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LIMITATION OF THERMAL INVERSE ALGORITHM AND BOUNDARY CONDITIONS RECONSTRUCTION FOR VERY FAST CHANGES ON BOUNDARY

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Summary: *Direct measurements of boundary conditions in application such as descaling or hot rolling in steel industry or in experiments that simulates these processes are impossible. Thus temperature histories are recorded inside the investigated body. The distance of the measurement point from the investigated surface strongly influences the shortest impulse that can be reconstructed by an inverse method. The time delay dependence on distance of the measurement point from the surface is shown in this paper. According to this time delay and quality of the measurement device a minimal number of forward steps are computed for sequential Beck's inverse algorithm. Based on the number of forward steps the degradations of reconstructed boundary conditions are presented.*

1. Introduction

For computational methods knowledge of boundary conditions is necessary. Those conditions can be computed for simple cases, however, they must be obtained from measurements in most cases. Boundary conditions can be measured directly on the surface or if not possible we can do the measurement inside the investigated body and then we have to use an inverse task to compute boundary conditions from measured values. In our case we concentrate on boundary conditions during water cooling of hot steel products or of hot working rolls (Horský 2005). In these cases it is not possible to measure cooling intensity directly on the surface and we have to measure temperature history inside the body and to compute boundary conditions using an ill-posed inverse task. The accuracy of the computed results is strongly dependent on two factors: distance of the thermocouple from the investigated surface and on the additional noise in the measured data.

2. Measurement

Hydraulic descaling is the process of removing oxide layers from a hot (typically steel) surface using high-pressure water jet that have flat water stream (see Figure 1). Experimental conditions were prepared in such way, which resembles as close as possible to the real mill conditions (Raudenský 2003). There are two basic parameters, which should be kept. The first is the initial temperature of tested sample and the second is the speed of sample motion. To measure boundary conditions a special experimental stand was developed for these tests.

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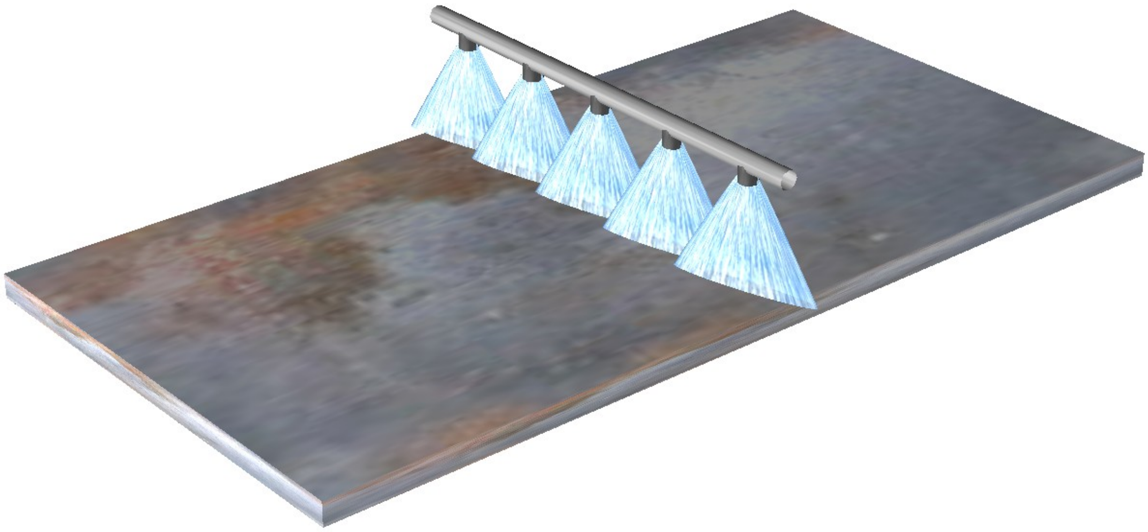

Figure 1 – High-pressure water nozzles with flat water stream removing oxide layers from a hot surface steel.

Experimental stand

The experimental stand was built to study the cooling of linearly moving objects. A six meter long girder carrying a movable trolley and a driving mechanism (see Figure 2) forms the basic part of the experimental device. An electronic device measuring the instant position of the trolley is embedded in the trolley. The driving mechanism consists of an electric motor controlled by a programmable unit, a gearbox, two rollers and a hauling rope. The girder is divided into three sections. The marginal sections are used for the trolley's acceleration or deceleration. The velocity of the trolley is constant in the mid-section and it is here where the spray nozzles quench the measured sample.

