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MOTION OF ROTOR SUPORTED ON AERODYNAMIC BEARING

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Summary: *The tilting pad bearings are very often applied in high speed rotating machines. The exact solution of bearing characteristics, including the motion of tilting pads, is very difficult and not yet completely elaborated. An approximate method for calculation of rigid rotor motion with large amplitudes limited only by the bearing clearance is presented. The models of stiffness and damping forces valid for the entire area of journal motion in the aerodynamic bearing's clearance are based on the transformation of linear stiffness and damping characteristics calculated for two cases of stationary state and extended to the whole bearing field by using the correction function $f_{cor}(r)$. Differential equation of motion of rotor modeled by a mass point supported on aerodynamic bearing with proposed nonlinear characteristic is numerically solved and resulting time histories of journal motion and its plane trajectories are presented. The influence of eccentricity of rotor on its dynamic response are analysed as well.*

1. Introduction

The radial tilting-pad journal bearings are very often used in modern high-speed rotation machinery. The properties of such kind of bearings are more complicated than those of simple cylindrical bearings and therefore the tilting pad bearings are now topics of a lot of research. .

Particularly the behaviour of tilting pad aerodynamic bearing is not yet sufficiently studied. The exact solution of bearing characteristics, including motion of the tilting pads is very difficult and only special eases are known and therefore simplified methods, mostly strongly linearized, are usually applied. The linearized dynamic characteristics of aerodynamic tilting pad bearings loaded by constant force were calculated in Techlab Ltd. in the form of evolutive stiffness and damping matrices. They have been used in IT ASCR for investigation of response curves and modal analysis of experimental rotor. Due to the linearity, this model can be applied only for solution of small oscillations around equilibrium state (Pust, Kozanek 2005a) given by static deformation $x = r_0$ produced by the rotor weight.

The extension of force properties from one point $r = r_0$ to the whole bearing field was then realized by using the correction function $f_{cor}(r)$, which ensures that the force characteristics in the equilibrium point r_0 agree with the before calculated values. Correction function $f_{cor}(r)$ has zero values in the centre of bearing. This function was applied in some publication (Pust,

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Kozanek 2005b,c) and interesting results were achieved. However the assumption of zero stiffness in the centre of bearing does not correspond to the real journal property and therefore a more sophisticated correction function considering also the non-zero stiffness and damping matrices in the bearing centre was derived and applied for solution of journal motion.

In this contribution, we present an approximate method for calculation of rigid rotor motion with large amplitudes limited only by the bearing clearance r_h. The approximate models of stiffness and damping forces - valid for the entire area of journal motion in the aerodynamic bearing's clearance - are based on the transformation of linear stiffness and damping characteristics from two points into the entire area. The assumption that the rheological properties of 3-tilting pads bearing are centrally symmetric is also used. The stiffness and damping matrices on the field of motion $r = (0, r_h)$, $-\pi < \varphi \leq \pi$ are described by the combination expressions in the equilibrium point and in centre of bearing.

For detailed study of the nonlinear bearings properties, let us focus here only on one bearing, without any couplings with the other bearing of rotor. The inertia of rotor is concentrated into mass point in the centre of bearing. Differential equations of motion of this mass are numerically solved and resulting time history of journal motion and their plane trajectories will be presented and analyzed.

2. Motivation of research

The application of air as lubricant in aerodynamic bearings is very advantages in various branch of technical praxis, particularly in chemical and food industry. Aerodynamic principle is simple, it does not need any auxiliary apparatus and accessories and is suitable for very high revolution, e.g. for high-speed compressors.

For these reasons, several types of aerodynamic bearings were designed in Techlab Ltd. and investigated in the Institute of Termomechanics ASCR and in CKD – NEW ENERGO a.s.. The laboratory prototype of rotor supported on two aerodynamic tilting pad bearings is shown in Fig. 1. This experimental set is based on the high–frequency synchronous motor running up to 50 000 rpm. Its rigid rotor (length l = 0,32m) is supported in two identical radial aerodynamic bearings (diameter 50 mm, clearance $r_h = 0{,}05\,\text{mm}$) with three tilting pads in each bearing - Fig. 2. The greater machinery – turbo compressor equipped with aerodynamic bearings (120 mm, 100 kW) is developed in CKD.

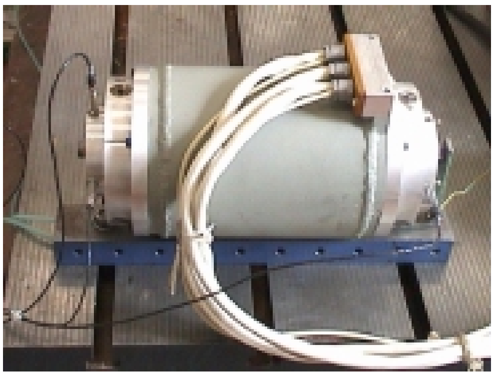

Fig. 1

Fig. 2

Theoretical background of this experimental research is provided in the project GAČR No. 101/06/1787 where the analytical and numerical investigation of simplified mathematical models of rotor excited by centrifugal force from the unbalanced rotor is solved.

