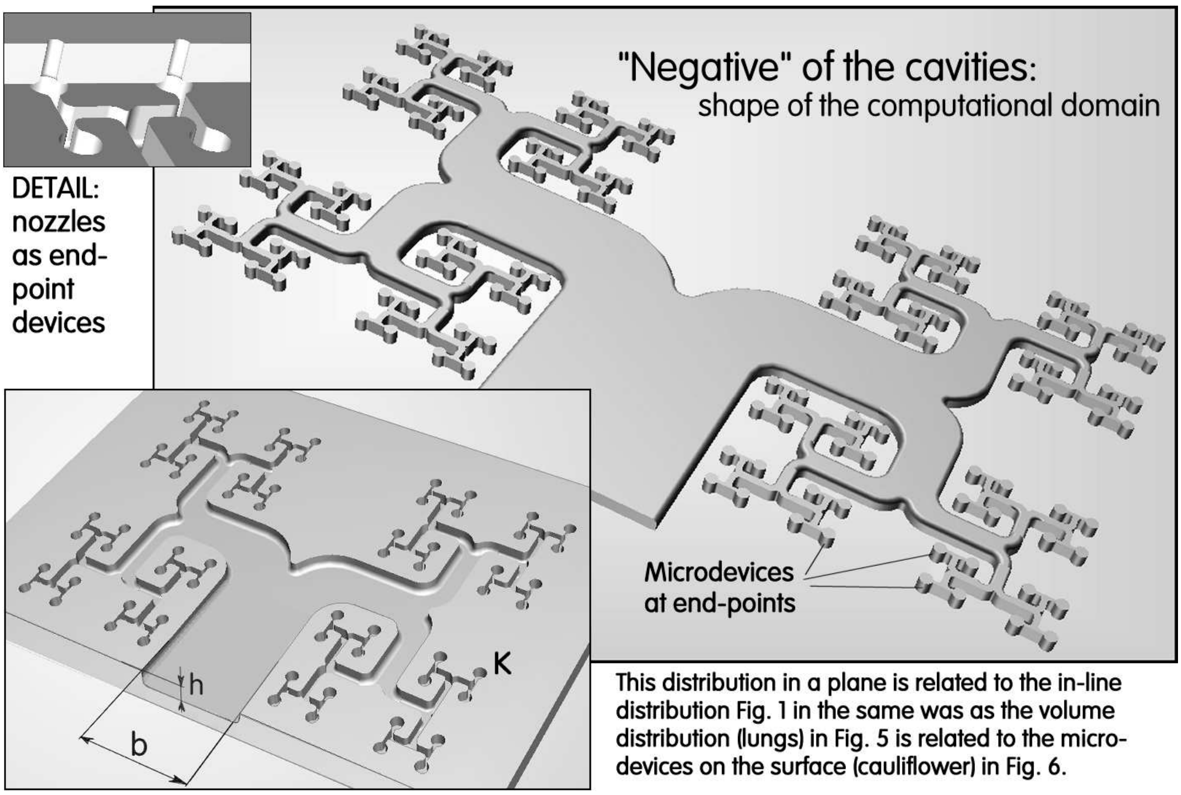


Fig. 12 The H-scheme of bifurcating supply channels into microfluidic devices at end-points K distributed over the plane (rather than in-line as in Fig. 1). If the fluid is not stored, consumed, or left to leave (irrigation, impinging micro-jets), the design is complicated by the necessity of multi-layer configuration providing the space for the exit network in another parallel plane.

This, however, cannot be the decisive criterion alone as it would simply lead to the channel cross sections as large as possible together with short channel lengths. This would jeopardise the paramount requirement of equal distribution of the flows – because it would tend to lead to the membrane case Figs. 2, 5 with its problematic flow distributions among the channels.