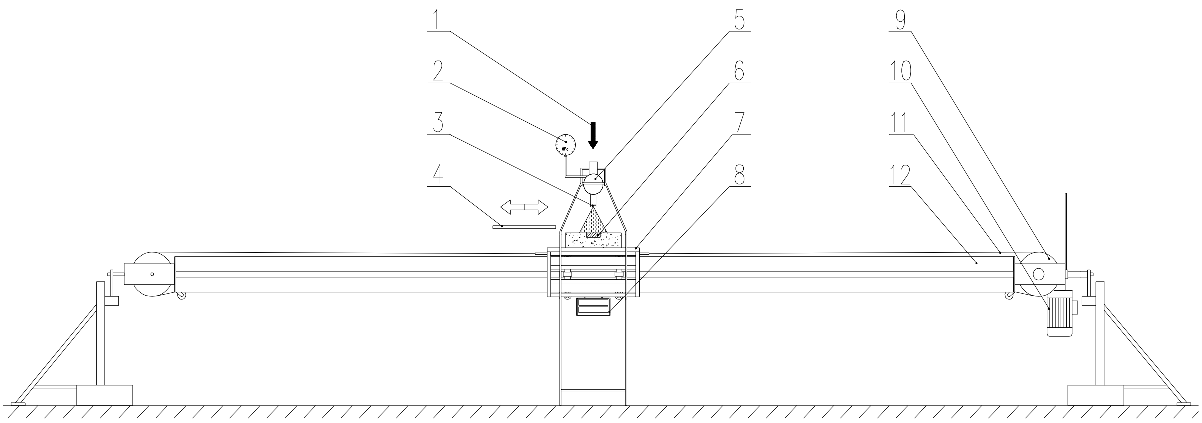

Figure 2 – Principal scheme of the linear test bench (1-cooling medium supply, 2-pressure gauge, 3-nozzle, 4-moving deflector, 5-manifold, 6-tested sample, 7-moving trolley, 8-datalogger, 9-roller, 10-electric motor, 11-hauling steel wire rope, 12-girder).

The procedure of the experiment is as follows:

- An electric heater heats the test plate to an initial temperature of the experiment.
- The plunger water pump is switched on and the pressure is adjusted.
- A driving mechanism moves the test plate under the spraying nozzles. After recovering the temperature field in the plate, the movement of the plate under the spraying nozzles is repeated.
- The temperature is measured using K-thermocouples (see Figure 3) inside the investigated steel plate at a depth of 1 mm from the cooled surface and the temperature is recorded into data logger memory.
- The positions of the test plate and the thermocouples (in the direction of movement) are recorded together with the temperature values. The record of instant positions is used for computation of instant velocities and positions while moving under the spray.

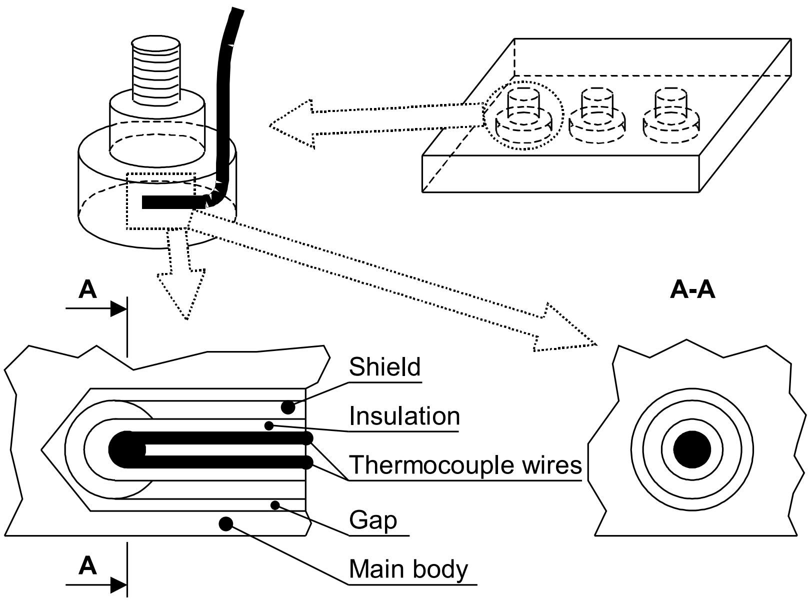

Figure 3 – Application of sensor, and its structure in detail.

3. Evaluation using inverse task

The pass under the nozzle causes temperature drop in the material sample. This information together with material properties and calibration characteristics of temperature sensor is used as an input for the inverse heat conduction task. We use sequential Beck's approach (1985). The results of computation are surface temperature, heat flux and HTC.

The main feature of Beck's approach is sequential estimation of the time varying boundary conditions. He demonstrated that function specification and regularization methods could be implemented in a sequential manner and that they gave in some cases nearly the same results as the whole domain estimation. Moreover the sequential approach is computationally more efficient. Beck's approach has been widely used to solve inverse heat conduction problems to determine unknown boundary or material property information.

