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DIFFERENCES IN APPLICATION OF THE WAVE OR SLEEVE SPRING TO THE VIBROISOLATION SYSTEM

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Summary: *In the paper is describing difference between application wave and sleeve springs in the dynamical system. Are bring in reason, witch complicated the application sleeve spring in to mathematical model of the dynamical system.*

1. Introduction

The guide mechanism of the vibro-isolation system (driver's seat, ambulance couch) makes it possible join to the air springs with a several nonlinear transmission, where, with knowledge of the burdening characteristic of the applied spring, given to designer possibility to the system tuning (natural frequency, etc.).

It is true, that it depends on the type of the pneumatic spring is using, since as model admittance, so experimental admittance, is different in application sleeve or wave spring.

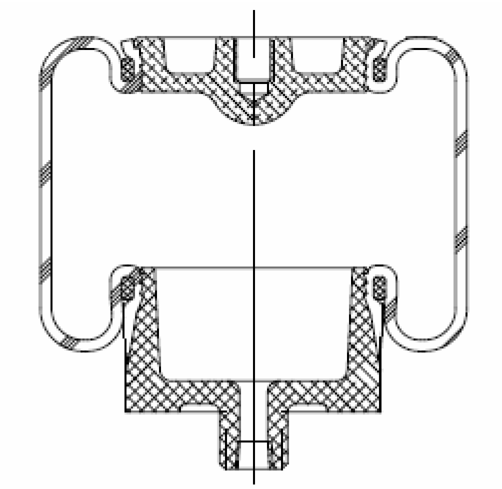
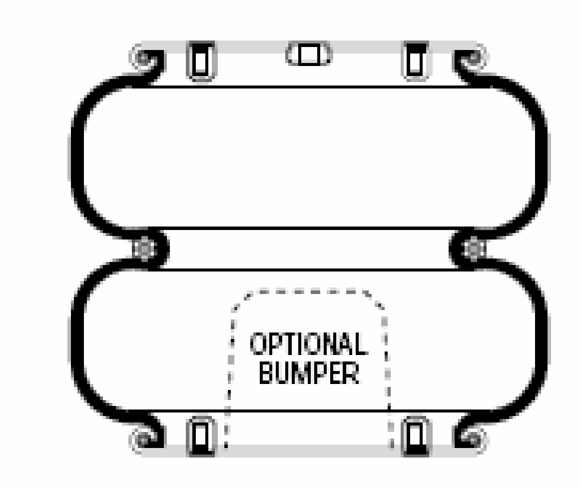


Fig.1 Sleeve air spring from Goodyear



Fif.2 Wave spring from Firestone

2. General relation

If we start from presumption, that duration of the spring, generally connected between two components of the guide mechanism, is function generally coordinate α and for conversion γ is valid:

$$l = l(\alpha) \qquad \gamma = \frac{dl}{d\alpha} \tag{1}$$

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For both springs are true, if we suppose the isothermal process, current equilibrium:

$$p\frac{dV}{dt}+V\frac{dp}{dt}=R_pT\frac{dm}{dt} \tag{2}$$

Where volume $V=V(p,l)$ if function of the gauge pressure p and length of the spring l, R_p gas constant, T absolute temperature and dm/dt is mass flow (flow to $(+)$ or from $(-)$ the spring). If is spring closed, is right side of the equation zero and differentials dV and dp are frozen by the relation:

$$p\left[\frac{\partial V}{\partial p}\cdot\frac{dp}{dt}+\frac{\partial V}{\partial l}\cdot\frac{dl}{dt}\right]+V\frac{dp}{dt}=0 \tag{3}$$

and it is valid:

$$dp=\frac{-p\dfrac{\partial V}{\partial l}}{p\dfrac{\partial V}{\partial p}+V}\cdot dl \tag{4}$$