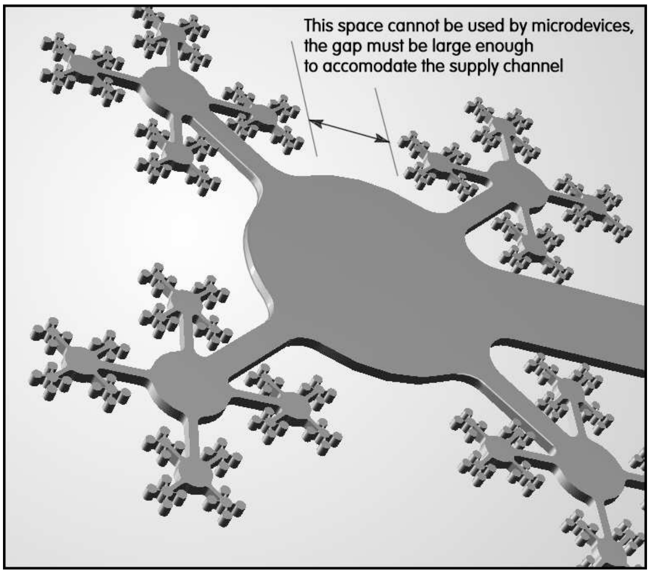
In fact, the large width of the distribution channels is likely to cause design problems. This is particularly obvious for the microdevices distributed in the plane, Figs. 12, 13, rather than in line as was the case in Figs. 1, 15. The devices in Fig. 12 are cooling micro-nozzles, generating impinging jets. Ideally, the endpoints of the distribution system should cover the plane uniformly. However, if the widths of the supply channels are large, it is difficult to fit them between the nozzles. The designer is forced to concentrate the nozzles in clusters with voids between the clusters – if the distribution in the plane is to be regular and the con dition of equal path lengths to the nozzles is to be met. The problem is more obvious for larger number n of daughter channels, Fig. 13. A non-isokinetic branching law $\beta > 1/n$, making the placement of the supply channels easier, is welcome even if it means increased total dissipative loss.

Another reasonable optimality criterion is the small total ***volume*** of the distribution network. It may be a useful design goal because of the usual supreme requirement placed on microfluidic systems - their small overall dimensions. Under the common condition of incompressibility, the small total volume is equivalent to minimum *residence time*. This is may be an important criterion especially for transitional regimes like start-up, rinsing, change of reagents in microchemistry, or depressurisation. Characteristically, an optimisation with respect to this criterion alone would lead to extremely narrow channels, the very opposite to the consequences of applying the pressure loss criterion.

Obviously, a compromise choice is needed. Seemingly taking into the account both aspects is minimising the power density – power dissipated per unit volume. This is equivalent to requesting as the criterion C

$$n\,\alpha / (n\,\beta\,\gamma) = u^2 = 1 \qquad \text{... (21)}$$

This, unfortunately, is equivalent to the da Vinci's isokinetic $u = 1$, $\beta = 1/n$, already dismissed as leading to the excessively large substrate area occupied by the low k level channels.



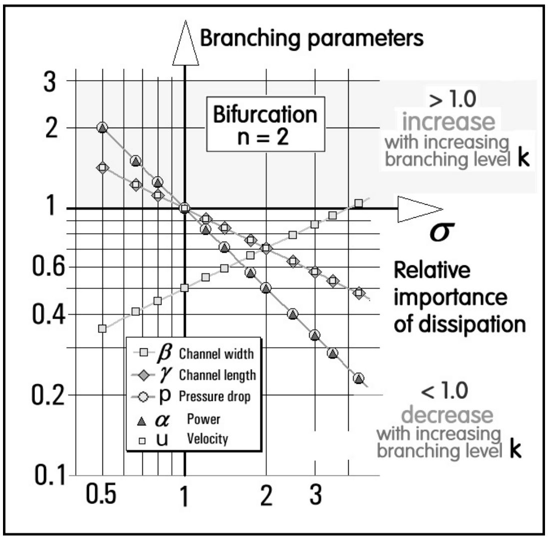


Fig. 13 (Left) A layout with end-points across a plane: quadrifurcation $n = 4$ scheme of alternating "+ "and "x" crosses (shown here again in the "negative"). Similar to Fig. 13, the necessary space for the wide supply channels makes achieving an equidistant end-points distribution practically impossible if the flowpaths are to be of the same lengths.

Fig. 14 (Right) Values of the branching parameters evaluated with the compromise criterion C with different dissipation weights σ for the bifurcating branchings.