The method uses sequential estimation of the time varying boundary conditions and uses future time steps data to stabilize the ill-posed problem. The heat transfer coefficient (HTC) is found after determining the heat flux at the surface. To determine the unknown surface heat flux at the current time t^m, the measured temperature responses $T_i^{*,m}$, are compared with the computed T_j^m from the forward solver (e.g. FDM, FVM, FEM, etc.) (Patankar 1980), using n_f future time steps

$$SSE = \sum_{f=m+1}^{m+n_f} \sum_{j\equiv i;i=1}^{n_T} (T_i^{*,f} - T_j^f)^2 \,. \tag{1}$$

Using the linear minimization theory, the value of the surface heat flux that minimizes Eq. (1) is

$$\hat{q}^m = \frac{\sum_{f=m+1}^{m+n_f} \sum_{j\equiv i;i=1}^{n_T} (T_i^{*,f} - T_j^f|_{q^m=0})\zeta_i^f}{\sum_{f=m+1}^{m+n_f} \sum_{i=1}^{n_T} (\zeta_i^f)^2} \tag{2}$$

where $T_j^f|_{q^m=0}$ are the temperatures at the temperature sensor locations computed from the forward solver using all the previously computed heat fluxes, but without the current one q^m. The ζ_i^f is the sensitivity of the i^{th} temperature sensor at time t^f to the heat flux pulse at time t^m. These sensitivity coefficients are mathematically the partial derivatives of the computed temperature field to the heat flux pulse, but in this case they physically represent the rise in temperature at the temperature sensor location for a unit heat flux at the surface. The sensitivity coefficient of our interest is defined as

$$\zeta_j^m = \frac{\partial T_j^m}{\partial q^m} \tag{3}$$

Once the heat flux is found for the time t^m, the corresponding surface temperature T_0^m may be computed using the forward solver. When the surface heat flux q^m and surface temperature T_0^m are known, the heat transfer coefficient is computed from

$$h^m = \frac{\hat{q}^m}{T_\infty^m - (T_0^m + T_0^{m-1})/2} \,. \tag{4}$$

This approach is limited to linear problems. However, it can be extended to nonlinear cases. The modification of this procedure involves an outer iteration loop which continues until the computed temperature field is unchanging. The nonlinearity requires iteration only to determine the present value of the heat flux, while the computations to determine the surface temperature and heat transfer coefficient need only be performed once for each time t^m. The sensitivity coefficients are also nonlinear, due to the dependence of the thermal properties on the temperature field, and they must be computed for each iteration.

Once the heat transfer coefficient at the "present" time is computed, the time index m is incremented by one, and the procedure is repeated for the next time step. For n measured time steps only $n - f$ can be computed owing to the use of future data as a regularizing approach.

4. Limitation of the inverse method

Due to the impulse propagation delay from the surface to the measuring sensor the method can not be used for investigation of extremely fast changes on boundary. The more distant the sensor is from the surface the slower changes can be investigated.

Suppose a hot steel plate of 1000°C cooled down by water of 0°C. The velocity of the steel plate is 1 m/s and the cooling impulse last 0.01 s. This corresponds to the spray depth of the flat descaling nozzle. The spray depth is usually smaller than 10 mm. The applied boundary condition, computed surface temperature and temperature 2 mm under the cooled surface is shown in Figure 4. It is obvious that the temperature 2 mm under the cooled surface starts falling 0.05 s after the beginning of the cooling impulse. This means that the temperature starts falling when the measuring sensor is already 40 mm behind the flat water spray.

