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ON THE TURBULENT SPOT AND CALMED REGION

P. Jonáš*

Summary: *The purpose of the contribution is to collect information on the generation, propagation and coalescence of turbulent spots and calmed regions in a transitional boundary layer. The appreciation of the behaviour of both turbulent spot and calmed region under turbulent free stream is the focus of this study.*

1. Introduction

Prediction of the transition from laminar to turbulent flow in boundary layers is of a great practical interest. The particular relevance of the problem is in changes of skin friction and momentum and scalar diffusion caused by the transition. So a steady effort is devoted to understand the process of boundary layers transition. It is well known, e.g. Morkovin (1969) that scenarios of the road from laminar state to turbulence are quite different in wall bounded shear flows and in free shear layers. Transition of bounded flows – layers in open systems (Saric et al., 2002; different upstream or initial amplitude conditions) is composed of very different sequences of instability events. We will restrict ourselves to 2D-boundary layers under different external flow conditions.

The mechanism of how the boundary layer is forced by disturbances penetrating into the layer from environment is named receptivity of boundary layer nowadays. It accounts for turbulence-vorticity sound disturbances and their interactions with leading edge shape-curvature, the effects of discontinuities in surface curvature, surface in-homogeneities-roughness, surface-vibrations, wall temperature etc. Amplitude, frequency and phase of the initial perturbations entering boundary layer determine the initial conditions and the route from laminar state to the turbulent one.

In the extreme case of very small amplitude (≤ 0.1% U_e) the initial disturbances generate instability waves (TS-waves) inside the layer that die away until they reach the location x_{ind}, where the local Reynolds number Re attains at least the value of indifference Reynolds number Re_{ind} (e.g. $Re_{ind} = 8.83\ 10^4$ for Blasius boundary layer). The TS waves frequencies and longitudinal wave number assume the values (Walker, 1989 and Kozlov et al., 1990)

$$\frac{2\pi f \upsilon}{U_e^2} = 3.2\, Re_1^{1.5};\quad k_x \simeq \frac{1}{2\delta_1};\quad Re_1 = \frac{\delta_1 U_e}{\upsilon};\quad Re = \frac{x U_e}{\upsilon} \qquad (1)$$

here U_e, f, k_x, x, δ_1 and ν are external flow velocity, frequency, longitudinal wave number, stream-wise coordinate, displacement thickness and kinematics viscosity.

* RNDr. Pavel Jonáš, DrSc., Institute of Thermomechanics v.v.i.; Dolejškova 5, 182 00 Praha 8; tel.: +420 266 052 025; fax: +420 286 584 695;e-mail: jonas@it.cas.cz

At Re > Re_{ind} the disturbances start to grow exponentially with time because boundary layer lost stability. The indifference Reynolds number Re_{ind} is the minimum value at which a neutral perturbation still exists. Linear stability theory describes this period of transition process modulated by pressure gradient (if $dU_e/dx > 0$ then increase of Re_{ind}; $dU_e/dx < 0$ produce decrease of Re_{ind}), suction (make the layer thinner), wall heat transfer (wall cooling/heating produce increase/decrease of Re_{ind}). Initially 2D-instability waves possess wave lengths several times the boundary layer thickness and their wave speed is about 0.3 U_e. As the wave amplitude grows (0.5% to 1% of U_e) nonlinear and 3D interactions occur with a span-wise scale similar to the TS waves length.

Then the secondary instability started. The region of secondary instability is considerably shorter then the linear one. However many phenomena successively occur in the region. Three-dimensionality of waves produces a rapid amplification of disturbances. They occur: twist of vortex filaments resulting in secondary motions in x = const. planes, origin of the peak-valley structure, occurrence of staggered (small amplitudes) or aligned (large amplitudes) pattern of vortex loops - hairpin eddies - Λ vortices, high-shear layers associated with profiles of momentary velocity having an inflection point, intense spikes in the velocity signal especially in the peak region, sudden increase of waves amplitudes accompanied with high frequency velocity oscillations (of order higher then frequency of TS-waves) and finally the origin of first turbulent spots (at intermittency factor ~ 0.05). For details and more data see

