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APPLICATION OF PASSIVE DAMPER FOR SUPPRESSING FOOTBRIDG MOTIONS

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Summary: *This paper describes the types of footbridges which theoretical and physical models are solved. The highly sensitivity to dynamic load is decreased due to ball vibration absorber or due to liquid damper. Effectiveness comparison of the two types of dampers is recommended.*

1. Introduction

Concrete stress-ribbon footbridges, having very simple structural behaviour, are highly sensitive to dynamic loads because of their low bending rigidity, mass, natural frequencies and damping. The vibrations of footbridges cause the feeling of discomfort to pedestrians, in the case of major amplitudes they may result in damages of the footbridge pavement. Such vibrations may be caused by pedestrians, by wind or vandalism, and for this reason the stress-ribbon footbridges were subjected to the research of these loads.

2. Characteristic of footbridges

In the period of the stress-ribbon footbridges application were built in several variants. The original simply supported stress-ribbon suspended from one support to another was soon supplemented by the longer ones, supported with one or more intermediate rocking supports. Subsequently the stress-ribbon was strengthened by a rope stretched between the ends of the principal span. Finally, a system was designed in which the stress-ribbon was supported by an arch of ordinary bending rigidity. The dynamic behaviour of these systems is, of course, different.

There were three types of stress-ribbon footbridges used till now (see Fig. 1):

1. Prestressed ribbon hanging in the catenary form, stretched between the massive blocks on both banks of the river (one-span or several continuous spans).

2. Stretched stress-ribbon hung on a bearing cable; the centres of curvature of the ribbon and of the bearing cable are on opposite sides, under and over the pavement.

3. Stretched stress-ribbon supported by a classical arch structure near the midspan.

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2.1. Footbridges of the 1st type

The footbridges of the 1st type were used very often in the Czech Republic for spans of 60-80 m, continuous ribbons were used with even greater spans. There is one bridge of this type, spanning 126 m (Stráský & Pirner, 1983), another, by the same authors, was built in the USA, spanning (127.40 m) over the Sacramento River (Redfield, 1990).

The natural frequencies of simple ribbon-footbridges drop linearly with their spans. Fig. 2 shows the lowest natural frequency of the footbridge plotted against its span for various cases and according to various authors. The envelope of the gathered data can be described by the relation

$$f = \left\langle \frac{217}{l^{1.431}} - \frac{112}{l^{0.925}} \right\rangle \text{ for (m), (Hz).} \qquad (1)$$