3. Mathematical model of the rotor motion

The weight of the experimental rotor ÚT AVČR supported on aerodynamic bearings is *mg*=78 N and the inertia moment to the axis perpendicular to the axis of rotation is I = 0,10024 kgm^2. The inertia properties defined by *m* and *I* can be replaced also by effects of three virtual mass points (Brepta, 1994) m_1, m_2 situated in the centers of bearings and the third mass m_3 in the middle of the length *l* between the bearings. These masses are

m_1 =1,616 kg, m_2 =2,376 kg, m_3 =3,608 kg.

The motion of rigid rotor are described by equation

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{K}(\omega)\dot{\mathbf{X}} + \mathbf{B}(\omega)\dot{\mathbf{X}} = \mathbf{O}(\omega t) me\omega^2 + \mathbf{F} \,, \qquad (1)$$

where

$$\mathbf{X} = [x_1 \quad x_2 \quad y_1 \quad y_2]^T \,,$$

$$\mathbf{M} = \begin{bmatrix} m_1 + m_3/4 & m_3/4 & 0 & 0 \\ m_3/4 & m_2 + m_3/4 & 0 & 0 \\ 0 & 0 & m_1 + m_3/4 & m_3/4 \\ 0 & 0 & m_3/4 & m_2 + m_3/4 \end{bmatrix}, \qquad (2)$$

$$\mathbf{K}(\omega) = \begin{bmatrix} k_{xx}(\omega) & 0 & k_{xy}(\omega) & 0 \\ 0 & k_{xx}(\omega) & 0 & k_{xy}(\omega) \\ k_{yx}(\omega) & 0 & k_{yy}(\omega) & 0 \\ 0 & k_{yx}(\omega) & 0 & k_{yy}(\omega) \end{bmatrix}, \qquad \mathbf{B}(\omega) = \begin{bmatrix} b_{xx}(\omega) & 0 & b_{xy}(\omega) & 0 \\ 0 & b_{xx}(\omega) & 0 & b_{xy}(\omega) \\ b_{yx}(\omega) & 0 & b_{yy}(\omega) & 0 \\ 0 & b_{yx}(\omega) & 0 & b_{yy}(\omega) \end{bmatrix};$$

$$\mathbf{O}_{(\omega t)} = [(1/2 - a/l)\cos\omega t \quad (1/2 + a/l)\cos\omega t \quad (1/2 - a/l)\sin\omega t \quad (1/2 + a/l)\sin\omega t]^T$$

$$\mathbf{F} = [mg/2 \quad mg/2 \quad 0 \quad 0]^T$$

The stiffness $\mathbf{K}(\omega)$ and damping $\mathbf{B}(\omega)$ matrices contain elements strongly depended on angular velocity ω of rotor and also on the static force and displacement of stationary state from the bearing's center.

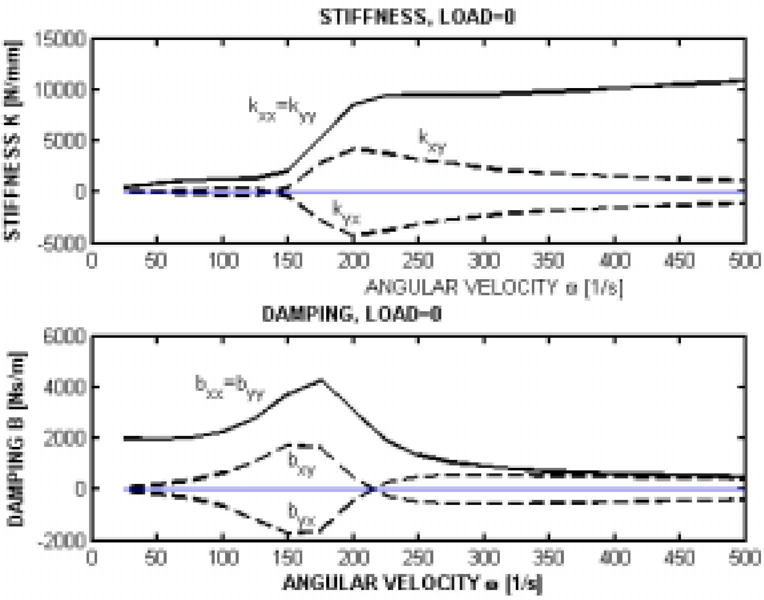

Fig. 3

A program for calculation dynamic characteristics **K, B** of aerodynamic bearings at different revolutions with considering of inertia properties of tilting pads was elaborated in Techlab Ltd. This program gives discrete values of elements of stiffness $\mathbf{K}(\omega)$ and damping $\mathbf{B}(\omega)$ matrix in steps of 2500 rpm (approx. 250 1/s).