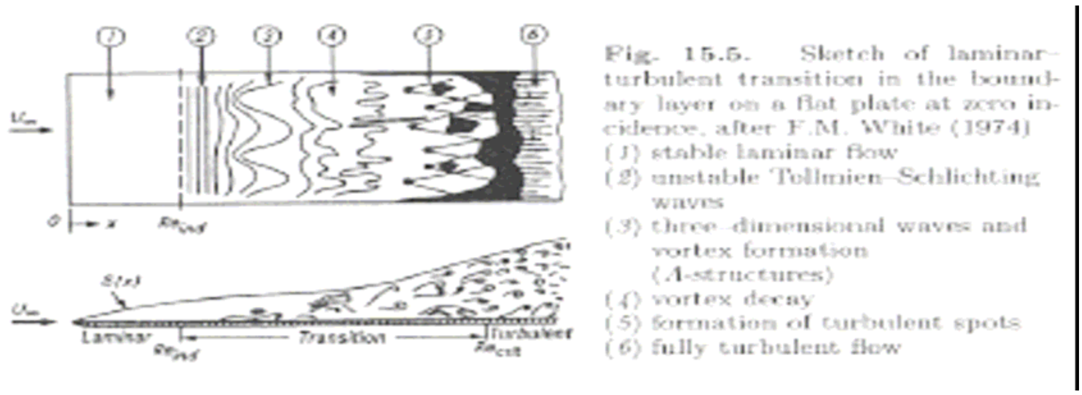

Figure 1 Sketch of the simplified scenario of boundary layer transition.
(copied from Schlichting & Gersten, 2000)

also e.g. Gad-el-Hak et al (1981), Herbert (1988), Jonáš, P. & Thang (1989), Kachanov (1994), Saric et al. (2002). Schlichting & Gersten (2000) elaborated a fair summary of the stability theory. The sketch of the described simplified scenarios of different paths to turbulent boundary layer is shown in Figure 1 (copied from Schlichting & Gersten, 2000).

Medium amplitude of the initial perturbations, ~ (0.1 ÷ 0.7)% of U_e, cause some differences in the above described path to turbulence. Disturbances containing the vorticity $\vec{\omega}\left(\omega_x, \omega_y, \omega_z\right)$ penetrate into the entire layer since leading edge from environment. They undergo transient growth, decay exponentially with time but simultaneously they can excite TS-waves or produce span-wise modulations of the already existing TS-waves. Each TS-wave packet participates to its own local transition process and thus accelerates transition to turbulent spots. At the same time, because the disturbances are moving stream-wise faster (~0.8 U_e) than TS-waves, they can themselves produce Λ− structures and high-shear layers and generate turbulent spots if their "energy" is high enough.

The opposite extreme of the first discussed case is the action of very strong free stream disturbances that are greater than say 0,7% of U_e. Then, because the “energy” of disturbances is high enough, the linear instability period can by-passed (e.g. Morkovin, 1969 and 1993) and non-linear stages are triggered directly. Sub critical instabilities and turbulent spots production occur and the transition process is soon afterwards completed. Greg et al. (1991) ascertained that generation of TS-waves is possible also at high level of outer flow disturbances. TS waves pass through the above-described evolution and thus support the by-pass mechanisms.

Space limitations permit outline only simplified scenarios of transition process. We ignore the assorting of different routes to break down by various attributes as are K-mode, N-mode etc., because various authorities use different assortments; compare e.g. Herbert (1988), Kachanov (1994), Morkovin (1991) and Saric et al. (2002). Author does not feel himself competent to explain in few words sophisticated arguments for various selections of attributes.

Now it is possible to make a conclusion: the final phase of laminar turbulent boundary layer transition starts with the occurrence of first turbulent spots regardless of initial conditions. Spots are an essential feature of transition to turbulence, they appear as the building blocks of boundary layer turbulence, Vinod & Govindarajan (2004). So it is meaningful analyse this phenomenon more deeply.

2. Turbulent spot and calmed region

Emmons (1951) first reported spots. He observed them in the course of water table experiments and proposed the model of transition based on formation and growth of turbulent spots later on. Spots are isolated regions of strong fluctuations, which have properties close to the turbulent flow. A visualisation of a turbulent spot is shown in Figure 2. Spots are stream-wise carried within a laminar shear flow and at the same time growing in size and coalescing with neighbours. The creation rate, growth characteristics and the merger of turbulent spots lead to fully developed turbulent flow, they control the length of the transition region. Turbulent spots appear irregularly in time and at arbitrary locations of the boundary layer.

Figure 2. Spot flow visualisation showing the sublayer streaks (Cantwell et al., 1978; copied from Krishnan & Sandham, 2006).