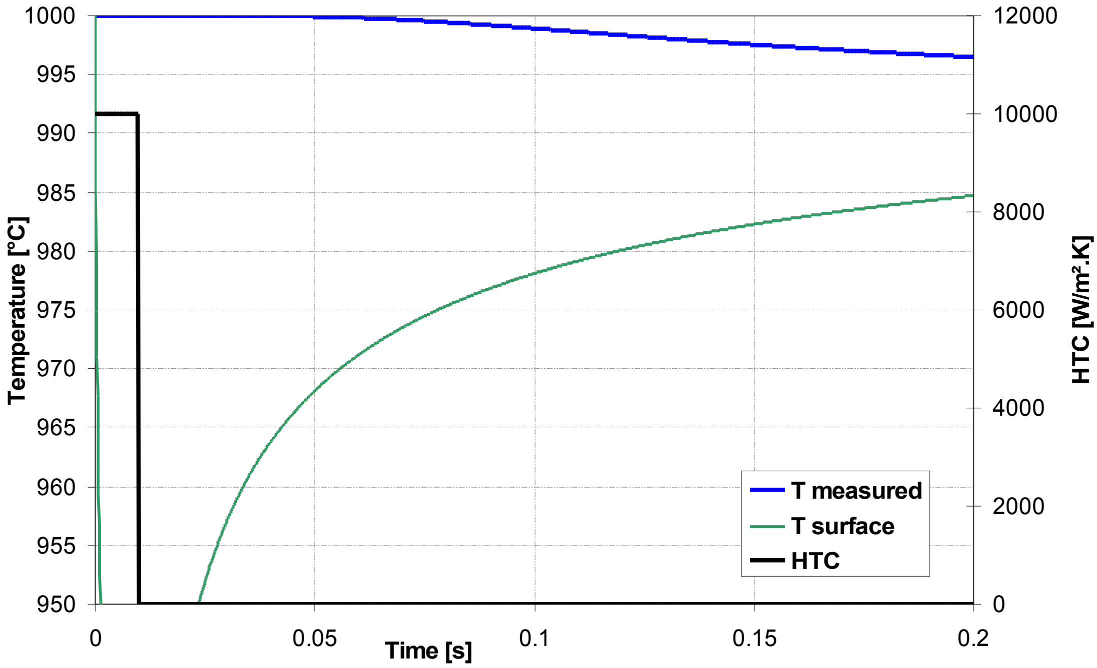

Figure 4 – Computed temperature histories for the surface temperature and the temperature 2 mm under the cooled surface (T measured).

We can compute time delay of temperature change in several distant points under the cooled surface using an analytical equation (Incropera 1996)

$$\frac{T(x,t)-T_\infty}{T_0-T_\infty}=\sum_{n=1}^{\infty}\frac{4\sin\zeta_n}{2\zeta_n+\sin(2\zeta_n)}\cdot\exp\left(\frac{-\zeta_n^2\cdot\alpha\cdot t}{L^2}\right)\cdot\cos(\zeta_n\cdot x)\ . \tag{5}$$

where the thickness of plate is L. The thickness must be small with regard to the width and height of the wall. In this case, we can assume that the conduction occurs only in the x direction. The wall is initially at uniform temperature T_0 and is suddenly exposed to a constant heat transfer coefficient h and constant ambient temperature. The discrete values of ζ_n are positive roots of the transcendental equation

$$\zeta_n\tan\zeta_n=\frac{h\cdot L}{k}\ . \tag{6}$$

5. Results

Time delays

The computed time delays for stainless steel are summarized in Figures 5–6. The temperature threshold 0.01°C in Figure 5 is used as common accuracy of forward solvers of heat conduction. Now assume that we use measuring equipment that enable us to measure temperature range 0-1000°C with 12-bits resolution. It means that that maximum accuracy (temperature step) is 0.25°C. The time delays for this temperature threshold are summarized in Figures 6.

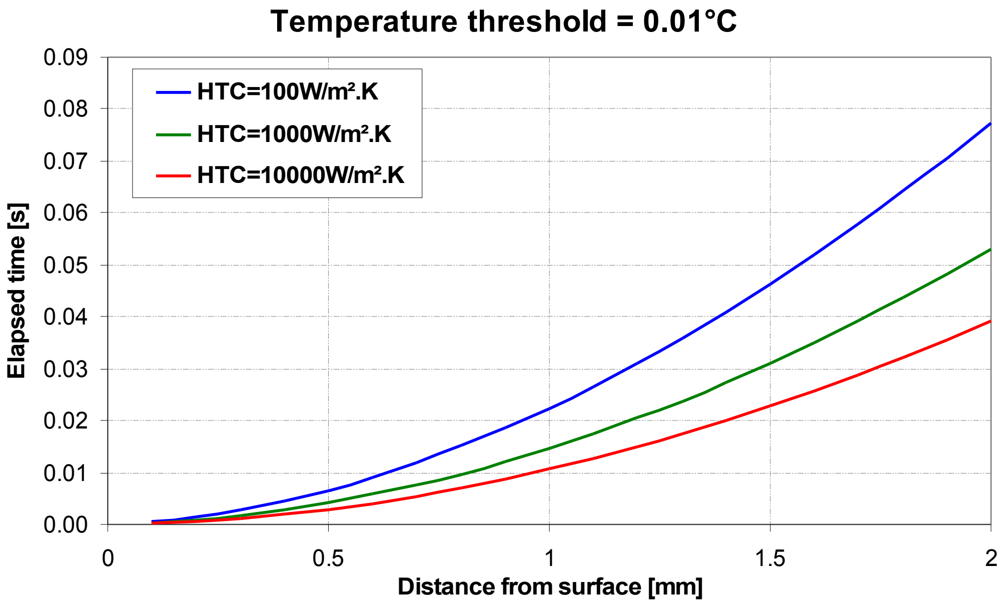

Figure 5 – Computed time delays for temperature threshold 0.01°C and three values of HTC

