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CAM CURVE DESIGN USING POLYNOMIAL FUNCTION

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Summary: *This paper proposes one of the approaches to cam curve design and adverts to cam curve description using polynomial function. Serious properties of the polynomial curves are continuities. It is possible to deal simultaneously with the dynamical optimization of the cam curves. The example of the construction of cam curve is applied to set up polynomial displacement law of cam mechanism of beating-up mechanism in weaving machine.*

1. Introduction

Cam mechanisms transmitting a mechanical motion from the constant speed to a periodic variable speed is used in many types of the modern machines. The cam profile is usually defined in a non-dimensional form called normalized displacement laws or cam curve, i.e. maximum interval of motion and also upraise reach to value one (Koloc). Cam curve is defined the transmission relationship between cam and follower.

In dependence on different requirements on design cam curve exist several methods, e.g. cubic spline or B-spline functions (Qiu), harmonic functions etc. The advantages such as high speed, rigidity and accuracy impact on dynamical behavior of the cam mechanisms and also whole machines. With respect the dynamic behavior cam mechanisms is underlined the basic characteristic of the displacement law - continuity.

2. Polynomial cam curve

The conditions for assurance of required course of displacement law are continuity of not only 1. and 2. derivatives, but also 3. derivative especially the continuity of higher value in end points of intervals of motion. Setting up of the cam curve results from 3. derivative of displacement law (jerk) (Petríková). There is combination of binomials of higher-orders in dependence on cam curve properties. Some important types of polynomials are described below.

The jerk in non-dimensional form is proportional to

$$\eta''' \sim (1-\xi)^n \tag{1}$$

in interval (-1, 1) derive continuities in end points and the cam curve is proportional to

$$\eta \sim (1-\xi)^{n+3}, \tag{2}$$

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i.e. n+2-th order of the highest derivative continuous in point -1, 1 and the acceleration in a non-dimensional form has continuity of n-th order (Fig.1).

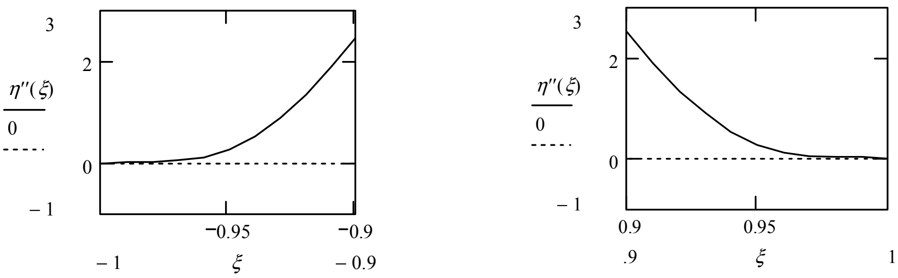

Fig. 1 Second derivative of displacement law in end points

For reduction of maximum acceleration is used polynomial

$$\eta''' \sim \left(\xi^2 - q\right)^m \ , \qquad (3)$$

where $q = \xi_q^2$ and ξ_q is point of contact with the tangent of order m at diagram $\eta''(\xi)$ (Fig. 2) and m+2 order the highest derivative equals zero in point ξ_q.

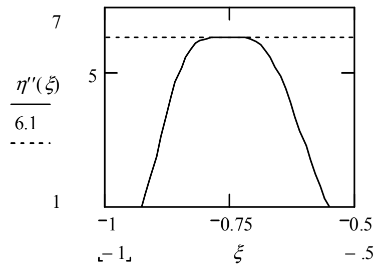

Fig. 2 Second derivative of displacement law in contact point with tangent

The binomial

$$\eta''' \sim \left(\xi^2 - p\right)^k \qquad (4)$$

provides k+2 order of the highest derivative equals zero in point ξ_p ($p = \xi_p^2$). The contact with the tangent is k+1 order at diagram $\eta'(\xi)$ and the contact in inflexion point $\xi_p, -\xi_p$ with tangent is k-th order at diagram $\eta''(\xi)$ (Fig.3).