The dependence used for road bridges, viz. $f = 92 l^{0.9}$

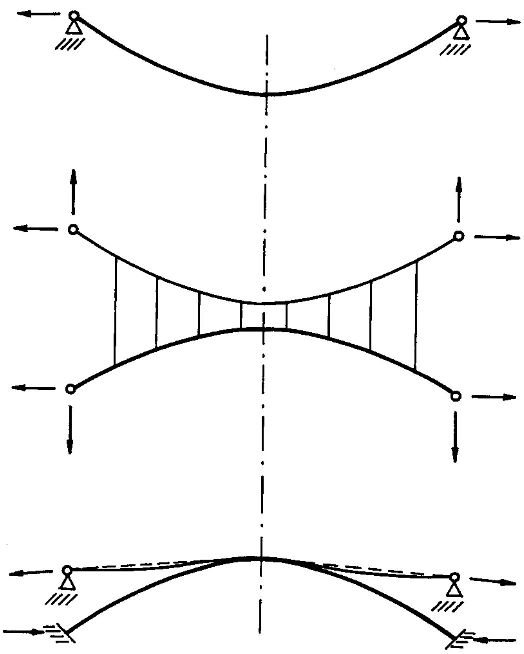

Fig. 1 Three types of stress-ribbon footbridges

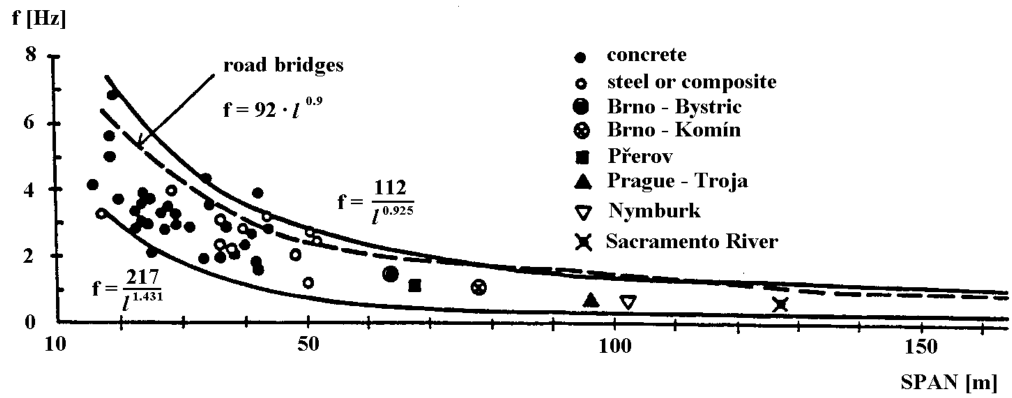

Fig. 2 Natural frequency stress-ribbon footbridges plotted against the span

The lowest natural frequency of a freely hanging ribbon corresponds to the antisymetrical mode-shape, when the middle point is at rest. The frequency is approximately equal to the 2nd frequency of a stretched spring, viz.

$$f_2 = \frac{1}{2l}\sqrt{\frac{H}{m}} \tag{2}$$

The continuous ribbon with intermediate rocking supports may change the span lengths during vibrations due to the horizontal displacements of the hanging points, which results also in the changes of the rope forces. The ribbon may vibrate in every span at an odd number of half-waves, similar to the ropes of a guyed mast. The natural vibrations of such a system must be analysed as a whole structure, e.g. using the conditions of dynamic equilibrium of forces acting on every intermediate support - see ch. 8 in Koloušek et al., (1983).

2.2. Footbridges of the 2nd type

The footbridges with greater spans would be too sensitive to wind fluctuations and to other dynamic effects, also the preservation of reasonable sags would require too high stressing forces, so that their anchoring in abutments would be difficult. In such a case, it is adequate to add a bearing rope above the footbridge on towers of such height as would make its sag and corresponding stressing force feasible, and to suspend the footbridge deck from this load bearing rope. If the deck as a whole were not stressed, the structure would actually behave as a classic suspension bridge, the rigidity of which is only due to the low bending rigidity of the ribbon. In such a case the footbridge could suffer excessive local deformations under concentrated loads. For a live load of the order of 0.01 of the local dead load these deflections can be expected to be of the order of centimeters, as such a system is indetermined referring to its form.

A more adequate solution is to design the footbridge deck as a curve, with the centre of curvature below the deck, on the opposite side to the centre of curvature of the bearing rope, and stress in sufficiently. This stressing would increase the loading of the bearing rope, but will increase the rigidity of the whole system. The deflection of such a system under the live load is of 1-2 orders smaller than in the above given example. Such a stiffening of the ribbon alone using the upper or lower bearing elements is also useful for the reduction of amplitudes of vibrations, excited by pedestrians' movement or by vandals.

The theoretical values of the natural frequencies and modes of the type 2 footbridges (coupled system consisting of the suspension ropes and the bridge deck with prestressing tendons) can be determined according the theory of prestressed networks, based on the integro-differential equation for equilibrium in the vertical direction (Pirner, 1994a). This theory makes it possible to determine the above-mentioned quantities for planar vibrations, provided that the suspension points of the ropes remain at rest during vibrations. This condition is approximately satisfied at these type of structures.

The vibration mode requiring the lowest amount of energy pertains to the frequency

$$f_{(2)} = \sqrt{\frac{1}{m}\frac{H_{\mathrm{r}} + H_{\mathrm{d}}}{l^2}} \tag{3}$$

here H_r is the force in the bearing rope, H_d is the stressing force of the bridge deck, m is the mass per unit length. The bending rigidities of the suspension rope and of the bridge deck have been neglected. Higher natural frequencies are given similarly

$$f_{(j)} = \frac{j}{2}\sqrt{\frac{1}{m}\frac{H_r + H_d}{l^2}} \tag{4}$$

If bending rigidities of the ropes and of the deck are taken into account, Eq. (4) will become more complicated (Pirner, 1994a)

$$f_{(j)} = \frac{j}{2}\sqrt{\frac{1}{m}\left(\frac{H_r + H_d}{l^2} + \frac{E_r I_r \pi^2 + E_d I_d \pi^2}{l^4}\right)} \tag{5}$$

For the fundamental natural mode (whose corresponding natural frequency is higher than Eq. (3), in which the suspension ropes and the bridge deck are getting longer and shorter) it holds (Pirner, 1994a):

$$f_{(1)} = \frac{j}{2}\sqrt{\frac{1}{m}\left(\frac{H_r + H_d}{l^2} + \frac{E_r I_r z_r^2 \pi^2 + E_d I_d z_d^2 \pi^2}{2l^4}\right)} \tag{6}$$

In these expressions E_r, A_r, I_r are the modulus of elasticity, area and moment of inertia of the cross-section of the rope, E_d, A_d, I_d are the same values for the bridge desk. z_r is the sag of the suspension rope, z_d is the rise of the bridge deck.

