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SIMILARITY OF BEARING CAPACITY OF BORED PILES ACCORDING TO ČSN 73 1002

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Summary: *Now, the translation in Czech and National Annex of EC7-1 (EUROCODE 7 - Geotechnical design, Part 1: General rules) are made. The principal difference exists between inputs according to EC7-1 and inputs according to the Czech basic standard ČSN 73 0031 "Structural reliability – Basic requirements for design". In general, the input difference leads to too conservative (ineffective) designs accordin to EC7-1 and more effective (more optimistic) designs according to ČSN. A comparative numerical analysis of the both design procedures appears useful.*

The paper deals with a similarity solution of bearing capacity of bored piles according to ČSN 73 1002 "Pile foundations" (1987). The solution should provide a part of the comparing analysis base for a simpler analysing.

1 Introduction

The theoretical base of the EC7-1 Limit State Design is very dissimilar, more conservative, than the one of the Czech basic standard ČSN 73 0031 (Structural reliability - Basic requirements for design) which is proved by the long-term successful practice. Mostly, this fact is due to the different definitions of material inputs, i.e. characteristic/standard and design values. While the EC7-1 defines for characteristic values the probability of 95% the standard values according to ČSN 73 0031 are defined like the average (probability of 50%). The definitions of design values according to the both regulations are the same but, of course, due to different characteristic/standard value definitions the material partial factors differ.

Due to it the analyses of slopes, shallow foundations and earth pressure concept have been carried out and the analysis pile design appears necessary too. The analysis need a wide concept for numerical comparison. The suitable way for it is a similarity solution of the design procedure and this is the object of the Paper.

Czech standard ČSN 73 1002 "Pile foundations" (1987 in force) does not involve a numerical design procedure but directly the tables of the design bearing capacity of different pile types. The standard numerical design procedure for piles is presented in ČSN 73 1004 "Drilled piles" (1981) which is not yet in force. However, the last mentioned standard procedure is only the Czech one for numerical design and the Paper applies it. The Paper deals with a similarity solution of bearing capacity of shallow foundations according to ČSN 73 1004 with a number of parameters. An easier approach of the problem solution appears theory of

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similarity. The solution should provide a part of the comparing analysis base for a simpler analysing.

2 Bearing Capacity According to ČSN 73 1004

Czech standard ČSN 73 1004 “Drilled piers” is based on the Limit State Design (LSD) theory like EC7-1. The pile bearing capacity has two components: bearing capacity of toe and rear surface shear strength. A general homogeneous mass and vertical loading force are supposed.

The geotechnical design values (c_d, ϕ_d) are derived by usual LSD procedure from "standard" (characteristic) values, however that is basically different than the procedure of EC 7-1 deriving design values from characteristic values. These procedures act no role for similarity and they will not distinct in the next text.

2.1 Toe force

The base of solution is considered according to ČSN 73 1001 “Subsoil under shallow foundations”. The procedure need following 7 inputs:

- r - radius of the pile,
- h - depth (height) of the pile,
- δ - deviation of loading force from vertical direction,
- γ_1 - unit weight of soil above pile toe,
- γ_2 - unit weight of soil below pile toe,
- ϕ_d - angle of soil resistance - design value,
- c_d - cohesion - design value.

All values should be considered as design ones. The prior three quantities are geometrical, the last four ones are material properties.

The solution is based on three following dimensionless functions of the angle of soil resistance those characterise influences of foundation width, cohesion and angle of soil resistance:

$$N_b = 1.5\,(N_b - 1) * tg\,\phi_d \qquad (1)$$

$$N_c = (N_b - 1) * cotg\,\phi_d \qquad \phi_d > 0 \qquad (2)$$

$$N_c = 2 + \pi \qquad \phi_d = 0 \qquad (2`)$$

$$N_d = tg^2\,(45 + \phi_d/2) * exp(\pi * tg\,\phi_d) \qquad (3)$$

Bearing capacity stress under the toe is given by equation

$$R_d = c_d * N_c * s_c * d_c * i_c + \gamma_1 * h * N_d * s_d * d_d * i_d + \gamma_2 * 0.5 * b * N_b * s_b * d_b * i_b \qquad (4)$$