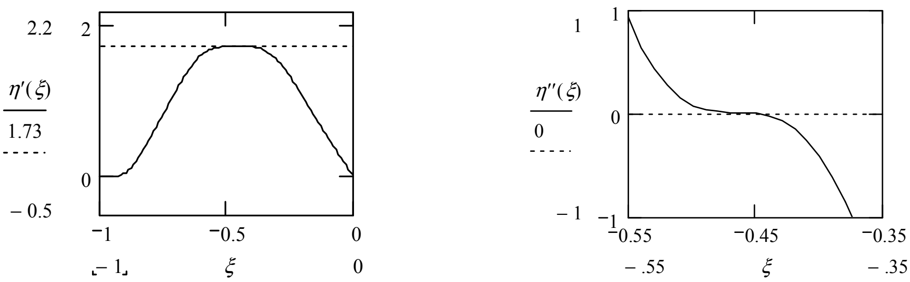

Fig. 3 First derivative of displacement law in contact point with tangent and second derivative in inflexion point with tangent.

The equation of third derivative of displacement law for cam mechanism of beating-up mechanism at weaving machine is set up in following form

$$\eta'''(\xi) = A\xi^{\ell}\left(1+\alpha\xi^2\right)\left(\xi^2-q\right)^3\left(1-\xi^2\right)^4 \tag{5}$$

or

$$\eta'''(\xi) = A\xi^{\ell}\left(1+\alpha\xi^2\right)\left(\xi^2-p\right)^2\left(\xi^2-q\right)^3\left(1-\xi^2\right)^4. \tag{6}$$

The binomial

$$\left(1+\alpha\xi^2\right) \tag{7}$$

decreases maximum value of the non-dimensional acceleration in the middle of a working interval. The term ξ^{ℓ} performs the contact η'' in $\xi = 0$ with top tangent ℓ-th order. The course of displacement law according equation (5) and its the first, second, third derivatives $\eta(\xi)$, $\eta'(\xi)$, $\eta''(\xi)$, $\eta'''(\xi)$ are shown in the following figure (Fig.4).

