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A NEW METHOD OF THE ONE CLASS ROTOR CRITICAL SPEED DETERMINATION

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Summary: *The paper deals with a new approach to critical speed determination of rotors by means of the simple method against the so far used iterative method and a method utilizing a Campbell's diagram. A presented method enables to analyse the rotor system mathematical model described by constant and linear dependent matrices on the angular speed of revolution like e.g. the gyroscopic matrix. The resulting critical angular velocities are obtained as a single solution of the eigenvalue problem.*

1. Introduction

There are two so far used basic approaches to solution of the rotor critical speed of revolution [1]. The first is the iterative method taking imaginary part of some eigenvalue from foregoing iteration as angular speed of the rotor for the next iteration. It means that each critical speed is obtained as a result of iterative process. The total desired computing time is a sum of individual time streches for each calculated critical speed. The second way is based on a grafical crossing search of eigenvalue imaginary part dependence on angular speed with the axis of quadrant. This dependence is known as a Campbell's diagram. Both mentioned methods are very time consuming because of multiple solution of the eigenvalue problem. This paper offers a very simple method for solution of presented problem which is less time consuming because the eigenvalue problem is solved only ones.

2. A mathematical model

Let suppose that the free vibrating rotor behaviour is represented by the equation of motion

$$\mathbf{M}\ddot{\mathbf{q}}(t)+\left[\mathbf{G}_0(\omega)+\mathbf{B}\right]\dot{\mathbf{q}}(t)+\mathbf{K}\mathbf{q}(t)=\mathbf{0}, \tag{1}$$

where $\mathbf{M},\mathbf{G},\mathbf{B},\mathbf{K}\in\mathbf{R}^{n,n}$ are mass, gyroscopic, damping and stiffness matrices, respectively and $\mathbf{q}(t)\in\mathbf{R}^{n}$ is a vector of generalized displacements. Adding a trivial identity

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$$\mathbf{M}\ddot{\mathbf{q}}(t)-\mathbf{M}\ddot{\mathbf{q}}(t)=\mathbf{0} \tag{2}$$

to (1) we can rewrite both equations into compact form

$$\mathbf{N}(\omega)\dot{\mathbf{u}}(t)-\mathbf{P}\mathbf{u}(t)=\mathbf{0}, \tag{3}$$

where

$$\mathbf{N}(\omega)=\begin{bmatrix}\mathbf{G}(\omega)+\mathbf{B}, & \mathbf{M}\\ \mathbf{M}, & \mathbf{0}\end{bmatrix},\ \ \mathbf{P}=\begin{bmatrix}-\mathbf{K} & \mathbf{0}\\ \mathbf{0} & \mathbf{M}\end{bmatrix},\ \ \mathbf{u}(t)=\begin{bmatrix}\mathbf{q}(t)\\ \dot{\mathbf{q}}(t)\end{bmatrix}. \tag{4}$$

This approach is well known. Solution of the equation (3) can be found in the form $\mathbf{u}(t)=\mathbf{u}e^{\lambda t}$. Substituting the supposed solution to (3) the original problem can be transfered into the eigenvalue problem

$$[\mathbf{P}-\lambda\mathbf{N}(\omega)]\mathbf{u}=\mathbf{0}. \tag{5}$$

which can be rewritten into form

$$[\mathbf{A}(\omega)-\lambda\mathbf{I}_{2n}]\mathbf{u}=\mathbf{0}, \tag{6}$$

where $\mathbf{I}_{2n}\in\mathbf{R}^{2n,2n}$ is the identity matrix of the corresponding order and

$$\mathbf{A}(\omega)=\mathbf{N}^{-1}(\omega)\mathbf{P}=\begin{bmatrix}\mathbf{0} & \mathbf{I}_n\\ -\mathbf{M}^{-1}\mathbf{K}, & -\mathbf{M}^{-1}[\mathbf{G}_0(\omega)+\mathbf{B}]\end{bmatrix}. \tag{7}$$

The last matrix can be decomposed to two parts

$$\mathbf{A}(\omega)=\mathbf{A}_1+\tilde{\mathbf{A}}_2(\omega). \tag{8}$$

The first matrix in (8) is independent of frequency and the second one depends linearly on frequency. These two matrices take a form

$$\mathbf{A}_1=\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{M}^{-1}\mathbf{K}, & -\mathbf{M}^{-1}\mathbf{B}\end{bmatrix},\ \ \tilde{\mathbf{A}}_2(\omega)=\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0}, & -\mathbf{M}^{-1}\mathbf{G}_0(\omega)\end{bmatrix}. \tag{9}$$

