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EMPIRICAL MODEL RESPECTING A LOCAL EXTREME OF A COURSE OF THE VISCOSITY FUNCTION

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Summary: *Almost all empirical models dealing with the flow curves of non-Newtonian fluids can be applied only in the case of monotonous course of these curves. Exceptionally it is possible to model viscosity behaviour of rheologically more complex materials if there appears, in the course of a flow curve, a global extreme. The 8-parameter model proposed enables a description of viscosity behaviour in dependence on shear rate or shear stress also for the case when an extreme (maximum or minimum) is only local.*

1. Introduction

Phenomenological modelling of shear viscosity against shear stress or shear viscosity against shear rate has hitherto concentrated almost entirely on the category of shear-thinning and shear-thickening materials. Nevertheless as it is apparent from the recent period, new materials used in practice and/or research do not exhibit monotonous behaviour in the above-mentioned relations. As far as the present authors are aware literature to this point is very scarce.

First substantial restriction - in proposing new constitutive relations respecting non-monotonous course of viscosity in dependence on shear stress or shear rate – is the number of model parameters. In the case of a two-digit number the resulting relation cannot claim any practical impact to the rheological use as the individual parameters usually strongly interfere with each other and their physical substantiation is also fuzzy (cf. Guillou & Makhloufi (2002)). Second restriction consists in sufficiently simple mathematical expressions used in the proposed constitutive model.

Respecting these two restrictions there are – roughly speaking – two approaches how to cope with non-monotonicity of the relations that are looked for. For both approaches the number of parameters should not exceed the one for the description of monotonous course of viscosity, i.e. approximately eight parameters (see Roberts et al. (2001)). First approach comes out from the classical constitutive models depicting solely shear-thinning (thickening) materials but enriched with an additional multiplicative function ensuring non-monotonous behaviour (see Zatloukal et al. (2002)). The second approach considers completely new forms of the proposed models (see David & Filip (2004)).

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Six-parameter model presented in David & Filip (2004) obeys both restrictions stated above, however it exhibits two basic imperfections. First, functional behaviour of a viscosity function along both sides separated by its global maximum cannot be modelled with a sufficient flexibility. Second, modelling of the so-called intermediate Newtonian plateau (or its mild suspicion) is possible in a close vicinity of the global viscosity maximum not in a larger distance from it.

The present contribution aims at presentation of a new eight-parameter empirical model including non-monotonous behaviour of viscosity exhibited by bituminous and polymer materials and eliminating the shortcomings of the model presented in David & Filip (2004).

2. Eight-parameter empirical model

The following 8-parameter model relating viscosity with shear rate is proposed

$$\eta = \frac{\eta_0 \exp\left(-\log(c_0\dot{\gamma})^{p_0}\right)}{b_0 + \exp\left(\log(c_0\dot{\gamma})^{p_0}\right) + \exp\left(-\log(c_0\dot{\gamma})^{p_0}\right)} + \frac{\eta_\infty \exp\left(\log(c_\infty\dot{\gamma})^{p_\infty}\right)}{b_\infty + \exp\left(\log(c_\infty\dot{\gamma})^{p_\infty}\right) + \exp\left(-\log(c_\infty\dot{\gamma})^{p_\infty}\right)} \quad (1)$$

Analogously the relation viscosity vs. shear stress (for discussion which term - shear stress or shear rate – is more ‘fundamental’ see e.g. Lomellini & Ferri (2000)) is proposed in the form

$$\eta = \frac{\eta_0 \exp\left(-\log(c_0\tau)^{p_0}\right)}{b_0 + \exp\left(\log(c_0\tau)^{p_0}\right) + \exp\left(-\log(c_0\tau)^{p_0}\right)} + \frac{\eta_\infty \exp\left(\log(c_\infty\tau)^{p_\infty}\right)}{b_\infty + \exp\left(\log(c_\infty\tau)^{p_\infty}\right) + \exp\left(-\log(c_\infty\tau)^{p_\infty}\right)} \quad (2)$$