A more general alternative is to place a different importance weight on the dissipation by chosing

$$\sigma\,\alpha / (\beta\,\gamma) = 1 \qquad \text{... (22)}$$

- with the weight factor σ adjustable according to particular conditions. This adjustability permits covering the whole range between the the minimum volume $\sigma=1$ or even $\sigma<1$ and the more emphasis placed on minimal losses with $\sigma>1$. The latter, of course, is more important in devices spending most operation time in steady regime where the minimum residence time is of secondary concern. The corresponding values of the branching factors u, β, γ, α, p are plotted as a function of the chosen σ in Fig. 14, valid for the bifurcating network. Note that the factor u of velocity distribution in the branches varies with σ equally as the length ratio factor γ, also the pressure and power changes coincide. The velocity change in the branching is

$$u = 1/\sqrt{\sigma} \qquad \text{... (23)}$$

- so that velocity is lower in the daughter channels for $\sigma>1$, corresponding to the larger channel widths factor

$$\beta = \sqrt{\sigma}\,/n \qquad \text{... (24)}$$

A different requirement replacing the filling of the plane may be used if the task is to distribute the flows into a device array positioned along a line, such as in Fig. 1. A detailed example of such branching network with typical channel bends at inlet as well as at the outlet is shown in Fig. 15. Contrary to the placement in the plane, here is no strong motive for the non-isokinetic branching law $\beta > 1/n$. It is easy to derive the expressions for the channel length l_k decrease with progressing branching level k. If the relative pitch a/b_o of the endpoints is within reasonable limits, the rounded geometry – surprisingly perhaps – has no effect on the bifurcation factors γ and β, the values of which are the same as derived above for the optimality condition of filling the plane.

Similarly usually negligible effects exhibit also other alternatives with sophisticated geometry – such as e.g. with inclined channel axes and different relative magnitudes of the rounding radii.

6. Interaction of flows in the nodes

All discussions above use the simplicistic assumption of constant-pressure behaviour of the flow division – and, on the downstream side, summation - nodes. The fact that the consequences agree reaso-

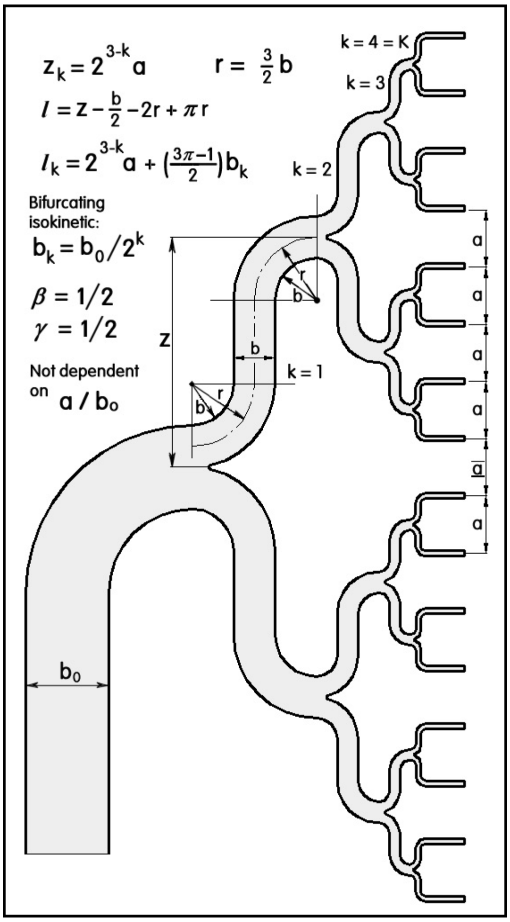
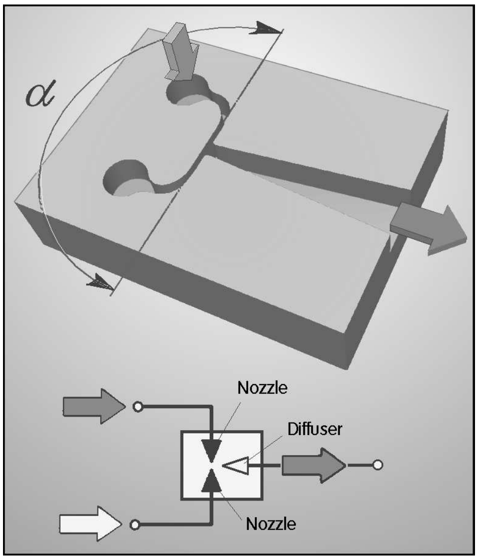


