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PARAMETRIC DESCRIPTION OF THE DEVELOPMENT OF SECANT MODULUS OF ELASTICITY OF CONCRETE UNDER CYCLIC LOADING

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Summary: *Fatigue is a process of permanent progressive changes in the structure of a material exposed to cyclic loading. This paper describes the strain development in concrete structures under cyclic loading and presents a simplified method for assessing the remaining secant modulus of elasticity under cyclic loading for both single and multiple load levels.*

1. Introduction

Fatigue is a process of permanent progressive changes in the structure of a material exposed to cyclic loading. Research related to fatigue of metals started in 1840's with the construction of railways. Fatigue of concrete and concrete structures was first described at the beginning of the 20th century and became a significant topic in 1920's with the development of highways. Nowadays, the use of high strength materials results into design of more slender structures with high live load proportion of the total load. High stress ranges in structures like bridges or crane-ways can result into accelerated crack propagation, higher deflections, structural stiffness reduction and consequently into fatigue failure.

2. Concrete failure mechanism and strain development under cyclic loading

Concrete is a heterogeneous three-phase material. This material is full of flaws and initial stress concentrations. Fatigue process in such material is much more complex than in homogeneous ferrous materials. Strain and secant modulus of elasticity development in concrete will be described further in the text.

Cyclic load acts on a material with a slightly changed structure in every new load cycle. The cracks do not close during the unloading phases. Stress concentrations at crack tips cause damage in every load cycle. The cyclic load causes further development of existing cracks. Cracks propagate, unite and finally they develop in the whole specimen section. Finally they cause the element to fail, though the stress it was subjected to was lower than the static strength of the material.

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The crack patterns under static and fatigue loading are similar, however the crack density under cyclical loading seems to be higher.

The failure sequence under cyclic loading is the same as the failure sequence under static loading.

3. Strain development under cyclic loading - The Cyclic Creep Curve

The cyclic creep curve (Figure 1) plots the change of strain or deformation with the number of load cycles the specimen is being exposed to.

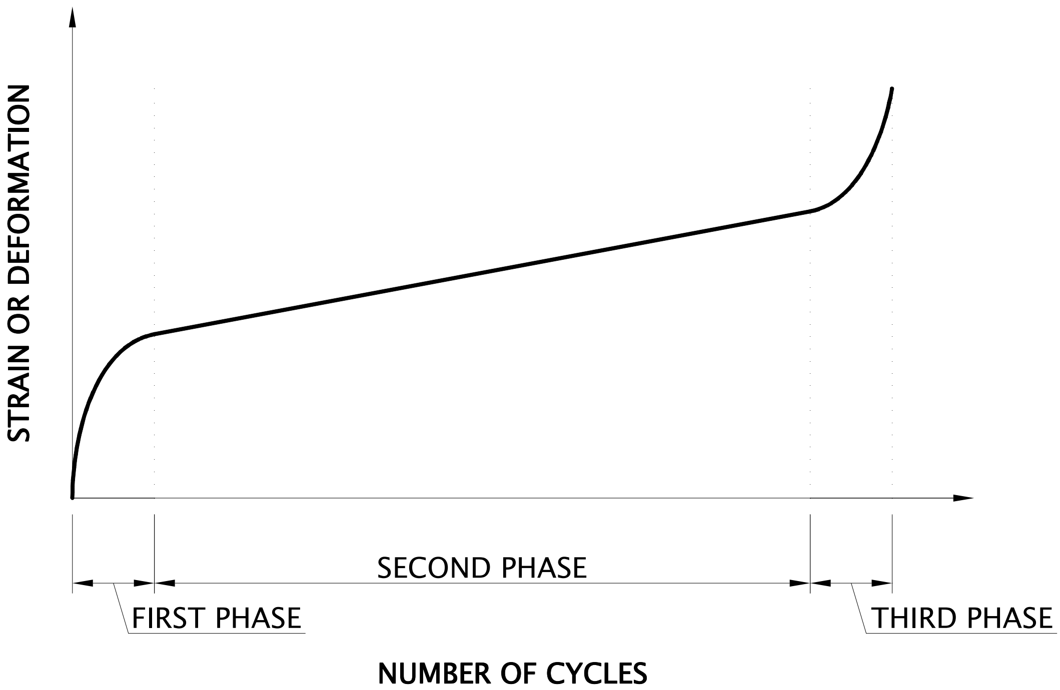

Figure 1 Cyclic creep curve

The cyclic creep curve consists of three phases which are similar to the fatigue phases of ferrous materials.