The non-negative parameters η_0 and η_∞ determine the first and second Newtonian plateau, respectively. The parameters c_i and p_i (i=0, ∞) are supposed to be positive, choice of the parameters b_i (i=0, ∞) is restricted by the condition b_i>-2 to ensure positiveness of the denominators in the relation (1). If both b_i are negative then rel.(1) (and (rel.(2)) enables to describe non-monotonous course of shear viscosity (involving extreme points). Other choice of b_i (one non-negative and the other one in the interval (-2,0] or both non-negative) may lead either to the monotonous or non-monotonous course of a viscosity function.

Figs.1-3 illustrate the application of the empirical model (1,2), especially the examples where the 6-parameter model published in David & Filip (2004) partially fails.

3. Conclusion

The number of parameters - eight - in the model presented exceeds the corresponding number in most classical empirical models, nevertheless its flexibility balances this shortcoming - possibility to describe non-monotonous behaviour of viscosity, respecting a possible existence of a local extreme.

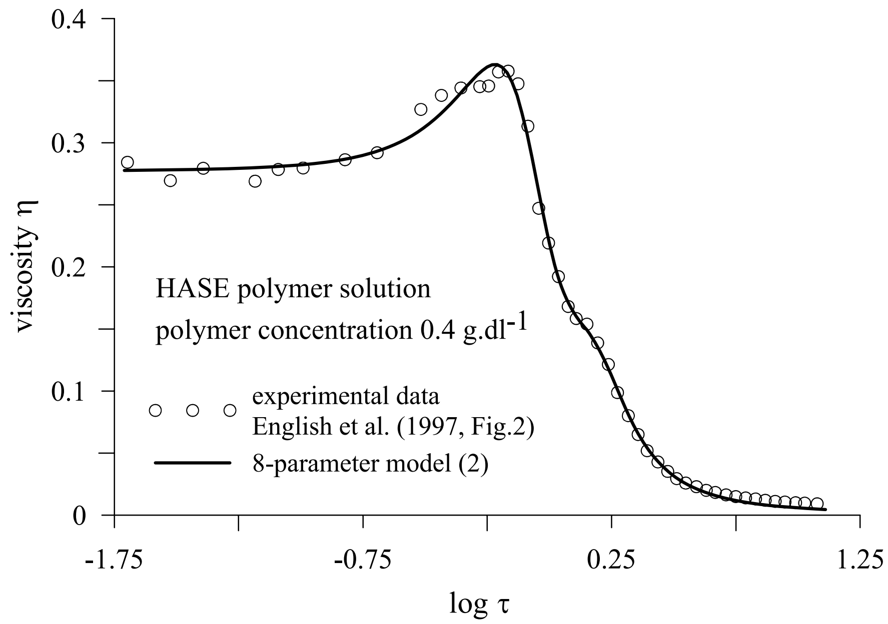

Fig.1 Modelling of a flow curve for HASE polymer solution with polymer concentration 0.4 $g.dl^{-1}$ (English et al. (1997)), parameters in rel.(2): η_0=0.276, b_0= -0.86, c_0=1.18, p_0=5, η_∞=0.00008, b_∞= -1.9993, c_∞=0.67, p_∞=0.13.