Starting from Schubauer & Klebanoff (1955) a lot of investigations was done on turbulent spots observations under various circumstances. Only examples of the relevant references are shown in parenthesis further.

External flow: fluid was water (Gad-el-Hak et al., 1981) or largely air; air velocities from of the order $\mathcal{O}(1^0 \div 1^1)$ m/s (most frequently) up to Mach number ~ 2 (Clark et al., 1994); investigated boundary layer was on a flat plate and the suction side of turbine cascade blades or compressor stator blades; the simulated pressure distributions had pressure gradient from strong favourable gradient (Koyabu et al., 2005) up to the strong adverse one (Gostelow et al., 1997);

Type of initial disturbances: external flow turbulence (mostly grid turbulence) level from 0.03 percent (Schubauer & Klebanoff, 1955) up to 7 percent (Fransson et al., 2005); periodic passing of wakes (Gostelow & Thomas, 2005); various methods of wave pockets triggering (Schröder & Kompenhans, 2002);

Applied methods of investigations: hot-wire / wall hot-film anemometry widespread since beginning of transition studies; fluorescent dye, visible only when excited by a strong light source of the appropriate wavelength and light sheat (Gad-el-Hak et al., 1981); thermochromic liquid crystals (Kittichaikarn et al., 2001); Particle Image Velocimetry, e.g. multi-plane stereo PIV technique (Schröder & Kompenhans, 2002); solution of theoretical models (Brown & Smith, 2005); Direct Numerical Simulations, DNS (Mathew & Das, 2000 and Krishnan & Sandham, 2006).

Velocity traces show clearly the occurrence of a spot – a burst with distinctly high both frequency and amplitude and the connected smooth calmed phase; see e.g. records labelled by 1 and 2 in Figure 3. This character of the instantaneous velocity record allows the use of intermittency measurement and analysis methods for the detection of spots. Narasimha (1985) composed a model describing the transition process and turbulent spots occurrence through

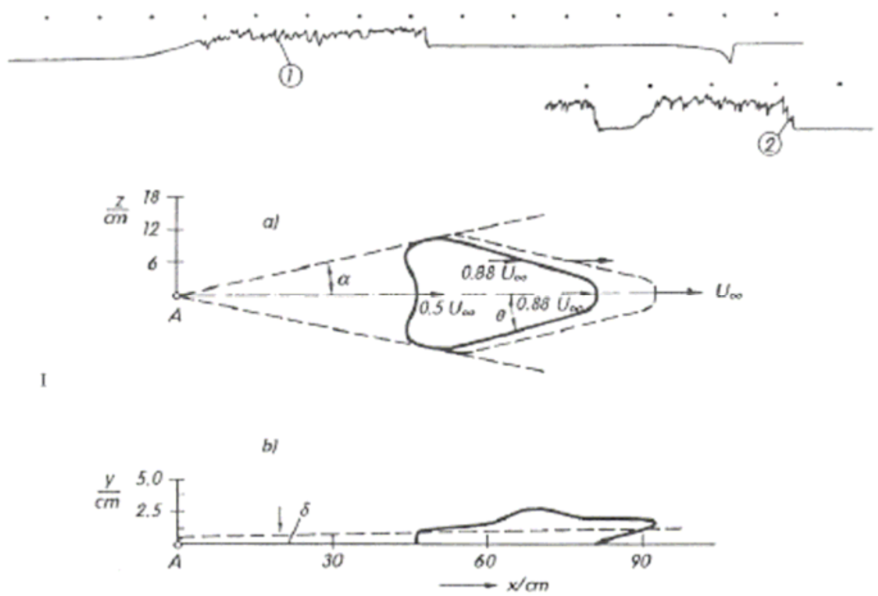

Figure 3. Examples of velocity traces, label 1 and 2 when a spot is passing and a) the floor projection and b) the side view of a triggered spot. (copied from Schlichting & Gersten, 2000)

the distribution of intermittency (fraction of time that the flow is turbulent at the given point). Model is based on Emmons (1951) notion that transition occurred through many *islands* of turbulence – spots surrounded by laminar flow. Schubauer & Klebanoff (1955) acquired the first quantitative data on the shape, growth and propagation of a spot, Figure 3 sub a) and b). The data confirmed qualitatively the Emmons concept however showed some quantitative discrepancies in the measured distributions of the intermittency factor and the distribution derived after Emmons assumptions. Narasimha (1985) removed the disagreement by means of the hypothesis of concentrated breakdown i.e. *spots form at a preferred streamwise location randomly in time and in cross-stream position*. After some clever considerations R. Narasimha derived intermittency distribution