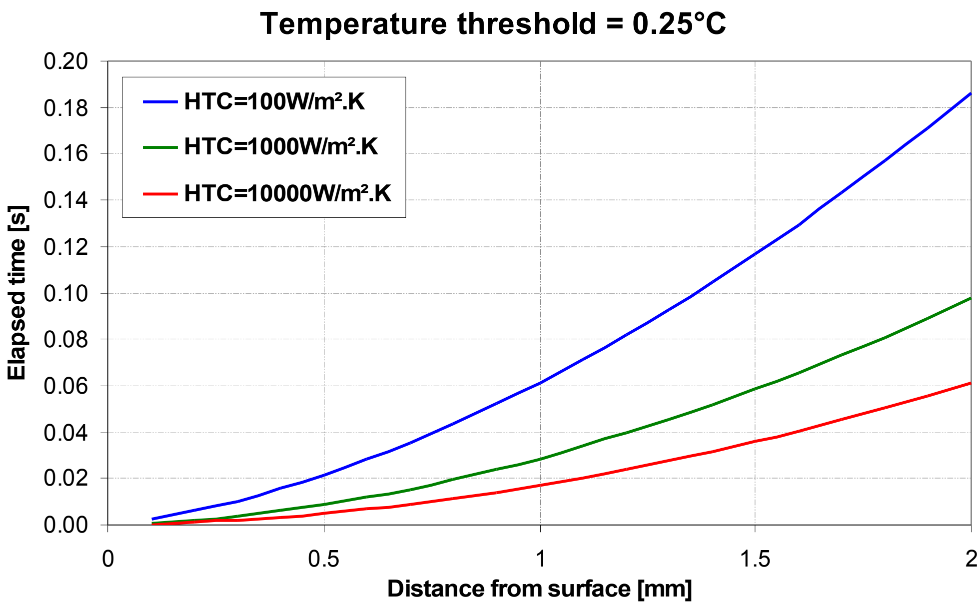

Figure 6 – Computed time delays for temperature threshold 0.25°C and three values of HTC

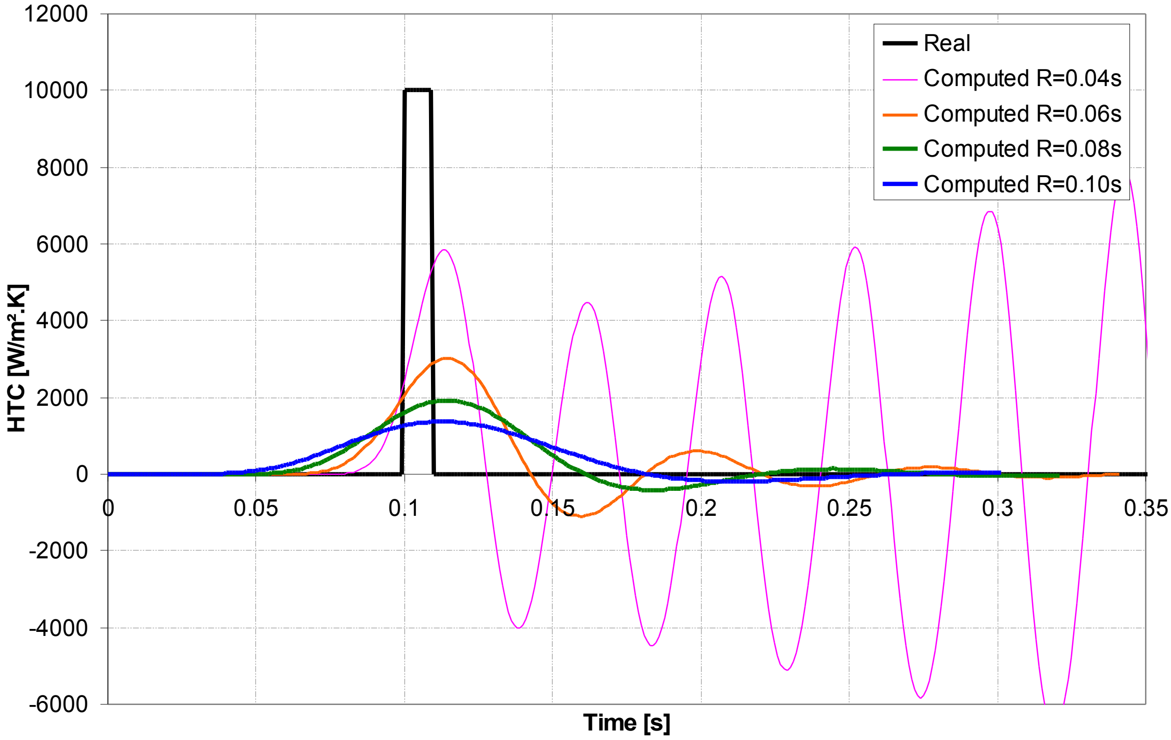

Figure 7 – Time dependent boundary condition reconstruction using various number of future time steps (recomputed to the time domain – R parameter) for short constant impulse of HTC.