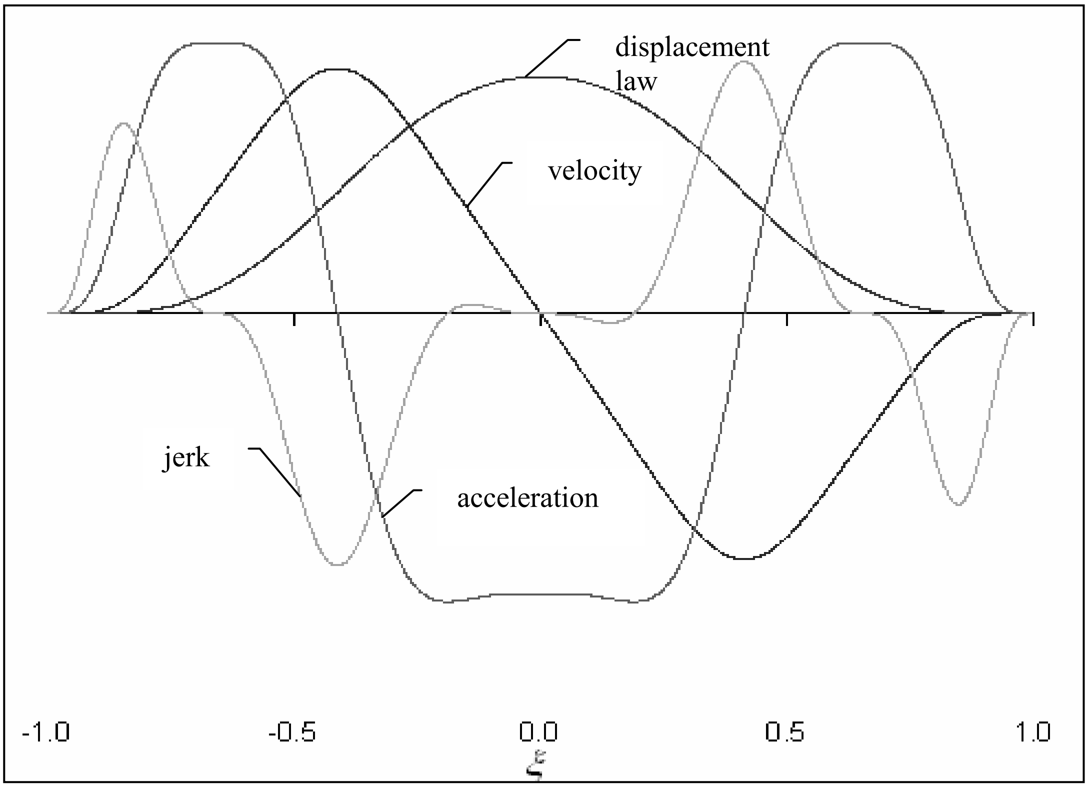

Fig. 4 Courses of displacement law, non-dimensional velocity, acceleration and jerk

Parameters A, q, p in equation (5) or (6) and integrative constants are derived by conditions in the end points of displacement law, non-dimensional velocity and acceleration intervals and maximum values of cam curve and non-dimensional acceleration

$$\eta(\pm 1) = 0, \tag{8}$$

$$\eta'(\pm 1) = 0, \tag{9}$$

$$\eta''(\pm 1) = 0, \tag{10}$$

$$\eta(0) = 1. \tag{11}$$

3. Dynamic analysis

The cam curve impacts against dynamical behavior of cam mechanisms. It is very important to control the residual vibration of the follower output motion and to adopt a suitable cam curve which results in less vibration than others.

For determination of dynamical properties designed cam is necessary to test displacement law in the simulation model of mechanism for that the cam designed. It is possible to create the universal model with single degree of freedom or with more degrees of freedom.

The displacement laws were tested by means of beating-up mechanism model with 14 degrees of freedom and compared both solutions - with polynomial No. I (5) and polynomial No. II (6). (Fig. 5)

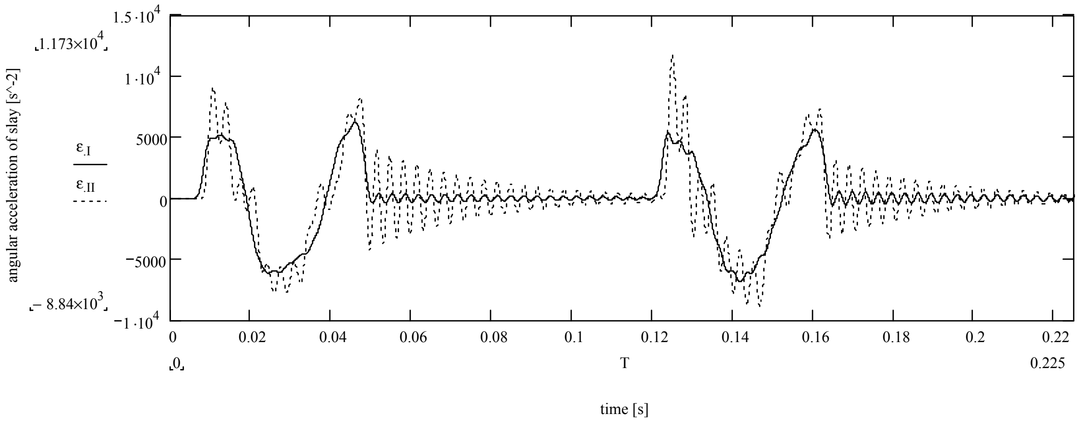

Fig. 5 Angular acceleration of sley with polynomial function I (—) and II (....) for cam curve design

4. Conclusion

Dynamic simulation with displacement law according equation (6) reflects much higher amplitudes and frequencies of vibrations through lift and higher residual vibrations after battening. The binomial (4) which supports uniform motion at change of angular acceleration sense has negative influence. While at the cam curve according equation (5) is possible increasing of speed about 8 % without elevation of the cam mechanism loading.

5. References

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