$$\gamma(x) = 1 - exp\left[-(x - x_t)^2 n\sigma / U_e\right] \quad \text{at} \quad x \geq x_t \quad \text{and} \quad \gamma(x) = 0 \quad \text{at} \quad x \leq x_t \qquad (2)$$

where x is the streamwise coordinate, n is the number of spots occurring per unit time and space distance at x_t and σ is a nondimensional propagation parameter (based on the assumption that the spot propagates with constant velocity and linearly in space and in time, i.e. the envelope of spot positions on the surface is a wedge of constant angle, Figure 3, a). The details of Narasimha (1985) fruitfull model are out of the scope this contribution. This topic is possible to conclude, that the investigations performed under various circumstances by applying different methods agree in general with the discussed model.

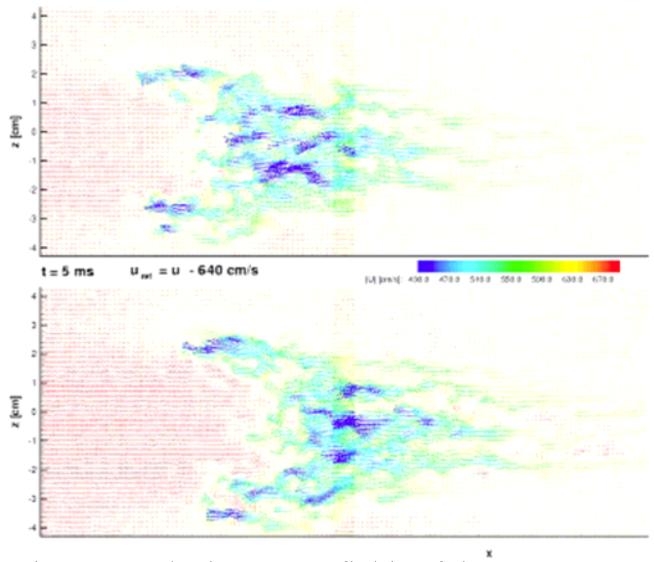

Figure 4 Velocity vector fields of the same spot with a time separation of t = 5ms at y = 4.5 mm and Ue = 7 m/s.
(copied from Schröder & Kompenhans, 2002)

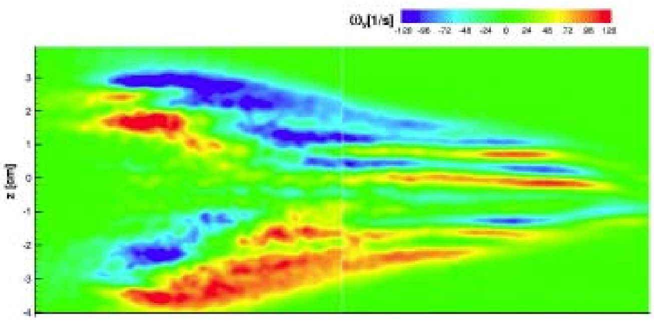

Figure 5 Ensemble average of y- component of vorticity. (copied from Schröder & Kompenhans)

Let us summarise the main features of turbulent spot. Spots drifted by flow (verage velocity 0.75 U_e) are growing in stream wise and spanwise directions. (The coordinate system was introduced in Figure 3: x and z are the streamwise and spanwise directions, y is the direction of normal to the wall.) Figure 4, borrowed from Schröder & Kompenhans (2002), illustrates this process. The spot has the shape similar to a curved arrowhead (Figures 2, 4 and 5) with the nose – leading edge, LE, oriented downstream. The height of a spot is about 3 times the boundary layer thickness, δ. The nose, LE, of the spot is above the laminar layer and like an overhang (Figure 3) including heads of vortex structures. In the Figure 5 is clearly shown that the structures are of the Λ - type, hairpin arising from the TE and near wall streaks. Spot remains self-similar as it grows for the given pressure gradient. The flat plate boundary layer case (grad P = 0) characterise following features. The lateral spreading half-angle α is defined in Figure 3. Its magnitude has been found α ≈ (7° ÷ 10°). Leading edge and the trailing edge, TE, are convected downstream with different velocities: U_{LE} ≈ (0.83 ÷ 0.90) U_e and U_{TE} ≈ (0.50 ÷ 0.62) U_e. The nondimensional spot propagation parameter σ includes both the streamwise and lateral spot growth

$$\sigma = \left(\frac{1}{U_{TE}} - \frac{1}{U_{LE}}\right) U_e \tan\alpha = F\left(\text{grad}P\right) \qquad (3)$$

Lower and upper limits of values of α, U_{LE} and U_{TE} reported on flat plate boundary layer in several references are inserted in the parenthesis.