3. Wave spring

In the case of the wave spring, given us her force relation

$$F=F(p,l)=p\cdot S_{ef}(l) \tag{5}$$

which is possible theoretically deduce (in ref. Krejčíř) and experimentally confirm. By introduction effective area S_{ef} are variables p and l separated. Next is valid

$$V=V(l)=\int_{l_0}^{l}S_{ef}\,dl+V_0 \qquad \frac{dV}{dl}=S_{ef} \tag{6}$$

Relation (4) is reduced on

$$dp=\frac{-pS}{V}dl \tag{7}$$

For the closed spring is

$$F(p,l)=p_0S_0+\left(p_0\frac{dS}{dl}-\frac{pS^2}{V}\right)dl \tag{8}$$

From relation for elementary work

$$dW=F(p,l)dl=M(p,\alpha)d\alpha \tag{9}$$

we deduce

$$M_\alpha=F(p,l(\alpha))\cdot\frac{dl}{d\alpha} \tag{10}$$

After substitution from (8) is

$$M_\alpha=p_0S_0\frac{dl}{d\alpha}+\left[p_0\left(\frac{dS}{dl}-\frac{S_0^2}{V}\right)\left(\frac{dl}{d\alpha}\right)^2\right]d\alpha \tag{11}$$

and ending torsion stiffness is

$$k_T = p_0 \left(\frac{dS}{dl} - \frac{S_0^2}{V} \right) \left(\frac{dl}{d\alpha} \right)^2 \tag{12}$$

4. Sleeve air spring

For sleeve air spring is impossible (add. equation (3)) variables separated, introduction of effective area is only formal and haven't practice sense. So equations (5) and (6) are invalid. Force and volume are function of the gauge pressure *p* and length *l*.

$$F = F(p,l) \qquad\qquad V = V(p,l) \tag{13}$$

If we expand expression (13) to Taylor series and confine to members to first order, get:

$$F(p,l) = F(p_0,l_0) + \left.\frac{\partial F}{\partial l}\right|_0 dl + \left.\frac{\partial F}{\partial p}\right|_0 dp \tag{14}$$

and for closed springs we express *dp* as (4). After substitute to (14) is

$$F(p,l) = F(p_0,l_0) + \left(\left.\frac{\partial F}{\partial l}\right|_0 + \frac{\partial F}{\partial p} \cdot p \frac{\partial V}{\partial l} \left. \left(p \frac{\partial V}{\partial p} + V \right)^{-1} \right|_0 \right) dl \tag{15}$$

If is a sleeve spring build-in mechanism, his location is describe by general coordinate α, is valid for elementary work equation (9), then for applied general moment is

$$M(p,\alpha) = F(p,l)\frac{dl}{d\alpha} = F(p_0,l_0)\frac{dl}{d\alpha} + \left[\frac{\partial F}{\partial l} + \frac{\partial F}{\partial p} \cdot p \frac{\partial V}{\partial l} \left(p \frac{\partial V}{\partial p} + V \right)^{-1} \right] \left(\frac{dl}{d\alpha} \right)^2 d\alpha \tag{16}$$

and for torsion stiffness is valid

$$k_T = \left[\frac{\partial F}{\partial l} + \frac{\partial F}{\partial p} \cdot p \frac{\partial V}{\partial l} \left(p \frac{\partial V}{\partial p} + V \right)^{-1} \right] \left(\frac{dl}{d\alpha} \right)^2 \tag{17}$$

5. Conclusion

For identification sleeve type spring is demand from experiments value establish two quantities $F = F(p,l)$ (in fig.3 are dependencies force on the pressure or length for one (from more) initial value) and $V = V(p,l)$(in fig.4 are dependencies volume on the pressure or length for one initial value), to be possible with solve of the ending stiffness determine appropriate to partial differentiation with gauge pressure and length.

In other site for identification wave spring is possible for one value of the gauge pressure locate the dependency between force and height and from here deduced dependency between effective area and height. In next step just in one point (preferably in base position) determine $V_0 = V(l_0)$. The formulas are valid only for axis loading of the spring. For non-axis loading are valid formulas in work Doc. Marvalová (ref. Marvalová).