♦ Phase 1

The first phase is the initiation phase. Within this one, microcracks develop in the weaker parts of the cement paste and the strain increases rapidly. Yet the rate of strain increase is decreasing progressively with the number of load cycles. The first phase is completed after approximately 5-10% of the total load cycles.

♦ Phase 2

In the second phase, the cracks propagate stably and the strain increases approximately linearly with the number of load cycles. The microcracks which developed in the initiation phase propagate slowly further until they reach their critical length. Some authors name this phase as microcracking – the growth of microcracks. The second phase usually takes about 80% of the total load cycles.

♦ Phase 3

The third phase represents the instable crack growth which leads to the fatigue failure of the specimen. It starts when there are already enough instable cracks present, i.e. cracks which have reached their critical length. These microcracks than unite into one macro crack which can cause the fatigue failure. In the third stage, the bearing capacity of the structural element is already severely weakened and the strain increases under progressively increasing rate until failure. The third stage takes the remaining 10-15% of the total load cycles.

4. Development of secant modulus of elasticity of concrete under cyclic loading

The development of secant modulus of elasticity under cyclic loading reflects the development of strain or deformation, et versa. The development of secant modulus of elasticity under cyclic loading was described by J.O. Holmen in 1979, see Figure 2.

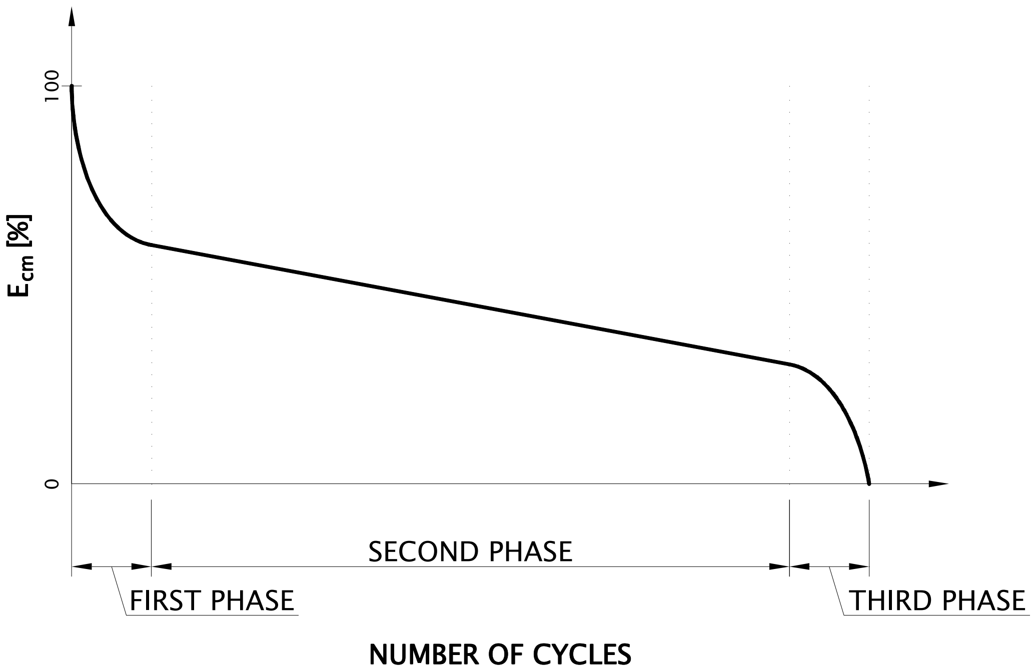

Figure 2 Development of secant modulus of elasticity of concrete under cyclic loading

In his research, Holmen used a loading frequency of 5Hz and minimum stress equal to $0.5*f_{cm}$. The maximum stress varied from $0.95*f_{cm}$ to $0.67*f_{cm}$. Higher values of the maximum stress gave smaller proportion of the first phase from the total number of load cycles (less than 10%) and a steeper slope in the second phase. Lower values of maximum stress had a bigger proportion of the first phase from the total number of load cycles (approximately 10-15%) and a flatter slope in the second phase of secant modulus of elasticity development. The first phase of development of secant modulus of elasticity finished at 75-95% of E_{cm}, the second phase at 65-75% of E_{cm} depending on the maximum stress applied.

5. Parametric description of the development of secant modulus of elasticity of concrete under cyclic loading

As stated in previous sections and plotted on Figure 1, the strain development under cyclic loading consists of three phases. Similarly, the development of the secant modulus of elasticity of concrete under cyclic loading consists of three phases, see Figure 2.