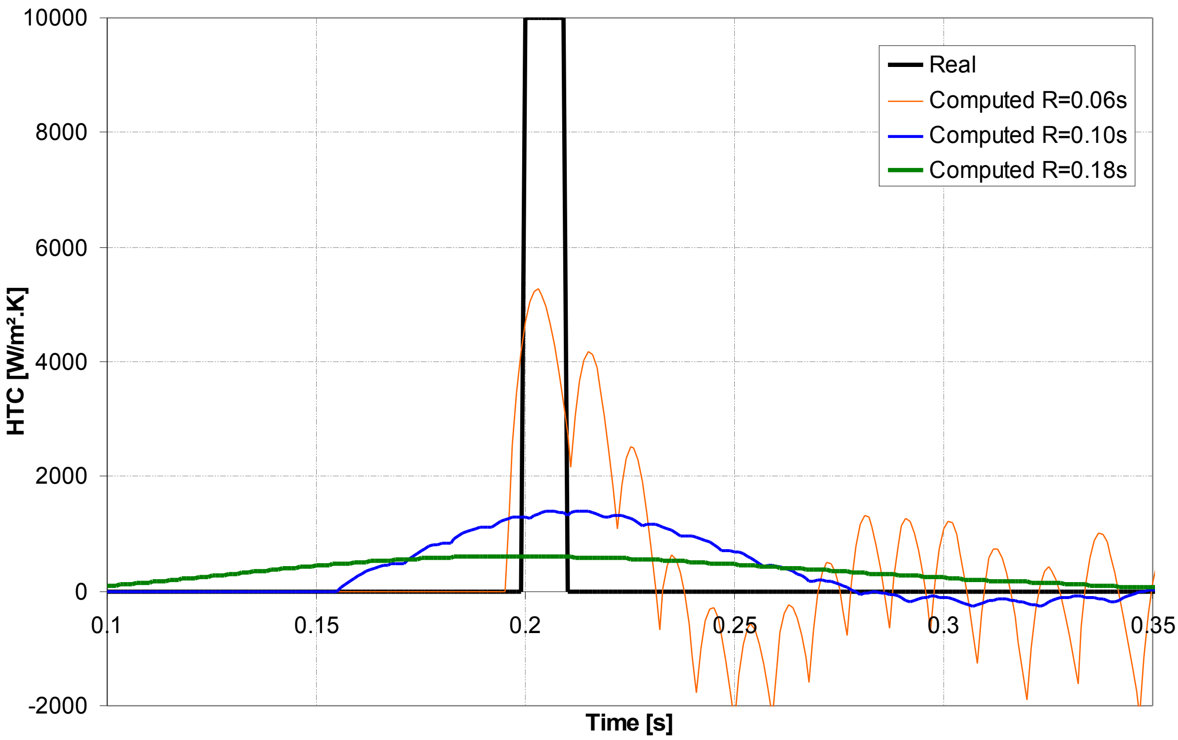

Figure 8 – Time dependent boundary condition reconstruction as in the Figure 7 but the input temperature history was assumed to be from the measuring equipment with 12-bits resolution.