These values vary very strongly at different revolutions. There are no monotone functions because the inertia of tilting pads causes resonance phenomena, superimposed on the monotone increase or decrease at variation of revolutions. The examples of these properties are in Figure 3 for unloaded journal that is for the position in the center of bearing. The stiffness k_{yy} is the same as k_{xx}, the cross stiffness k_{xy} and k_{yx} are of opposite sign. It means, that the stiffness matrix **K** is anti-symmetric. The same anti-symmetric properties has the damping matrix **B** whose elements are plotted in the bottom half of Fig. 3.

When the journal is vertically loaded by the force F = 40 N, the properties of both stiffness and damping matrices changed. It is seen from Fig. 4, where the curves k_{xx} and k_{yy} differ as well as the damping curves b_{xx} and b_{yy}. The cross stiffness k_{xy} a k_{yx} and cross damping b_{xy}, b_{yx} are not yet of opposite sign but their form differ. The peaks of curves shift to the higher velocities (from ω = 150-200 1/s to ω = 250-300 1/s) due to the increase of loading from 0 to 40 N.

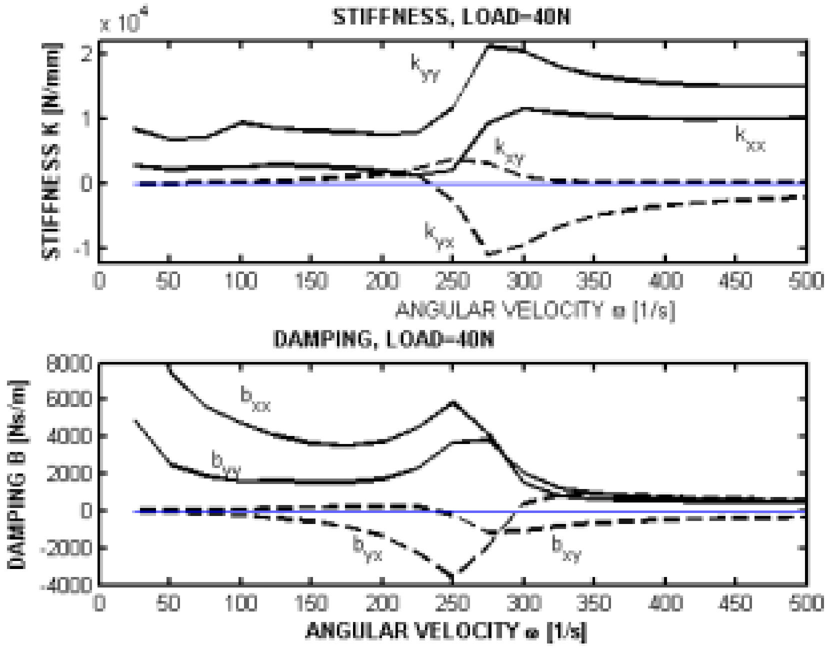

Fig. 4

4. Ascertaining of the correction function $f_{cor}(r)$

The stiffness properties are known only in several few points on the vertical *x* axis of the bearing. In order to extend the mathematical description of stiffness properties on the whole area of possible journal motion, two following steps must be done:

1) Define, at least approximately, the stiffness properties in the all points on the vertical radius $r \in (0, r_h)$.
2) Extend this definition from the vertical line to the all radiuses inclined with angle $-\pi < \varphi \leq \pi$.

The first task we try to solve by means of correction function.

The force field of bearings is central symmetric and the force magnitude depends only on the radius *r* from the geometric centre and is independent on the angle φ. The extension of force properties on the entire radius field $r \in (0, r_h)$, where r_h is the radial clearance

(r_h = 0.05 mm in our prototype), can be then realized by multiplying the force characteristics $(\mathbf{K}(\omega), \mathbf{B}(\omega))$ by a correction function $f_{cor}(r)$. The correction function $f_{cor}(r)$ in the previous calculations (Pust, Kozanek,2005a,b) was selected in the form of hyperbolic function, which approximately expresses the rise of stiffness of the bearing with increase of eccentricity described in literature (e.g.Tondl,1965):

$$f_{cor}(r) = \frac{s}{p + r_h - r} - h \tag{3}$$

and which ensured the identity of the force field characteristics in the equilibrium point $r = x_0$ with the previously calculated values .

This function gives zero value in the centre of bearing in order to ensure the continuity of function in the centre x = 0, y = 0 (or r = 0). The more detailed calculation of stiffness and damping matrices at various loading and various equilibrium positions shown that in the central bearing position at zero loading has the stiffness matrix non-zero elements, but it is anti-symmetric (see Fig. 3). Therefore the more exact correction has to be realized by expression consisting of two parts, one of them $f_{cor1}(r)\, k_{xx}(0)$ describing the properties in the centre of bearing i.e. at zero loading, the second of them $f_{cor2}(r)\, k_{xx}(r_0)$ depends on the value of stiffness k_{xx} in the equilibrium point $r_0 = x_0$

$$k_{xx}(r) = f_{cor1}(r) * k_{xx}(0) + f_{cor2}(r) k(r_0)\,. \tag{4}$$

The correctness of this form will be however finally justified (or modified) according to the further experimental or/and theoretical results on the bearing prototype.