2.3 Footbridges of the 3rd type

The adequately rigid arch supporting the stress-ribbon from underneath will act dynamically with considerable independence, similarly as both loose parts of the ribbon, stretched between the arch centre and abutments. It can be merely expected that the natural frequencies of the respective antimetric arch vibrations will be slightly higher due to the stresses induced from the deck ribbons. Also here the bridge deck will be risen; its stressing force will improve the transversal resistance of the structure, nevertheless the slenderness of the arch is limited with respect to its stability.

3. Damping of the footbridges

With respect to the large sensitivity to dynamic loads it is important to know the magnitude of dynamic response as early as at the stage of design. In the computation of the response of these structures it is important to consider the dependence of damping on displacement, since the logarithmic decrement is not constant.

Damping is one of the characteristics of the structures which cannot be derived, it is therefore necessary to check it with the use of dynamic loading tests. The measured damping values of reinforced concrete stress-ribbon footbridges (in the Czech Republic) corresponding to the vibrations in the lowest natural frequency lie within the limits of

$$\vartheta = \langle 0.010 - 0.120 \rangle \tag{7}$$

with maximum measured response of 0.75 ms^{-2}

The excessive dynamic response determines the limit state of serviceability; it can namely evoke an unpleasant feeling, which is usually defined by the vibration velocity of 24 mm/s. In (Tilly et al., 1984) the limit is given by the acceleration according to nondimensional formula

$$a_{\text{lim}} = \frac{1}{2}\sqrt{f_{(\text{lowest})}} \text{ for } (\text{m/s}^2),\ (\text{Hz}) \tag{8}$$

Although the amplitudes of the air flow and pedestrian induced vibrations do not threaten the structure in general, there were in two cases special vibration dampers designed for the bridge: for vertical vibration it was a classical vibration absorber consisting of a mass and spring moving inside the railing columns of the Sacramento river footbridge (Pirner, 1994a), for horizontal movement of the bridge spanning 252 m in South Moravia it was a new type of ball damper, mounted under the pavement of the bridge. In this second case, it is in fact a tuned mass damper, but due to the low frequency of the horizontal vibrations of the bridge the pendulum-type absorber cannot be used. The pendulum movement is substituted by rolling of a mass (ball) on the curved bottom, the radius of which corresponds to the desired frequency of the absorber motion.

The theoretical solution of this damper has been given in (Pirner, 1994b; Pirner, 1995). In the described case the natural frequency of the bridge was about 0.5 Hz, and even the horizontal movement with this frequency and amplitude of approximately 0.5 m, with wind blowing at the velocity of approximately 20 m/s was observed during the erection of the bridge, before the pair of cables had been attached to the bridge deck. Although the installation of the stretched cables certainly limits the horizontal motion, it was designed to increase the bridge damping by a system of low long boxes, which can be fastened transversally from beneath the deck (see Fig. 3). The bottoms of these boxes are curved so as to form cylindrical surface with the axis parallel to the longitudinal axis of the footbridge. The dimensions of the boxes are 250/250, length 1600 mm, on the curved bearing plates inside each box there is a rubber-coated sphere, with a mass of 24 kg.

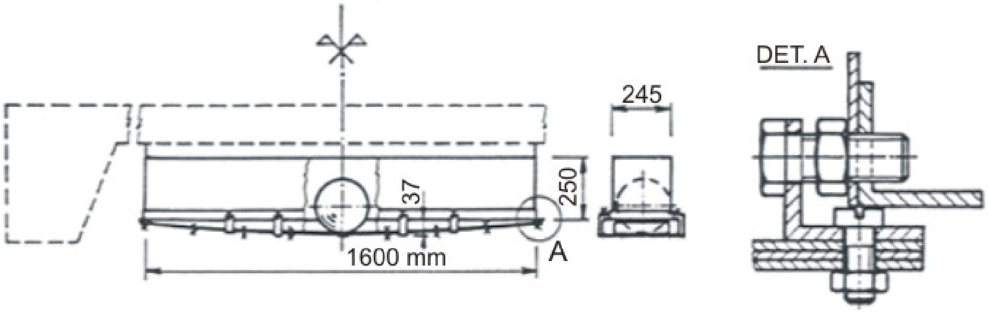

Fig. 3. Damper of horizontal vibrations. Left: view along the bridge axis; right: detail of the curved bottom plates. The cross-section of the bridge is shown by dashed lines.