Fig. 15 Planar microfluidic flow distribution network with end-points distributed along a line: bifurcating isokinetic $n = 2$, $u = 1$, $K = 4$ self-similar geometry complicated by the rounded channel entrances and exits. The derived expressions may be easily generalised to non-isokinetic cases with various channel shapes and parameters of the geometry.



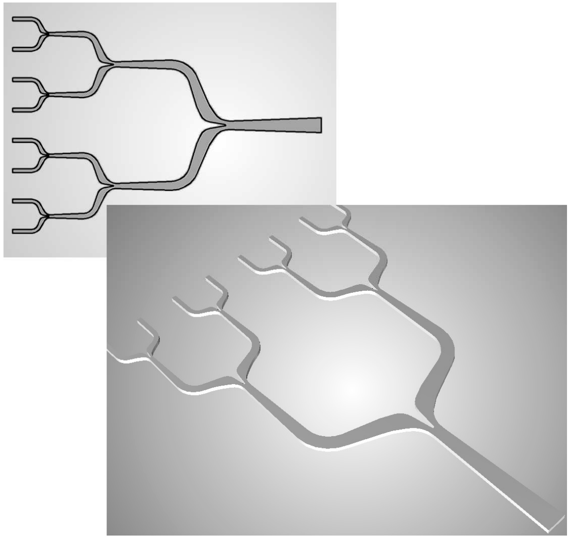
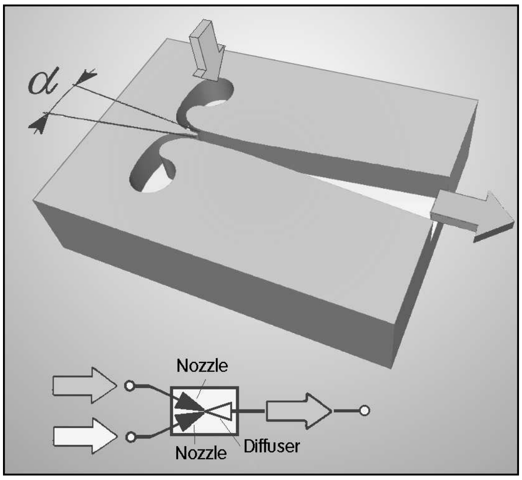


Fig. 16 (Above) Example of interacting flows in the downstream summation node: head-on collision with the opposing flow accelerated in the nozzle inhibits the neighbouring flowpath. This may be useful e.g. to keep the neighbouring upstream device flooded.

Fig. 17 (Right) Another dynamic interaction of flows in the summation node. Jet-pumping effect is used to promote the neighbouring flow. This way, the network shown on top generates cleaning flows to remove other fluids from the parallel paths in a fluidic sampling system [15, 16] to prevent cross-contaminations between various fluid samples.



nably with existing data are more a reflection of the current initial stage of the branching problem investigations rather than a non-importance of the real node behaviour. The need for a more precise description is likely to arise in foreseeable future. There is, fortunately, an already available theory for the node behaviour, e.g. [10, 11, 12, 13, 14]. In fact, the behaviour may be quite complex – and there is a number of useful operations which the specially adjusted nodes can perform. Usually such an operation takes place at higher Reynolds numbers, the flow interactions be of dynamic nature. For example, instead of just the passive flow summation in the downstream network, the flows may be accelerated in a nozzle and thus either mutually inhibit (Fig. 16) or promote (Fig. 7) themselves. This is of importance in those systems where the flows are not equal – mostly due to the effects taking place in the endpoint devices.