Let us choose the first correction function $f_{cor1}(r)$ monotone decreasing from value 1 in the bearing centre (r = 0) and zero value at $r \rightarrow r_h$:

$$f_{cor1}(r) = (1 - r / r_h)^2\,. \tag{5}$$

The second correction function $f_{cor2}(r)$ is selected in the form of hyperbolic function, which approximately expresses the stiffness rise of the bearing with increase of eccentricity. Free parameters s, p, h in equation (3) can be ascertained from the following conditions:

a) Zero values $f_{cor2}(0) = 0$ in the centre of bearing.

b) Value of entire stiffness $k_{xx}(r) = k_{xx}(r_0)$ in the point of stationary state.r_0.

c) The progressivism of characteristics i.e. the value of correction function infinitely near to the boundary $r \rightarrow r_h$ can be given by choosing the ratio $k_{xx}(r_h) / k_{xx}(r_{x0})$.

Condition a) gives $$\frac{s}{p + r_h} = h\,. \tag{6}$$

The full expression (4) has to be used for fulfilling the condition b). The load 40 N on the journal bearing causes the eccentricity r_0 = 0.25 r_h = 12.5 μm at 30000 rpm. The vertical stiffness of air film in this position is $k_{xx}(r_0)$ = 11520 N/mm. Unloaded journal has in the centre of bearing (r = 0) vertical stiffness $k_{xx}(0)$ = 9524 N/mm.

Introducing (4),(5), (6) into (3) gives in the point $r = r_0$

$$\mathrm{k_{xx}}(\mathrm{r_0}) = (1 - r_0 / r_h)^2 * \mathrm{k_{xx}}(0) + (\frac{s}{p + r_h - r_0} - \frac{s}{p + r_h}) * k_{xx}(\mathrm{r_0}) \ , \tag{7}$$

from which the parameter *s* can be determine as a function of parameter *p*:

$$s = \frac{[1 - (1 - r_0 / r_h)^2 k_{xx}(0) / k_{xx}(r_0)] * (p + r_h - r_0) * (p + r_h)}{r_0} \ . \tag{8}$$

The last condition c) can be formulated by means of equations (5), (6), in which we select the increase of stiffness $k_{xx}(r)$, near the contact of journal with the surface of bearing shell, e.g. $f_{cor2}(r_h) = 10$. Then

$$10 = \frac{s}{p} - \frac{s}{p + r_h}, \qquad s = \frac{10 p (p + r_h)}{r_h} . \tag{9}$$

The value of parameter *p* can be ascertained after introducing *s* into (8).:

$$p = \frac{[1 - (1 - r_0 / r_h)^2 k_{xx}(0) / k_{xx}(r_0)] * r_h (r_h - r_0)}{10 r_0 - [1 - (1 - r_0 / r_h)^2 k_{xx}(0) / k_{xx}(r_0)] * r_h} \ . \tag{10}$$

For the before mentioned values $f_{cor2}(r_h) = 10$, r_0 = 12.5 μm, r_h = 50 μm, $k_{xx}(r_0)$ = 11520 N/mm. $k_{xx}(0)$ = 9524 N/mm, the parameters *p, s, h* are *p* = 10.209 μm, *s* = 122.95 μm, *h* = 2.042 and correction functions:

. $f_{cor1}(r) = (1 - r / 50)^2$, $\qquad f_{cor2}(r) = \frac{122.95}{60.209 - r} - 2.042$.

The full correction function is

$$\mathrm{f_{cor}}(r) = (1 - r / r_h)^2 + (\frac{s}{p + r_h - r} - \frac{s}{p + r_h}) * k_{xx}(\mathrm{r_0}) / \mathrm{k_{xx}}(0) \ , \tag{11}$$

which after using above mentioned values *s, p*, r_h, $k_{xx}(r_0)$ and , $k_{xx}(0)$ gives

$$\mathrm{f_{cor}}(r) = (1 - r / 50)^2 - 1.6880 + 2.0326 / (1.2042 - r / 50) \ . \tag{11a}$$

The vertical stiffness of the aerodynamic bearing at 30000 rpm is

$$k_{xx}(r) = (1 - r/50)^2 * 9524 + (\frac{122.95}{60.209 - r} - 2.042) * 11520, \tag{12}$$

where $k_{xx}(r)$ is in N/mm, *r* in μm. This formula will be used in the further solution.

Corrections functions and vertical stiffness are shown in Fig. 5 and Fig. 6 both versus radius of deflection from the centre of bearings. Two other curves for $f_{cor2}(r_h) = 5$ and 15 are drawn there as well; it enables to estimate the possibility of progressive properties control.