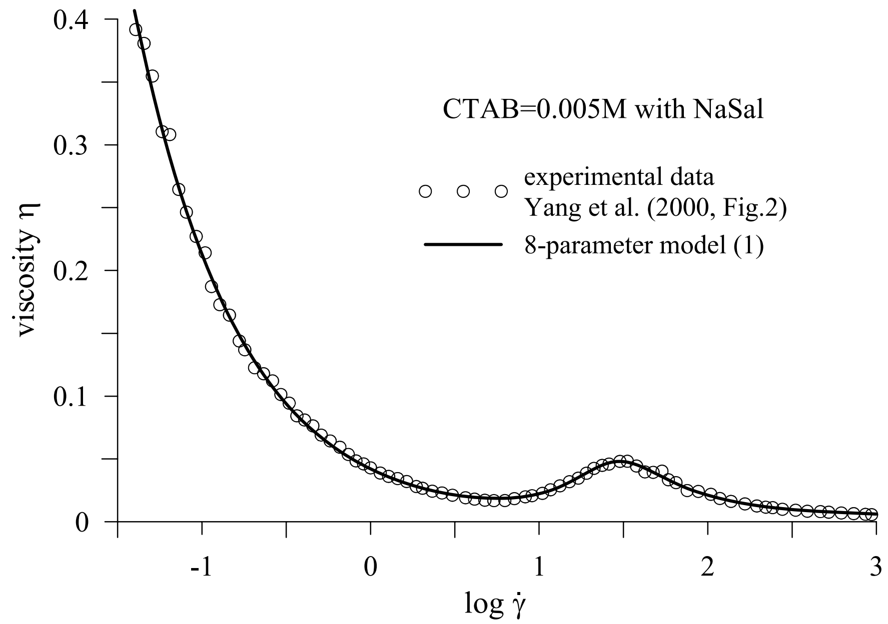

Fig.2 Modelling of a flow curve for cationic cetyltrimethylammonium bromide CTAB (concentration 0.005M) with the structure-enhancing additive sodium salicylate NaSal (Yang et al. (2000)), parameters in rel.(1): η_0=9, b_0= -0.1, c_0=1782, p_0=0.83, η_∞=0.0022, b_∞= -1.95, c_∞=0.036, p_∞=0.58.

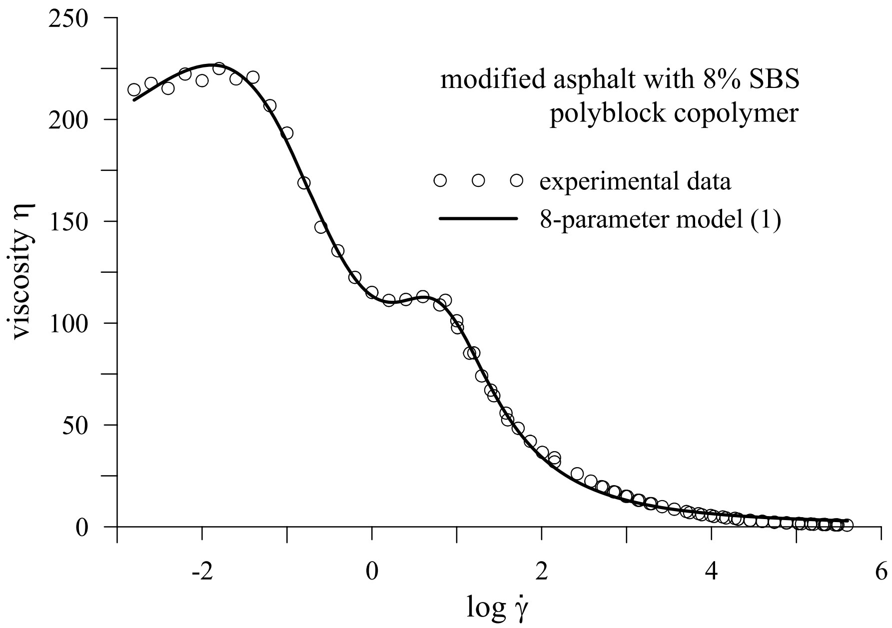

Fig.3 Modelling of a flow curve for asphalt modified with 8% styrene-butadiene-styrene polyblock copolymer, parameters in rel.(1): η_0=152, b_0= -1.11, c_0=9.3, p_0=0.596, η_∞=0.039, b_∞= -1.99954, c_∞=0.16, p_∞=0.0253.

4. Acknowledgement

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