Based on the experiments of J.O. Holmen, parametric description of the development of secant modulus of elasticity of concrete under cyclic loading is proposed.

For the purpose of the parameterization, the durations of both the first and the second phase of the strain development are assumed to be 10% of the total number of load cycles the structural element is able to resist.

5.1. Parametric description of the development of the secant modulus of elasticity of concrete under cyclic loading of a single load level

Based on the experiment of J.O. Holmen, the contribution of the first and the third phase of the development of the secant modulus of elasticity of concrete under cyclic loading to the total number of load cycles the structural element is able to resist seems to be constant for the applied maximum compressive stress level and can therefore be substituted by constants depending on the maximum compressive stress level.

Constants *a* and *b* are introduced, *a* for the decrease of the secant modulus of elasticity in the first phase of its development under cyclic loading, *b* for the remaining proportion of the original secant modulus of elasticity at the beginning of the third phase of its development. The graphical meaning of the constants is explained on Figure 3.

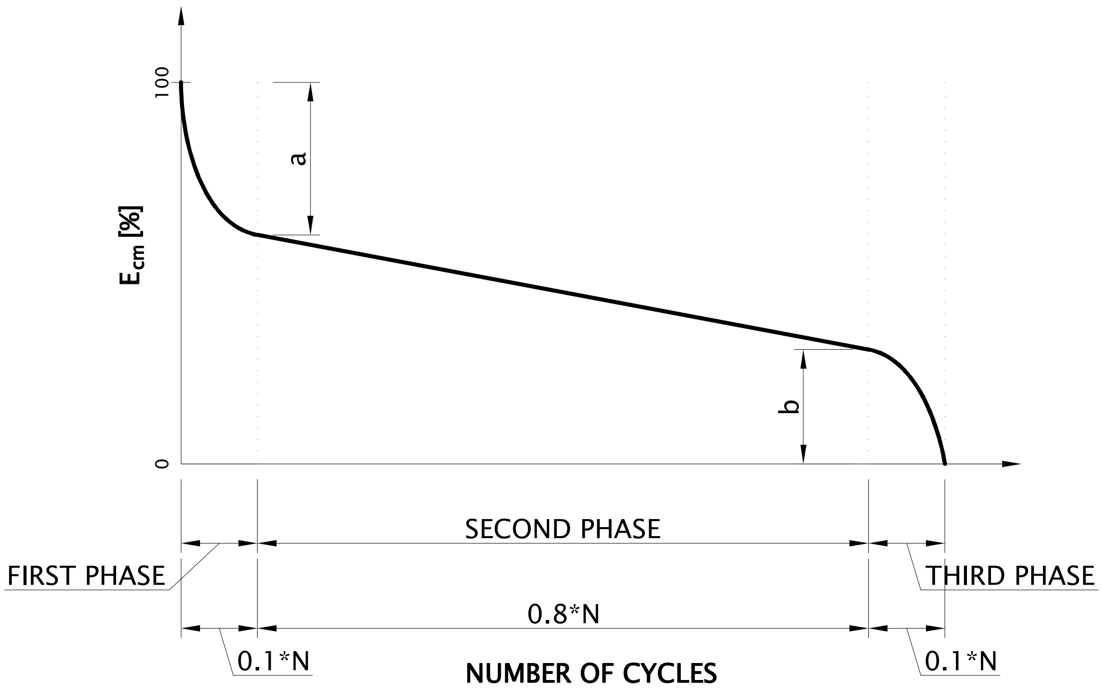

Figure 3 Development of secant modulus of elasticity of concrete under cyclic loading

The decrease of the secant modulus of elasticity in the first phase of its development under cyclic loading, the constant *a*, is bigger at lower stress levels. On the other hand, remaining proportion of the original secant modulus of elasticity at the beginning of the third phase of its development, the constant *b*, is smaller at lower stress levels. This means, expressed in a different way, that a structural element under lower stress level loses higher proportion of the secant modulus of elasticity during the first phase of its development than under higher stress level, while it is still operational at the beginning the third phase of the strain development with a smaller proportion of the original secant modulus of elasticity which would cause fatigue failure at higher stress levels.