Because the gyroscopic effect matrix can be written in form $\mathbf{G}_0=\omega\mathbf{G}$, the whole matrix $\tilde{\mathbf{A}}_2(\omega)$ can be also expressed in form

$$\tilde{\mathbf{A}}_2(\omega)=\omega\mathbf{A}_2=\omega\begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0}, & -\mathbf{M}^{-1}\mathbf{G}\end{bmatrix}, \tag{10}$$

where

$$\mathbf{A}_2 = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0}, & -\mathbf{M}^{-1}\mathbf{G} \end{bmatrix}. \tag{11}$$

If we take into account that the imaginary part of the eigenvalue corresponds to the critical angular speed of revolution $\lambda = \alpha + i\omega$ the equation (6) gives

$$[\mathbf{A}_1 - \alpha\,\mathbf{I} - \omega(i\mathbf{I} - \mathbf{A}_2)]\mathbf{v} = \mathbf{0}. \tag{12}$$

What is it critical angular speed of revolution? Critical speed is such angular velocity which is equal to some eigenfrequency of the rotor when even very small unbalance causes rise of a resonance stage. It means that the imaginary part of some eigenvalue corresponds to the speed of revolution. The method using a Campbell diagram looks for crossing of imaginary part of eigenvalue dependence on speed of revolution with axis of a quadrant $\mathrm{Im}\{\lambda_i\} = \omega$. It is well known, that in case of weakly damped system influence of damping to the eigenfrequencies is very small. From this reason the member $\alpha\,\mathbf{I}$ in (12) and $\mathbf{B}$ in (9) and (7) can be neglected. As can be seen the unknown critical speeds can be obtained by the single solution of eigenvalue problem

$$[\mathbf{A}_1 - \omega(i\mathbf{I} - \mathbf{A}_2)]\mathbf{v} = \mathbf{0}. \tag{13}$$

Then the real parts of the eigenvalues correspond to critical angular speeds of the rotor.

3. Numerical application

Let us take into account a compressor rotor depicted in fig. 1

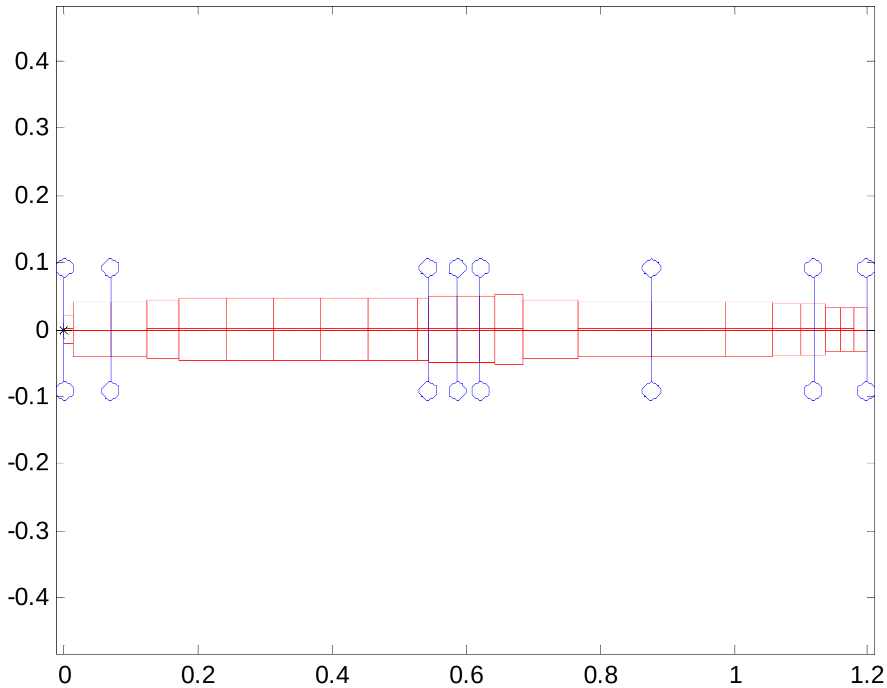

Fig. 1 A scheme of compressor rotor

This rotor was divided into 24 finite elements and the corresponding mathematical model has 150 degree of freedom. The first three lowest critical speeds and the total consumed time are written down in tab. 1.

Tab. 1 Comparison of the old and new method

i	$\Omega_{i,krit}\ [rad/s]$ old approach	$\Omega_{i,krit}\ [rad/s]$ new approach	Deviation [%]
1	144.78935	144.90868	0.08
2	203.88108	205.03607	0.56
3	531.57423	531.57473	9.4e-5
Time [s]	1.72	0.48	

As we can see the differences between these two approaches are small but will increase with growing number of critical speeds and number of degree of freedom. As a check of calculation the Campbell's diagram was performed and depicted in fig. 2. We can see that the crossing of quadrant axis with eigenvalue imaginary part dependence excellently correspond to the critical speeds obtained by means of the new approach.