7. Conclusions

Increasing complexity of microfluidic flow distribution networks places increasing importance on their effective design. It may be tempting to base their analysis on properties of fractals, with which the flow distribution networks often share some features, in particular the usual self-similarity. However, the branching level K in current microfluidics is rarely large enough to justify this approach. Present analysis actually shows that quite useful laws of optimum branching may be derived from rather simple criteria, labelled here A, B, C, D. The optimality there has the form of conservation condition for some physical property.

The validity of the applied point of view is demonstrated in this paper by showing a good agreement with known data on naturally evolved fluid flow branching networks in living organisms. The comparison shows that reasonable solutions may be obtained even with modest models which need not go very deep into the fluid mechanics of the branched flows.

8. References

[1] Tesař V., Allen R.W.K., Tippetts, J.R.: " MICROFLUIDICS - The Challenge of Low Re Flow Control", Proc. of FLUCOME 2000, Paper 052, Sherbrooke, Canada, August 2000

[2] Richter J. P.: "The notebooks of Leonardo da Vinci (1452-1519), compiled and edited from the original manuscripts", p. 499, Dover, New York, 1970

[3] Murray C.D.:"The Physiological Principle of Minimum Work", Proc. Nat. Acad. Sci U.S.A., p. 207, Vol. 12, 1926

[4] Liu T.: "Optimum Bifurcating-Tube Tree for Gas Transport", Trans. ASME, Journ. of Fluid Mechanics, p. 550, Vol. 127, 2005

[5] Weibel E.R.: "Morphometry of the Human Lung", Academic, New York, 1963

[6] West B.J. et al.: "Beyond the Principle of Similitude: Renormalization in the Bronchial Tree", Journ. Appl. Physiology, p. 1089, Vol. 60, 1986

[7] Schneck D.J.: „An Outline of Cardiovascular Structure and Function“, In: The Biomedical Engineering Handbook, Bronzino J. D., ed., CRC Press & Springer, 2000

[8] Bengtsson H.-U., Edén P.: "A Simple Model for the Arterial System", J. of Theor. Biology, p. 437, 221(3), 2003

[9] Luo L., Tondeur D.: "Optimal Distribution of Viscous Dissipation in a MultiScale Branched Fluid Distributor", Internat. Journal of Thermal Scinces, Vol. 44, p. 1131, 2005

[10] Tesař V.: "The Hypervector Characterisation of Ejectors and Related Jet-Type Fluidic Elements" Proceedings of the 9-th International Fluidics 'Jablonna' Conference Paper A- 1 Jablonna, Poland, September 1982

[11] Tesař V.: "An Inversion Algorithm for Quadratic Vector-to-Vector Mapping and its Application to Solution of Bifurcated Flow", Proc. of 8-th Fluidics and Fluid Engineering in Control Systems Conference Vol. I, Bucharest, Romania, June 1980

[12] Tesař V.: "The Equivalent Dissipance", Proc. of 8-th Fluidics and Fluid Engngineering in Control Systems Conference Vol.I, Bucharest, Romania, June 1980

[13] Tesař V.: "Use of Energetic Potential for Characterisation of Dissipative Three-Terminal Fluidic Elements and for Solution of Stationary Bifurcated Flows", Proc. of 7. 'Jablonna'- Fluidik – Konferenz, Dresden, Germany, November 1978

[14] Tesař V.: "Characterisation of Three-terminal Hydraulic and Pneumatic Elements by Means of Energetic Potential", Acta Polytechnica, No. 22 (II,3), 1979

[15] Tesař V.: " 'Fluid Plug' Microfluidic Valve for Low Reynolds Number Fluid Flow Selector Units", Journal of Visualization, Vol. 6, No. 1, ISSN 1343-8875 6(1)1 - 85(3002), p. 77 - 85, Japan, 2003

[16] Tesař V.: " Sampling by Fluidics and Microfluidics", Acta Polytechnica - Journal of Advanced Engineering, Vol. 42, No2/2002, ISSN 1210-2709, p. 41-49

[17] Tesař V., et al.: "Subdynamic asymptotic behavior of microfluidic valves", Journal of Microelectromechanical Systems, ASME + IEEE, U.S.A., 14 (2), pp. 335-347, April 2005,