4. Dampers

4.1. Ball damper

The principle of the function, the effectiveness and the design method of the ball vibration absorber, consisting of a heavy sphere rolling in a spherical dish, have been given elsewhere e.g. in Pirner, (1994b), Pirner & Fischer, (2000). Also its practical use on two 70 m high TV towers has been described. As this absorber system seems very promising due to both - its advantage of minimum necessary maintenance and its efficiency, it was decided to examine its behaviour in detail on the prototype. The same device (as to its dimensions and masses), which was used on both TV towers, was bought from the same producer and tested in the Laboratory of the Institute of theoretical and applied mechanics in Prague.

For the experimental investigation of the absorber behaviour a special stand had to be arranged. The dish of the 760 mm dia (depth 256 mm, R = 410 mm) was fastened to a steel table resting on 9 steel balls of ∅ 60 mm enabling the excitation of its movement by one, possibly also by two perpendicular forces; till now only the unidirectional excitation was tested, the movement of the table was restrained by lateral stoppers. The full steel sphere of the radius r = 300 mm and mass 840 kg rested freely inside the dish, its natural frequency of small vibrations being (Pirner & Fischer, 2000)

$$f_{0.th} = \frac{1}{2\pi}\sqrt{\frac{5g}{7(R-r)}} = 1.270 \text{ Hz} \tag{9}$$

The effectiveness of an absorber depends on its mass, natural frequency (tuning) and on the damping between the absorber mass and structure, as described elsewhere (e.g. Koloušek et al., 1984). Especially this last factor can be hardly controlled in the case of ball absorbers, in the described experiments only the rubber coating was applied. In agreement with theory, the device without rubber coating (with smaller damping) was more effective in resonance domain, but less effective outside it, while the absorber with rubber coating was effective in broader frequency band. The coating must be hard enough and resist to Theological effects, since the heavy ball will sink in and becomes immobile during longer periods without vibration. On the other hand, the coating prevents noike from rolling steel on steel, which can be sometimes undesirable.

4.2 Liquid damper

The motion of liquids in containers has been studied in the past few decades (see references in Pirner & Urushadze (2004b)). This article represents a continuation of Pirner & Urushadze, (2004b); it examines the influence of liquid viscosity, the direction of horizontal motion which is not parallel with the sides of the orthogonal tank; further it examines the influence of a perforated partition on damper effectiveness. The authors studied also the possibility of damping the movements of a structure the frequency of which was higher than the basic frequency of the translation motion of the liquid. The damper effectiveness was extended for the case of random excitation.

The effectiveness of a liquid damper installed on a real structure is expressed usually by the ratio of the displacements of the selected part of the structure in two states: 1 – damper excluded, 2 – damper active.

In our case the water damper was tested in the apparatus described in Pirner & Urushadze (2004b), the mass of its mobile (excited) part was 72 kg and the tank of which was designed on 1 : 1 scale to the damper intended for the actual structure.

The effectiveness of our damper is expressed by the relation

$$\varepsilon = \frac{RMS\ F_{\mathrm{H_2O}}}{RMS\ F_0} \tag{10}$$

where:

$F_{\mathrm{H_2O}}$ is the harmonic excitation force needed for the excitation of the required amplitude of the horizontal motion of the appliance with water

F_0 is the harmonic excitation force needed for the excitation of the required amplitude of horizontal motion of the appliance without water

The authors studied the effectiveness of direction of tank motion, the effectiveness of dynamic viscosity of the liquid, the effectiveness of obstacles in the tank.

5. Conclusions

The authors assessed also the effectiveness of all types of mechanical dampers. They have found that a liquid damper is more advantageous than a pendulum and a spherical (ball) damper because it can be tuned easily by the addition or subtraction of the liquid; in case of adequate support and tuning of rotation motion of the tank as stated in Pirner & Urushadze, (2004b) the damper damps both horizontal and rotation motion. Further details can be found in Pirner & Urushadze, (2004a) and Pirner & Urushadze, (2004c).

6. Acknowledgements

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