It follows from several references that the discussed characteristics vary substantially under pressure gradient (parameter λ_θ, θ denotes the momentum thickness) and external flow turbulence (level Tu). Gostelow (1997) interpolated the reported result by formulas:

$$N = n\sigma\theta_t^3 / \nu = 0.86\cdot 10^{-3}\exp\left[2.134\lambda_{\theta t}\ln\left(Tu\right) - 59.23\lambda_{\theta t} - 0.564\ln\left(\text{Tu}\right)\right]$$

$$\sigma = 0.03 + \left[0.37 / \left(0.48 + 3.0\exp\left\langle 52.9\lambda_\theta\right\rangle\right)\right] \qquad (4)$$

$$\alpha = 4.0 + \left[22.14 / \left(0.79 + 2.72\exp\left\langle 47.63\lambda_\theta\right\rangle\right)\right]$$

The streamwise and spanwise growth rates of turbulent spots are inhibited by a favourable pressure gradient and vice versa. Dramatic changes in lateral spreading of a spot are caused with the increase of the flow Mach number. Krishnan & Sandham (2006) advised a marked reduction of the lateral spreading of the spot from α = 5°to α = 1.7°with the increase from Ma = 2 up to Ma = 6.

The velocity fluctuations inside a spot have turbulent nature with high frequency fluctuation and with increasing both the streamwise velocity component and the wall shear stress as the spot is passing a point (fixed probe). Turbulence dissipation occurs almost exclusively in this zone. A calmed region is attached at the rear of a turbulent spot in a laminar or transitional boundary layer. Existence of such region, with essentially laminar flow, was already mentioned by Schubauer & Klebanoff (1956). The streamwise velocity profile relaxes from the high shear layer turbulent-like form to that of the original laminar boundary layer. The process of relaxation proceeds through a sequence of fuller profiles than that of the natural layer. Such profiles are more stable than the natural one. The calmed region produces more beneficial effects: the boundary layer is thinned, laminar separation is delayed or the already separated can reattach and the length of transition region increases. Brown & Smith (2005) derived these and

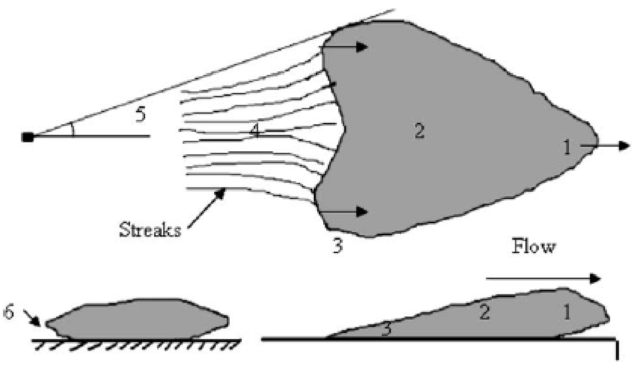

Figure 6 Turbulent spot and calmed region (4); front overhang (1), turbulence core (2), lateral wingtip (3),spreading half angle (5); spanwise overhang (6).
(copied from Krishnan & Sandham, 2006)

other knowledge by means of modelling the calmed region behind a spot. Results are generally in accord with experiments, e.g. Gostelow (1995 and 1997), Johnson (2001), LaGraff & Ashpis(1998 and 2002), Narasi,ha (1985 and 1998), Ramesh & Hodson (2002).

3. Conclusions

Boundary layer transition has many faces that's why turbulent spots with associated calmed region somewhat change with the initial / boundary conditions. The aim of the contribution was outline the problem, which is of great practical interest. On the workshop Minnowbrook II was estimated that 1% improvement in the efficiency of a low-pressure turbine would result in a saving of 52000 USD per year on a typical airliner. General meaning was that better flow over blades can contribute to improvement of efficiency i.e. namely deeper knowledge of boundary transition process, inclusive exploiting the calmed region trailing a spot, understanding the time-dependent forcing of transition in a turbine row because of wakes from upstream and the management of separation bubbles.

4. Acknowledgements

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