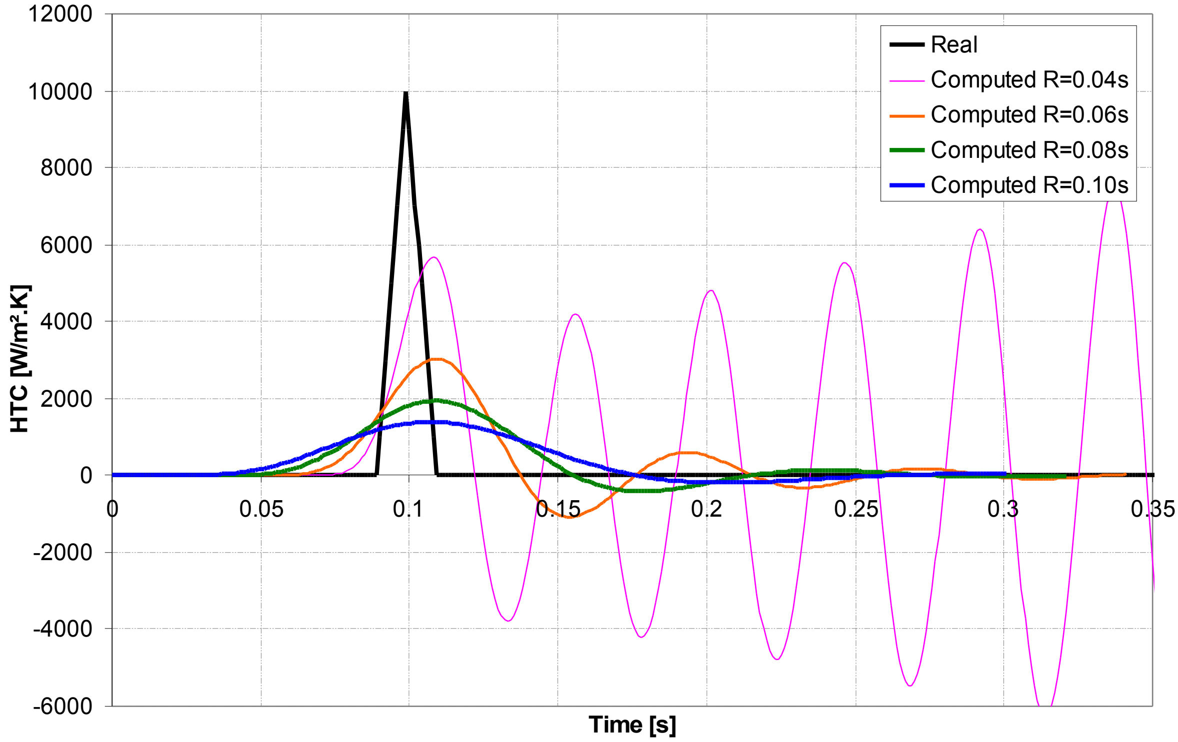

Figure 9 – Time dependent boundary condition reconstruction using various number of future time steps (recomputed to the time domain – R parameter) for short triangular impulse of HTC.

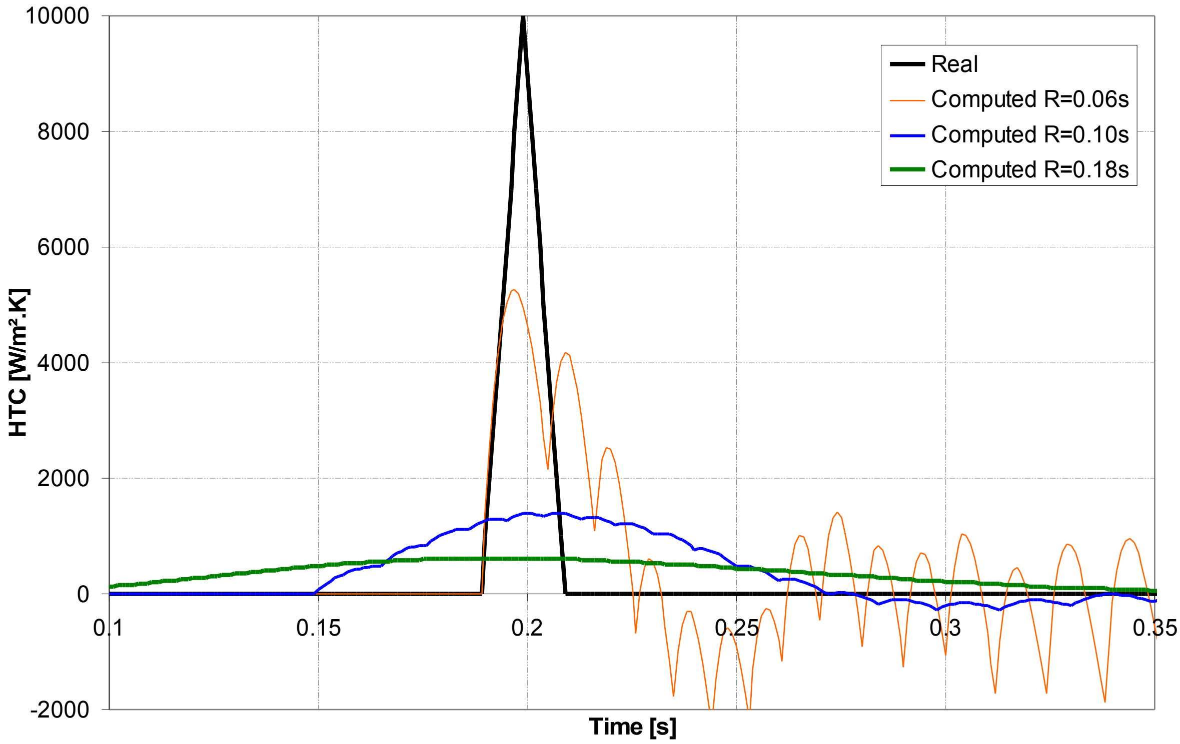

Figure 10 – Time dependent boundary condition reconstruction as in the Figure 9 but the input temperature history was assumed to be from the measuring equipment with 12-bits resolution.