The radial restoring aerodynamic force near to the position $r = r_h$ is given by the integral

$$F_r = \int_0^{r_h} k_{xx}(r) dr = \int_0^{r_h} [\ f_{cor1}(r) * k_{xx}(0) + f_{cor2}(r) k(r_0)] dr =$$

$$= k_{xx}(0) * \int_0^{r_h} f_{cor1}(r) dr + k_{xx}(r_0) * \int_0^{r_h} f_{cor2}(r) dr = k_{xx}(0) * g_{cor1} + k_{xx}(r_0) * g_{cor2} \tag{13}$$

where $$\vartheta_{cor1} = \int_0^{r_h} f_{cor1}(r)dr, \qquad \vartheta_{cor2} = \int_0^{r_h} f_{cor2}(r)dr. \qquad (13a)$$

The integration of (19a) gives

$$\vartheta_{cor1} = \int_0^{r_h} f_{cor1}(r)dr = \int_0^{r_h} (1 - r/r_h)^2 dr = r_h/3, \quad . \qquad (13b)$$

$$\vartheta_{cor2} = \int_0^{r_h} f_{cor2}(r)dr = \int_0^{r_h} (\frac{s}{p + r_h - r} - h)dr = s\ln((p + r_h)/p) - hr_h.$$

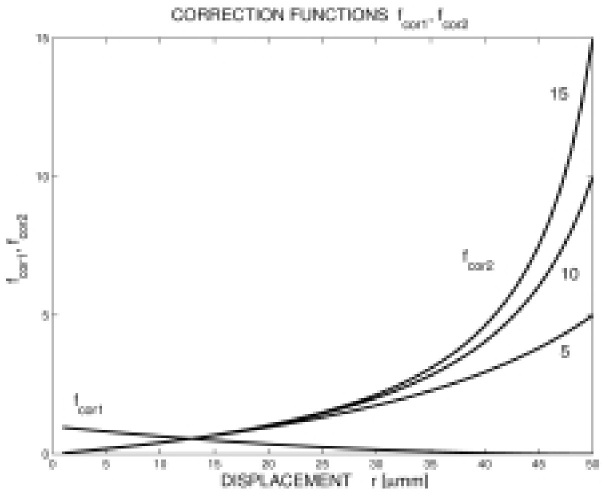

Fig. 5

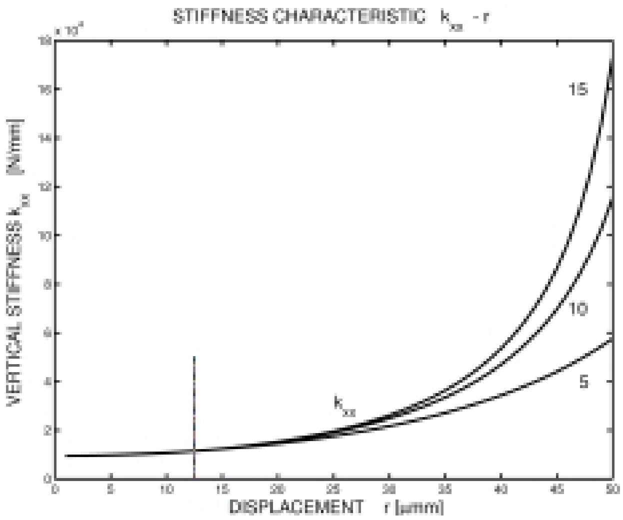

Fig. 6

The radial restoring aerodynamic force at extreme displacement $r = r_h$ is then

$$F_r = k_{xx}(0) * g_{cor1} + k_{xx}(r_0) * g_{cor2} =$$
$$= k_{xx}(0) * r_h / 3 + k_{xx}(r_0) * (s\ln((p + r_h)/p) - hr_h) = 1496 \text{ N}. \qquad (14)$$

The rubbing, i.e. undesirable contact of rotor journal with the inner surface of bearing bush occur when $r \geq r_h$. Aerodynamic forces are during this contact neglected against the elastic forces and therefore the characteristic of rubbing can be modeled by simple linear viscous-elastic matrix function:

$$\mathbf{F}_{imp} = (F_r + k_{rh}(r - r_h))\mathbf{X}/r - b_{rh} d\mathbf{X}/dr\,, \qquad \text{where is } \mathbf{F}_{imp} = \begin{bmatrix} F_x \\ F_y \end{bmatrix}, \quad \mathbf{X} = \begin{bmatrix} x \\ y \end{bmatrix} \qquad (15)$$

and where the slope k_{rh} is very great, several times greater than the maximum of vertical aerodynamic stiffness before the contact $k_{xx}(r_h) = k_{xx}(r_0)\, f_{cor2}(r_h) = 115200$ N/mm. Let it is k_{rh}=300000 N/mm. The characteristic of the aerodynamic forces (equations 11, 12) valid for range $r \in (0, r_h)$ must be completed by the characteristic (15) for the impacts of journal with the inner surface of bearing shell. One type of increase of vertical stiffness k_{xx} with displacement r is shown in Fig. 7.