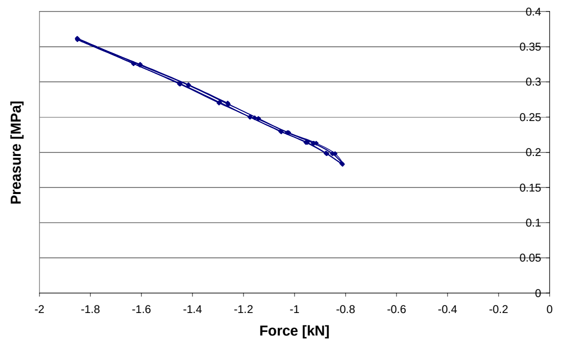


Fig.3a

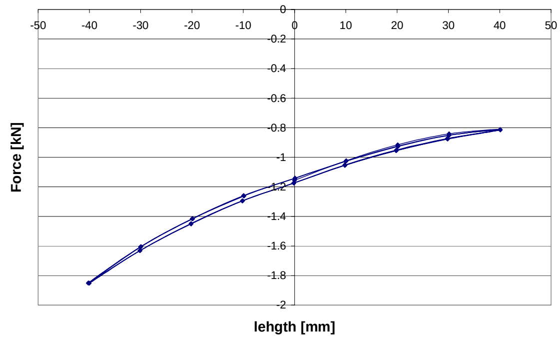


Fig.3b

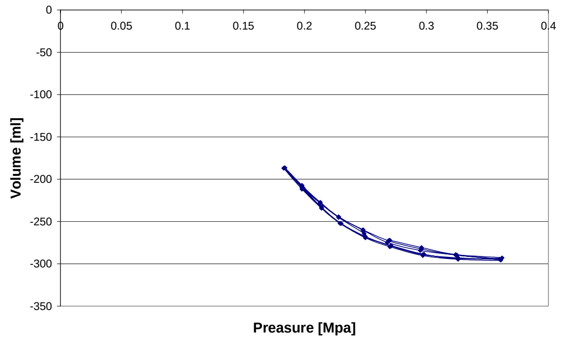


Fig.4a

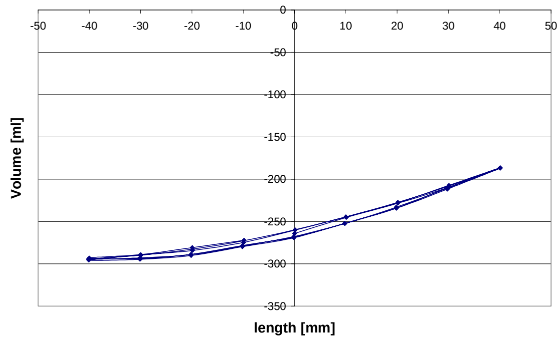
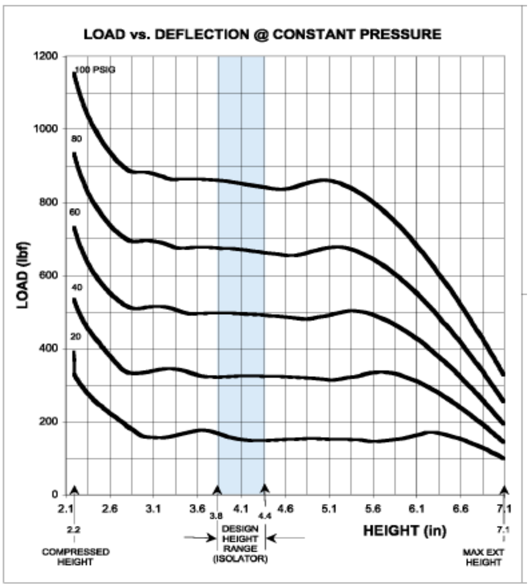


Fig.4b



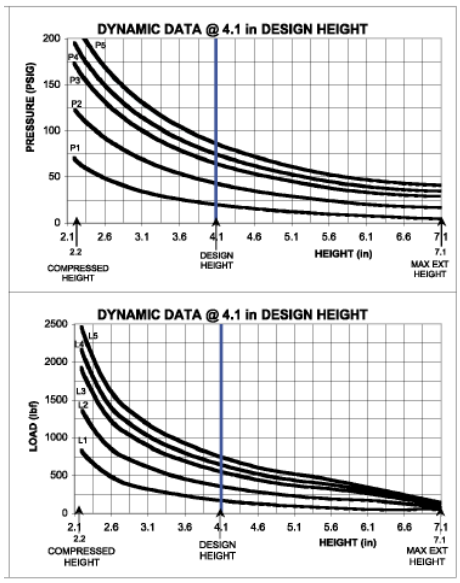


Fig.5 Constant pressure characteristic (sleeve)

Fig. 6 Dynamic characteristics (sleeve)

6. Acknowledgment

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7. References

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