The formulas for the constants *a* and *b* were obtained with the use of linear regression (see Figure 4):

$$a = 0.47 - 0.4 * S_{max} \qquad (1)$$
$$b = 0.57 + 0.17 * S_{max} \qquad (2)$$

where $$S_{max} = \sigma_{c,max} / f_{cd,fat}. \qquad (3)$$

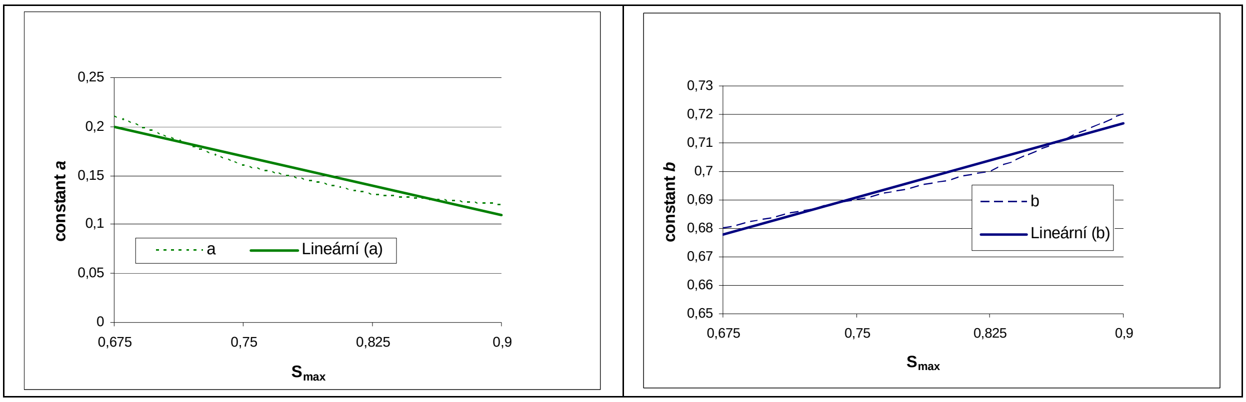

Figure 4 Estimation of the constants *a* and *b* using linear regression

The value of the secant modulus of elasticity during the second phase of the strain development under cyclic loading can be expressed by the following equation (4):

$$E_{s,cyc} = \left[(1-a) - \frac{(1-a-b)}{0.8 \cdot N} \cdot (n - 0.1 \cdot N) \right] \cdot E_{s,0} \le b \cdot E_{s,0} \qquad (4)$$

where $E_{s,cyc}$ is the value of the secant modulus of elasticity during the second phase of the strain development,

$E_{s,0}$ is the original value of the secant modulus of elasticity at the beginning of the cyclic loading,

N is the total number of load cycles the structural element is able to resist. This value can be calculated by the procedures given in Eurocode 2 or more conservatively by the procedures given in Model Code 1990,

n is the number of load cycles the structural element has already resisted,

a, b are the constants defined previously in the text.

5.2. Parametric description of the development of the secant modulus of elasticity of concrete under cyclic loading of variable load levels

The equation defined in the previous section can be easily adapted for cyclic loading of *m* variable load levels (5):

$$E_{s,cyc} = \left[(1 - a_{mean}) - \sum_{i=1}^{m} \frac{(1 - a_i - b_i)}{0.8 \cdot N_i} \cdot (n_i - 0.1 \cdot N_i) \right] \cdot E_{s,0} \le b_{mean} \cdot E_{s,0} \qquad (5)$$

where $E_{s,cyc}$ is the value of the secant modulus of elasticity during the second phase of the strain development,

$E_{s,0}$ is the original value of the secant modulus of elasticity at the beginning of the cyclic loading,

N_i is the total number of load cycles the structural element is able to resist at the particular load level. This value can be calculated by procedures given in Eurocode 2 or more conservatively by procedures given in Model Code 1990,

n_i is the number of the load cycles the structural element has already resisted at the particular load level,

a_i, b_i are the constants defined previously in the text for the particular load level,

a_{mean}, b_{mean} are mean values for the constants *a* and *b* for *m* multiple load levels,

$$a_{mean} = \frac{a_1 + a_2 + ... + a_m}{m}, b_{mean} = \frac{b_1 + b_2 + ... + b_m}{m} \quad .$$

6. Conclusions

This paper described the strain and secant modulus of elasticity development in concrete structures under cyclic loading and presented a simplified method for assessing the remaining secant modulus of elasticity of concrete under cyclic loading for both single and multiple load levels. This tool can help assessing the remaining useful life of concrete structures subjected to cyclic loading.

The influence of material deterioration on the fatigue performance of concrete in compression will be studied in the following research.

Acknowledgements

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