where coefficients *s, d, i* follow from

$$s_c = 1 + 0.2 = 1.2 \qquad d_c = 1 + 0.1\sqrt{h/r} \qquad i_c = (1 - tg\delta)^2 = 1 \qquad (5,6,7)$$

$$s_d = 1 + \sin\phi_d \qquad d_d = 1 + 0.1\sqrt{(h/r) * \sin 2\phi_d} \qquad i_d = (1 - tg\delta)^2 = 1 \qquad (8,9,10)$$

$$s_b = 1 - 0.3 = 1.3 \qquad d_b = 1 \qquad i_b = (1 - tg\delta)^2 = 1 \qquad (11,12,13).$$

The coefficient equations are lined according the standard. The limit bearing capacity force of the toe is given by equation

$$Q_{pu} = \frac{1}{4}\pi r^2 * R_d \tag{14}$$

2.2 Rear shear force

The base of solution is considered according to ČSN 73 1004 "Drilled piers". The procedure need following 4 inputs:

- r - radius of the pile,
- h - depth (height) of the pile,
- γ_1 - unit weight of soil above pile toe,
- ϕ_d - angle of soil resistance - design value,
- c_d - cohesion - neglected,

and 1 coefficient:

- m_1 - coefficient of the state $m_1 = 0{,}7$.

The prior two quantities are geometrical, the last three ones are material properties. All values should be considered as design ones. It is not prescribed in the standard how the design material values have to be derived. In the author opinion it is formally correct for this component the design material values according to ČSN 73 0037 "Earth pressure acting on structures ". Cohesion is neglected and bearing capacity function of the rear shear force is defined as follows:

$$Q_{su} = m_1 * 2\pi r * \frac{1}{2}\gamma h^2 * (1-\sin\phi) * \tan\phi \tag{15}$$

A distinction of the different design material values (according to ČSN 73 0037, ČSN 73 1001) is omitted for easier expression. An user should be careful to remember it.

3 Similarity solution

Similarity solution is intent above all on extraordinary variability of soils and their material properties. The solution does not consider an obliquity of load (angle δ of force) because it, from viewpoint of bearing capacity, appears less important. Thus, coefficients $i_b = i_c = i_d = 1$. The solution involves investigation of the effective stress state only, a solution of the total stress state does not appears substantial for a further geotechnical comparative analysis. With regard to the purpose of use for the geotechnical comparative analysis, the geometrical parameters except of depth h are defined as dimensionless quantities and are considered for the similarity solution like constants (see eqs. below). A soil mass is homogeneous. Consequently, the solution considers these variable inputs h, γ, ϕ, c and following constant ones:

$$\eta = h/r \qquad \kappa_\lambda = (c)/\gamma\, tg(\phi) \qquad \lambda = (c)/\gamma h\, tg(\phi) \qquad \text{(16a,16b, 16c).}$$

3.1 Toe force

The functional of bearing capacity stress of the pile toe according to the equation (4) is expressed by

$$R_{bd} = (c)* N_c(\phi)*s_c*d_c + (\gamma)*(h)*N_d(\phi)*s_d(\phi)*d_d(\phi) + (\gamma)*0.5*\eta*(h)*N_b(\phi)*s_b*d_b \qquad (17).$$