Estimation of necessary number of future time steps

The number of future time steps can be estimated from the Figure 5. If we are using sampling frequency 1000Hz the number of future time must be at least 40 (0.04 s) for the sensor at the depth of 2 mm to avoid divergence of the inverse algorithm (see Figures 7 and 9). To avoid significant negative HTC we need to increase the number to 80–100 (0.08–0.1 s).

If we use measuring equipment with 12-bits resolution the number of future time must be even higher. From the Figure 6 we can see that the minimal number increase to 60 (0.06 s) and the optimum number is approaching 180 future time (0.18 s). Of course that such a high number is not optimum for reconstructing our narrow HTC impulse. However, we can use Figures 5 and 6 for finding the maximum distance of the thermocouple from the cooled surface.

Estimation of maximum distance of thermocouple from investigated surface

First we need to use sampling frequency that is fine enough. For our case we should use the sampling frequency at least 1000Hz to have 10 samples per HTC impulse (see Figure 11). To reconstruct the impulse we should shorter future time than the duration of the impulse. Let's say 0.005 s. Now we can look in the Figure 5 and we can see that the maximum distance of the thermocouple from the cooled surface should be about 2 mm. The reconstructed HTC impulse for thermocouple 2 mm under the surface is shown in Figure 11. If we use measuring equipment with 12-bits resolution the distance must be even smaller.

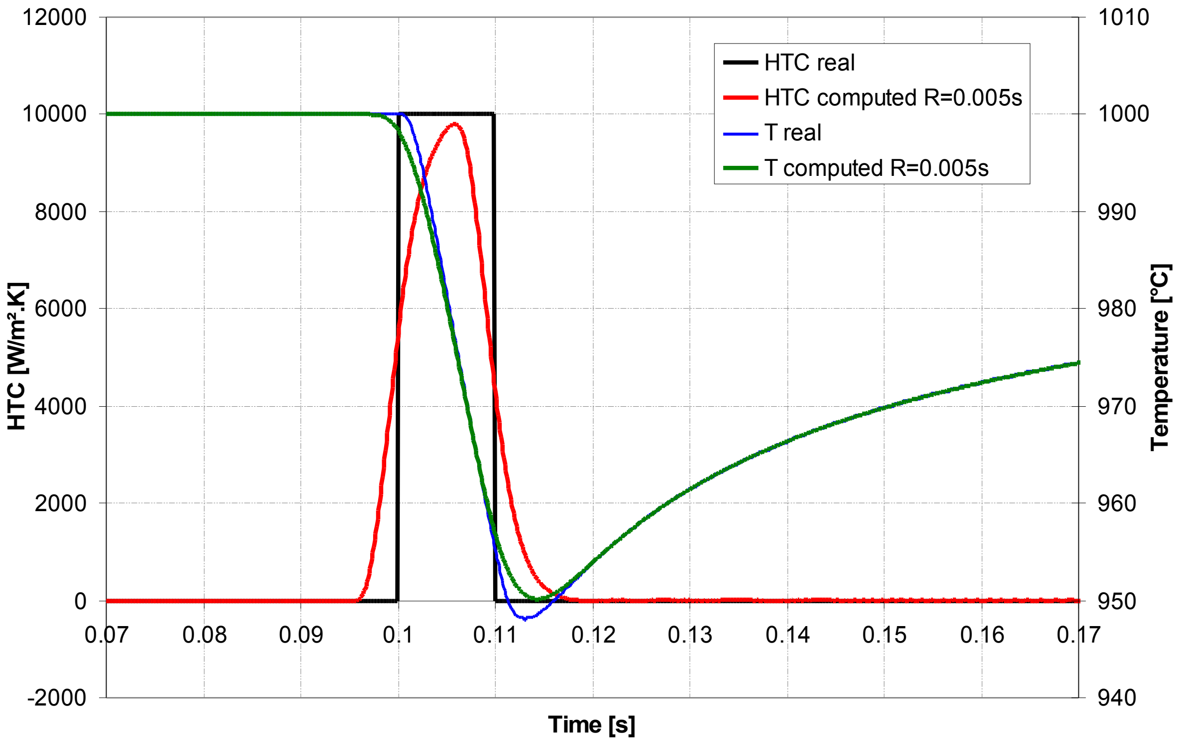

Figure 11 – Time dependent boundary condition reconstruction using thermocouple 2 mm under the investigated surface.

6. Conclusion

Mathematical procedures and precise inverse computations are used for evaluation of experimental results. Final output format of data are boundary conditions which can be used in numerical models of these processes.

The computational experiments have shown that Eq. (5) can be used for estimation of the future time steps for the inverse algorithm as well as for computation of the maximal distance of the thermocouple from the investigated surface. The self-adaptive design of computational models and knowledge of accuracy of computed results are necessary. This technology makes it possible for engineers and scientists to construct more realistic mathematical models of physical processes.

7. Acknowledgment

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8. Literature

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