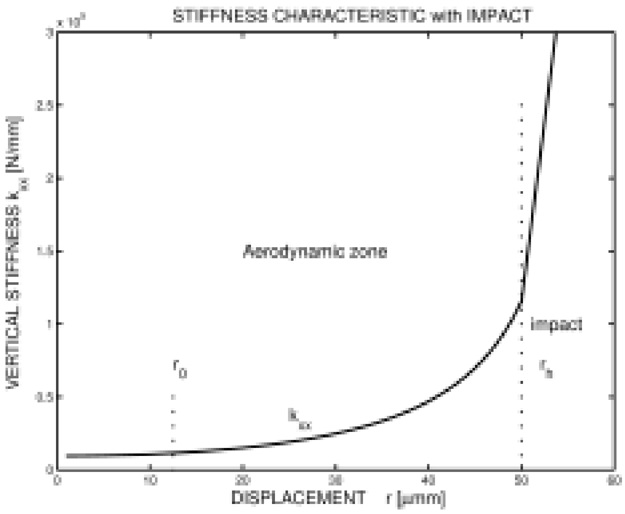

Fig. 7

The second task is the extension of stiffness description from the vertical radius r onto general position given by the angle $\varphi \in (0,2\pi)$. This can be realized multiplying stiffness and damping matrices by the plane rotation matrix

$$\mathbf{C} = \begin{bmatrix} \cos(\varphi) & \sin(\varphi) \\ -\sin(\varphi) & \cos(\varphi) \end{bmatrix},$$

which can be expressed in the coordinates x,y by

$$\mathbf{C} = \begin{bmatrix} x & y \\ -y & x \end{bmatrix} / r, \qquad r = \sqrt{x^2 + y^2}\,. \qquad (16)$$

The motion of mass point m in the aerodynamic bearing with taking also the impacts into account is

$$\mathbf{M}\ddot{\mathbf{X}} + f_{cor}(r)\mathbf{C}(x,y)\mathbf{K}(\omega)\mathbf{X} + f_{cor}(r)\mathbf{C}(x,y)\mathbf{B}(\omega)\dot{\mathbf{X}} = \mathbf{O}(\omega t) me\omega^2 + \mathbf{F}(t)\,, \qquad (17)$$

where the matrices $\mathbf{M}$, $\mathbf{X}$, $\mathbf{C}(x,y)$, $\mathbf{K}(\omega)$, $\mathbf{B}(\omega)$, $\mathbf{O}(\omega t)$, $\mathbf{F}(t)$ are defined in (2), (16) and $f_{cor}(r)$ in (11), (15).

5. Examples

The equation (17) enables to investigate the properties of mathematical model of aerodynamic bearing at various conditions. The next record is for eccentricity e = 5 μm.

In Fig. 8 are time histories of displacements x, y velocities $v = dx/dt$, $vy = dy/dt$ and one component of exciting force $F_x = me\omega^2 \cos\omega t$ during the first 12 revolutions. The same oscillations are plotted in the trajectories plane x, y in Fig. 9.

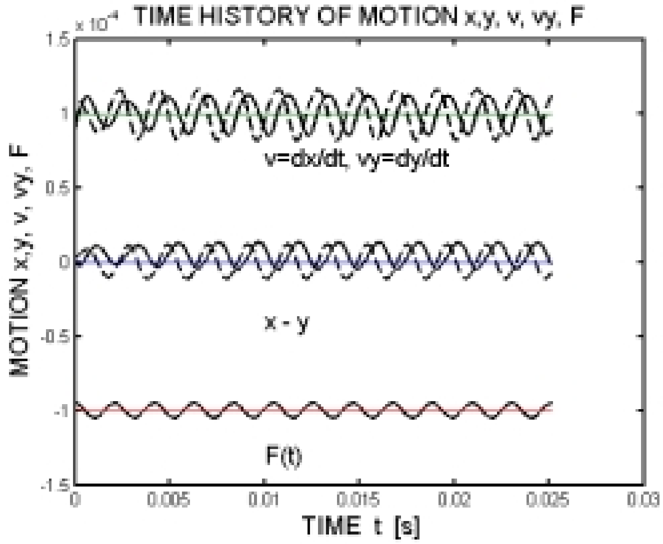

Fig. 8

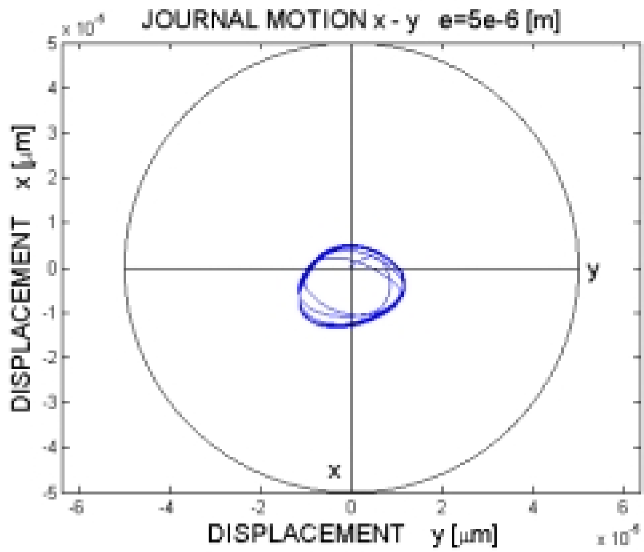

Fig. 9

It is clear the after some transient oscillations causes by initial conditions, the oscillations stabilize the periodic type, nearly harmonic.