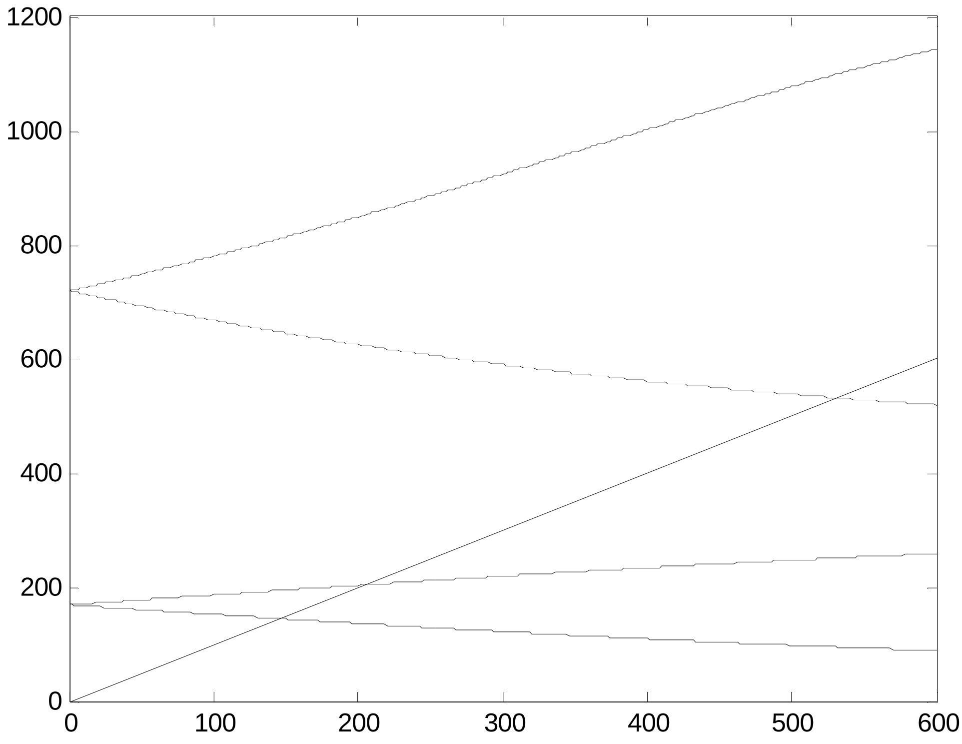

Fig. 2 The imaginary part of the eigenvalues

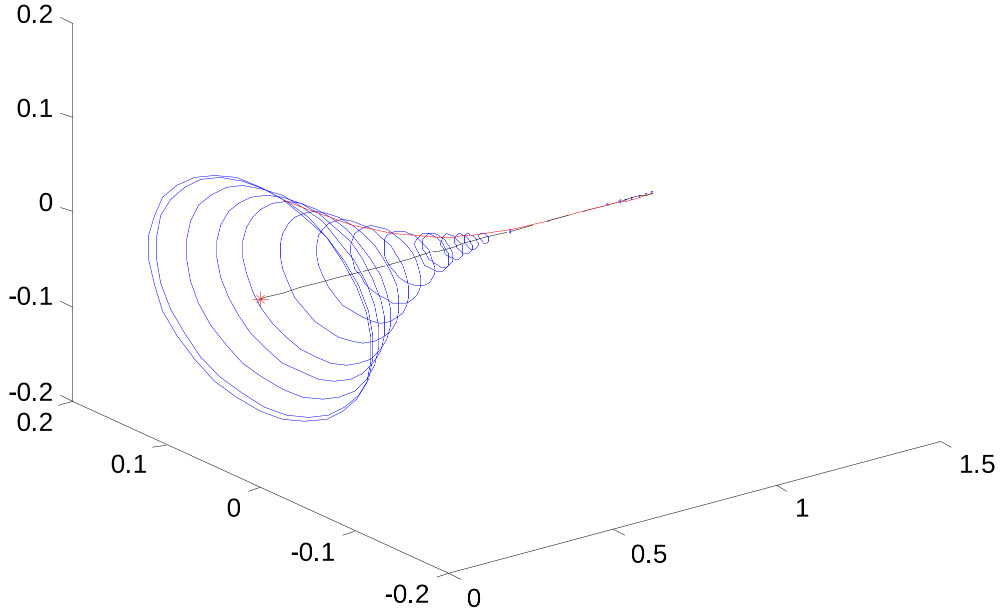

Fig. 3 The first critical mode shape

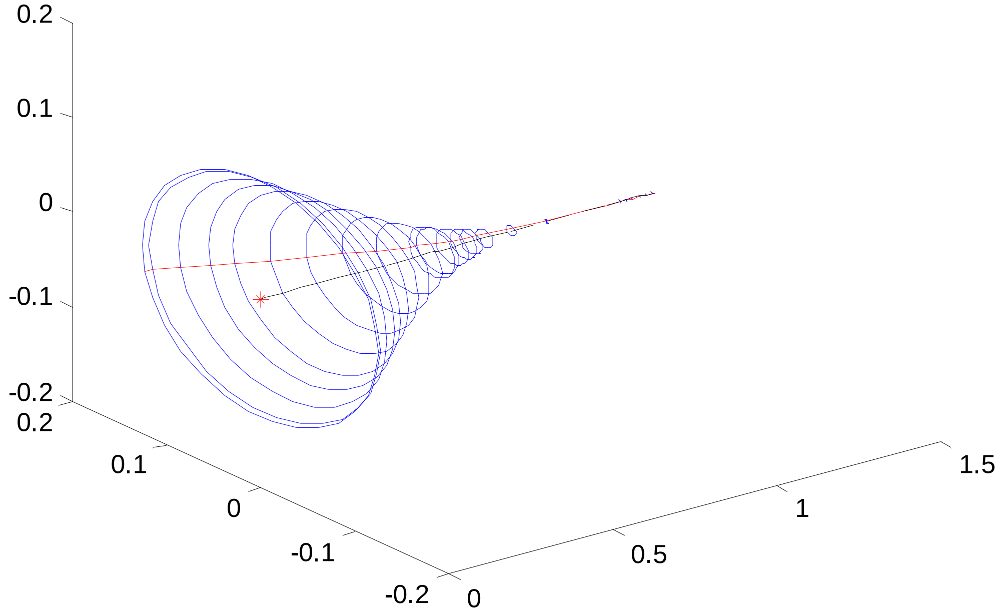

Fig. 4 The second critical mode shape

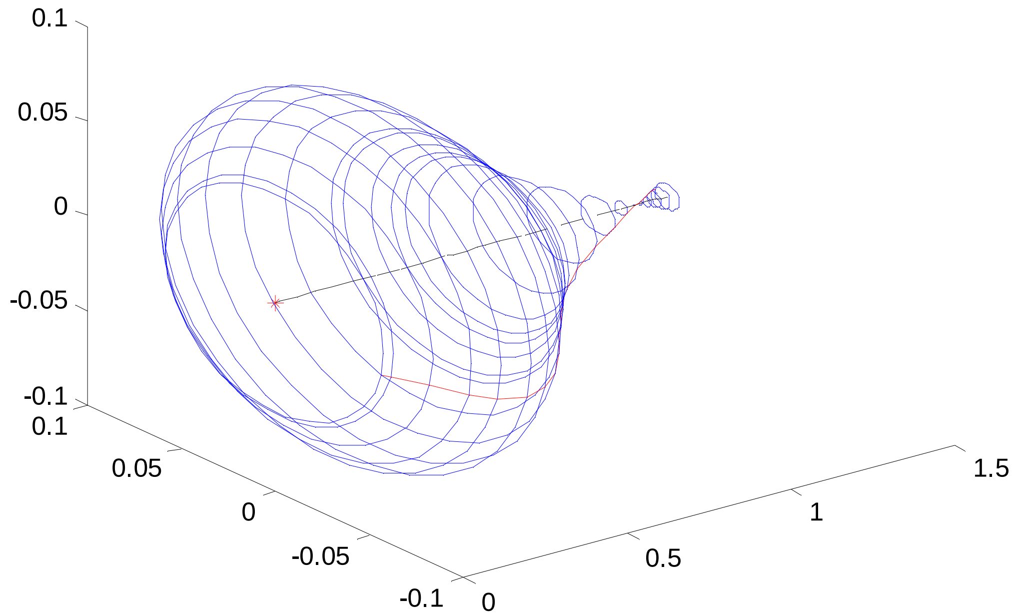

Fig. 5 The third critical mode shape

The first three mode shapes corresponding to the three lowest obtained critical speeds are depicted in fig. 4, 5 and 6. The difference between the first mode shape and the second one is in precission. The second mode shape corresponds to the parallel precission while the first mode shape corresponds to the counter-rotating precission.

4. Conclusion

As you can see the new method is much more faster then so far used methods. The time difference is the larger the number of DOF and number of desired critical speeds are higher. Nowadays more general modification of this approach is preparing.

5. Acknowledgement

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6. References

[1] Dupal, J. (2004) *Computational methods of mechanics.* Text book, University of West Bohemia in Pilsen, the 3-rd edition, Pilsen. (In Czech)