Using a simple set-up of former showed equations we can defined a more suitable similarity functional with functions $F_1(\phi, c)$, $F_2(\gamma,\phi, h)$, $F_3(\gamma,\phi, h)$:

$$R_{bd} = F_1(\phi, c) + F_2(\gamma,\phi, h) + F_3(\gamma,\phi, h) \tag{18}$$

where

$$F_1(\phi, c) = 1.2 * \left(1 + 0.1\sqrt{\eta}\right) * (c)(N_d(\phi) - 1) / tg(\phi) \tag{19}$$

$$F_2(\gamma, h, \phi) = (1 + \sin(\phi)) * \left(1 + 0.1\sqrt{2\eta \sin 2(\phi)}\right) * (\gamma) * (h) * N_d(\phi) \tag{20}$$

$$F_3(\gamma, h, \phi) = 0.75 * 0.7 * (1/\eta) * (h) * (\gamma) * (N_d(\phi) - 1) * tg(\phi) \tag{21}$$

and after little adjustments also the functional

$$R_{bd} = (\gamma) * (h)\left[\lambda * F_1^s(\phi) + F_2^s(\phi) + F_3^s(\phi)\right] \quad (22).$$

The functions F_i^s depend on the variable ϕ only according to formulations below

$$F_1^s(\phi) = 1.2 * \lambda * \left(1 + 0.1\sqrt{\eta}\right) * (N_d(\phi) - 1) \tag{23}$$

$$F_2^s(\phi) = (1 + \sin(\phi))\left(1 + 0.1\sqrt{2\eta \sin 2(\phi)}\right) * N_d(\phi) \tag{24}$$

$$F_3^s(\phi) = 0.75 * 0.7 * (1/\eta) * tg(\phi) * (N_d(\phi) - 1) \tag{25}$$

The last formulation of bearing capacity functional (22) involves two dimensional variable h and γ separated using dimensionless geotechnical similarity coefficient λ (Janbu`s) which can be considered constant for lot of (mathematically for infinitely) combinations . So a dimensionless similarity function of bearing shallow capacity is defined as is shown below.

$$\mathrm{P}_{bd} = \lambda * F_1^s(\phi) + F_2^s(\phi) + F_3^s(\phi) \tag{26}$$

The limit bearing capacity force of the toe is given according (14) and (22) by equation

$$Q_{bu} = \frac{1}{4}\pi r^2 * (\gamma) * (h) * \mathrm{P}_d \tag{27}$$

and after the adjustment $r = h/\eta$ the final formula for the toe component of the bearing capacity is expressed by

$$Q_{bu} = (\gamma) * (h^3) * \frac{1}{4} * \frac{\pi}{\eta^2} * \mathrm{P}_d \quad /28)$$

3.2 Rear shear force

The functional of bearing capacity of the rear shear component according to the equation (15) after easy adjustment for $r = h/\eta$ can be expressed only in dependence at angle of shearing resistance ϕ by

$$Q_{su} = (\gamma) * (h^3) * m_1 * \frac{\pi}{\eta} * (1 - \sin\phi) * \tan\phi \tag{29}$$

4 Results

The complete similarity analytical solution of the pile bearing Q_{pu} is sum of the both force components (30). The analytical solution has made possible the separation of the dimension parameters h and γ from dimensionless ones and definition of the dimensionless similarity functional P_d (31).

$$Q_{pu} = Q_{bu} + Q_{su} = (\gamma) * (h^3) * \left[\frac{1}{4} \frac{\pi}{\eta^2} * \mathrm{P}_{bd} + m_1 \frac{\pi}{\eta} (1 - \sin\phi) \tan\phi \right] \quad (30)$$

$$\mathrm{P}_{pu} = \left[\frac{1}{4} \frac{\pi}{\eta^2} * \mathrm{P}_{bd} + m_1 \frac{\pi}{\eta} (1 - \sin\phi) \tan\phi \right] \quad (31)$$

This formulations keeps the standard separation of cohesion, angle of soil resistance and geometrical influences at bearing capacity of foundations.

5 Conclusion

The presented analytical solution of pile designs according to ČSN 73 1004 leads to the relative simple formulae (26) and (28) for toe capacity component and (29) of rear shear component. The formulae of component create the base of the global formula (30) resp. (31), for further numerical analyses and a comparison of the LSD designs according to ČSN and EC7-1. Solution makes possible to consider different approaches for deriving of design values of geotechnical quantities.

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