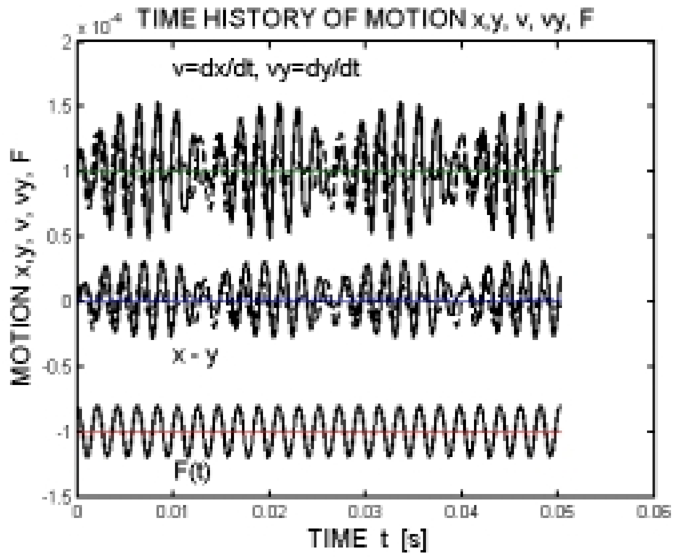

Fig. 10

The increasing of excitation force by means of eccentricity e=20μm changes the type of oscillation from periodic motion to the quasi-periodic or chaotic oscillations with beats as it is shown in Fig. 10 where again the motions x,y and velocities dx/dt, dy/dt are plotted in the time interval of 24 revolutions. The record in trajectories x,y plane fulfills now the whole area inside the boundary, what again proves the irregular motion.(see Fig.11)

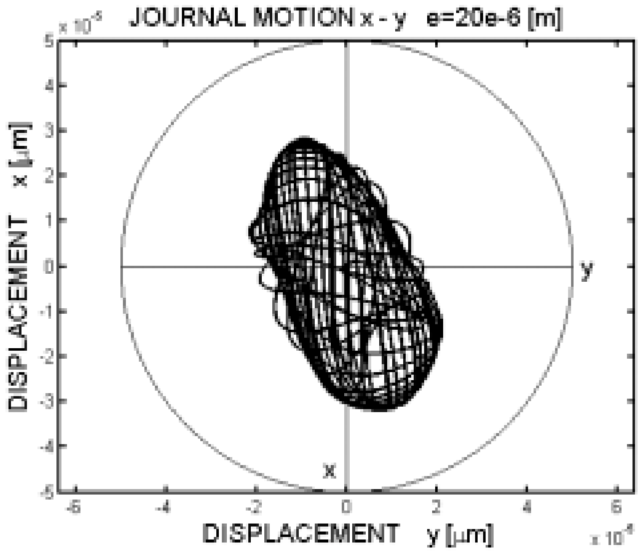

Fig.11

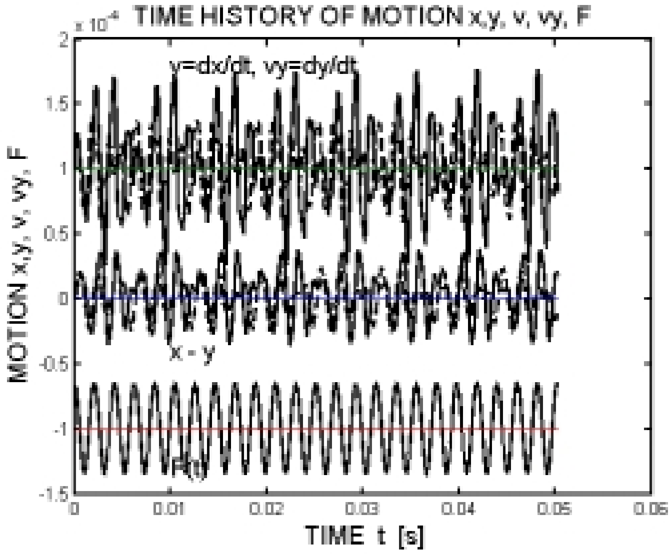

Fig. 12

The nonlinearity of the aerodynamic bearing accentuates at greater eccentricity *e*=35µm. The motion during the 24 periods of excitation is quasi-periodic with periods near to the $T \doteq 3\frac{2\pi}{\omega}$ as it is seen from Fig. 12. That this motion is not quite exact periodic it is evident from the Fig. 13, where a small revolutions of record-loops is clearly seen

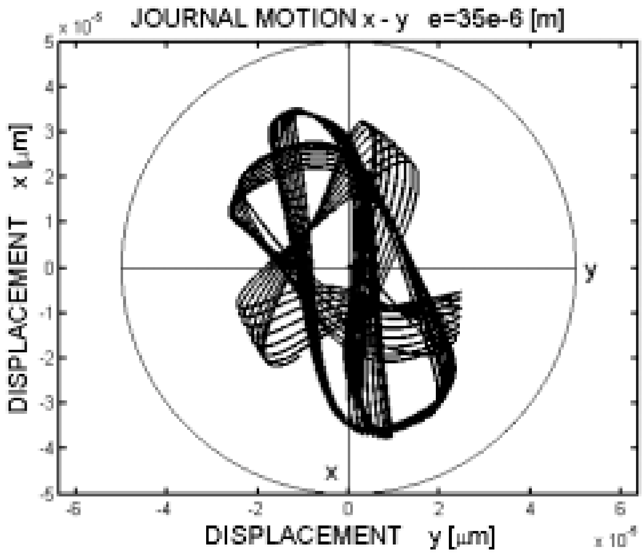

Fig. 13

The influence of nonlinearity can be studied also by means of increasing amplitude of exciting force. Three records in plane x, y are calculated for the static force F_{stat} = 600 N and three eccentricities e = 40; 60; 80 μm and are plotted in trajectories plane x, y in Fig. 14. The smallest loop belongs to the smallest eccentricity e = 40, the greatest loop to the greatest eccentricity e = 80 μm. All stationary states are periodic with the period of exciting force and certain one strong harmonic component of order 3, as it is seen from Fig. 15.

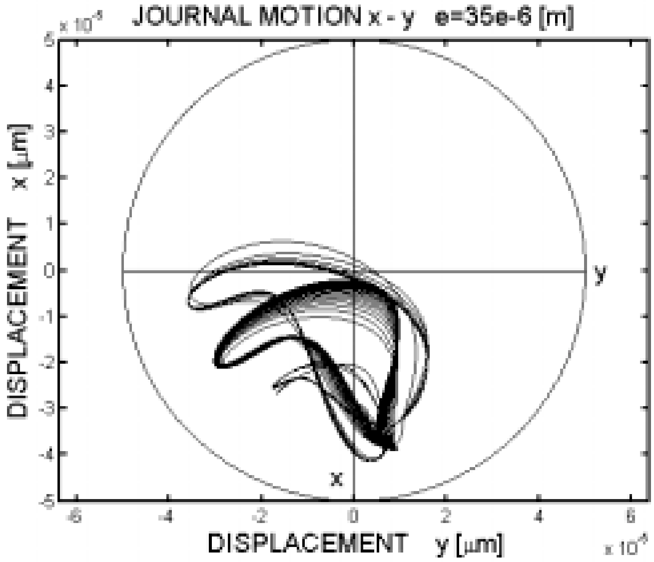

Fig. 14

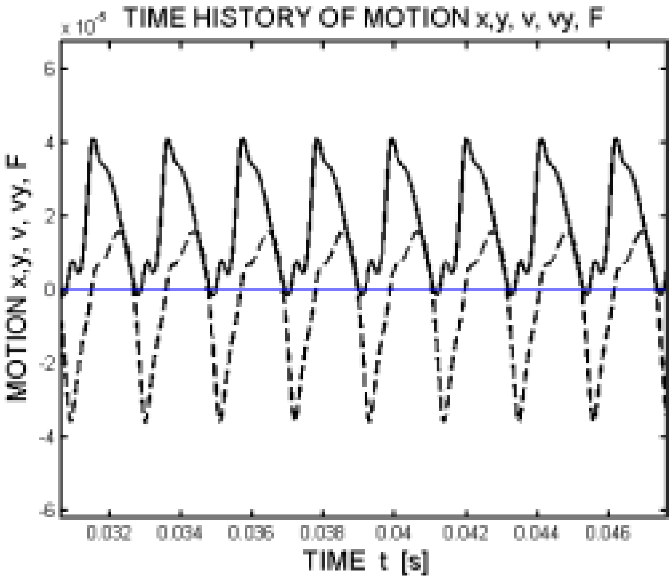

Fig. 15

6. Conclusion

Mathematical model of journal motion in aerodynamic bearings including properties of three tilting pads was derived for the large amplitudes in the clearance of bearing. Dynamic characteristics of such type of bearings are very strongly influenced by inertia properties of tilting pads – the stiffness and dynamic matrices are non-symmetric and their elements are non-monotonous functions of angular frequency ω.

Dynamic characteristics, given for the selected positions of journal in the form of linear stiffness and damping matrices were extended by using special correction function $f_{cor}(r)$ and by the plane rotation matrix **C** on the entire area of bearing clearance $0 < r < r_h$, and for the angular positions $-\pi < \varphi \leq \pi$.

This correction function consists of two parts, the first one proportional to the properties of unloaded bearing, the second one to the properties of loaded bearing.

Application of this approximate mathematical model of stiffness and damping properties of aerodynamic bearing is presented on examples, where the influence of various rotor eccentricity and external loading is shown.

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