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ON APPLICATIONS OF GENERALIZED FUNCTIONS TO CALCULATION OF BEAM-COLUMN DESIGN ELEMENTS

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Summary: *The classical mathematical model of beam-columns contains derivatives of functions for the bending moment, the shear force, the axial force, the slope, and the deflection, that are not defined at points in which internal supports or concentrated lateral and axial loading or internal hinges or internal sliding connections are situated. In order that the mathematical model of the beam-columns may be valid also at these points of discontinuity we use generalized functions and derive the generalized mathematical model in the form of a system of ordinary differential equations. We use the Laplace transform for solving this generalized model with constant axial tension force. The solution found is the generalization of the classical initial parameters method because it covers also discontinuous beam-columns, i.e. with internal hinges or internal sliding connections.*

1. Introduction

The classical mathematical model of beam-column bending in the form of the system of ordinary differential equations (SODE) contains classical derivatives that are not defined at points of discontinuity of unknown functions such as the shear force, the bending moment, the slope, and the deflection. Such discontinuities occur in calculation experience at points in which concentrated forces, concentrated moments, internal supports, internal hinges or internal shear-free connections are situated.

In order that the mathematical model for beam-column bending may hold true also at the points of discontinuity mentioned we use distributional derivatives for unknown functions and derive the generalized SODE. The generalized mathematical model for beam-column bending contains the Dirac distribution at various places so as to represent corresponding discontinuity. The discontinuity in the shear force may also occur at an internal hinge and the discontinuity in the bending moment may also occur at an internal sliding connection by virtue of the axial force. The discontinuity in the equivalent distributed force, in the flexural stiffness or in the axial force may be represented through the use of the Heaviside's unit step function.

The general solution to the generalized model of the beam-column bending may be found by means of the Laplace transform for prismatic beam-column with constant axial force.

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2. The Classical Mathematical Model of Plane Bending of Straight Slender Beam-Columns

This classical model (Němec, Dvořák, Höschl, 1989) may be formulated as the system of ordinary differential equations (SODE) of the first order Eqs. (1) to (4), and was derived under the following assumptions: (i) an infinitesimal element of a beam-column was cut out in the deformed shape, (ii) the Bernoulli-Navier hypothesis holds true, i.e. the cross sections of the beam-column remain always plane and perpendicular to the beam-column axis.

$$\frac{d}{dx}\mathrm{T}(x) = -q_n(x) + \left(\frac{d}{dx}(\mathrm{N}(x)\,\phi(x))\right) \tag{1}$$

$$\frac{d}{dx}\mathrm{M}(x) = \mathrm{T}(x) + q_m(x) \tag{2}$$

$$\frac{d}{dx}\phi(x) = \frac{\mathrm{M}(x)}{E\,\mathrm{J}(x)} \tag{3}$$

$$\frac{d}{dx}\mathrm{v}(x) = \phi(x) \tag{4}$$

where

$\mathrm{T}(x)$	shear force [*N*] (positive one causes an angle of turn clockwise of the tangent line to the beam-column axis)
$\mathrm{M}(x)$	bending moment [*Nm*] (positive one causing positive curvature)
$\phi(x)$	slope [*rad*] (positive direction counterclockwise)
$\mathrm{v}(x)$	deflection [*m*] (positive direction upward)
$q_n(x)$	equivalent distributed force [*N/m*] (positive direction downward)
$q_m(x)$	equivalent distributed moment [*Nm/m*] (positive direction clockwise)
$\mathrm{J}(x)$	area moment of inertia [m^4]
$\mathrm{N}(x)$	axial force of a beam-column (positive as tensile) [N]
E	Young's modulus [*Pa*]
x	longitudinal axis of a beam-column

Remark.

In place of the equilibrium Eqs. (1) and (2), the following Eqs. (5) and (6) are frequently used in technical literature

$$\frac{d}{dx}\mathrm{T}(x) = -q_n(x) \tag{5}$$

$$\frac{d}{dx}\mathrm{M}(x) = \mathrm{T}(x) + N\left(\frac{d}{dx}\mathrm{v}(x)\right) \tag{6}$$

though they give incorrect diagram of the shear force (where N(x)=N=const.), which may be checked up by means of the finite element method.

3. The Generalized Mathematical Model of Bending of Straight Bernoulli-Navier Beam-Columns

The Eqs. (1) to (4) contain classical derivatives that are not defined at points of discontinuity of the unknowns. In order to rectify this ineffectiveness, that comes out in practical calculations, we have used the distributional derivative for unknowns hence we have developed the generalized model of plane bending for straight slender beam-columns, Eqs. (7) to (10).

$$\frac{d}{dx}\mathrm{T}(x) = -q_n(x) + \left(\frac{d}{dx}(\mathrm{N}(x)\,\phi(x))\right) - \left(\sum_{i=1}^{n1} F_i\,\mathrm{Dirac}(x-a_i)\right) \tag{7}$$

$$\frac{d}{dx}\mathrm{M}(x) = \mathrm{T}(x) + q_m(x) + \left(\sum_{i=1}^{n2} M_i\,\mathrm{Dirac}(x-b_i)\right) + \left(\sum_{i=1}^{n4} \Delta_i\,\mathrm{N}(x)\big|_{x=d_i}\,\mathrm{Dirac}(x-d_i)\right) \tag{8}$$

$$\frac{d}{dx}\phi(x) = \frac{\mathrm{M}(x)}{E\,\mathrm{J}(x)} + \left(\sum_{i=1}^{n3} \Phi_i\,\mathrm{Dirac}(x-c_i)\right) \tag{9}$$

$$\frac{d}{dx}\mathrm{v}(x) = \phi(x) + \left(\sum_{i=1}^{n4} \Delta_i\,\mathrm{Dirac}(x-d_i)\right) \tag{10}$$

where

Dirac(x-a_i)	Dirac distribution (Dirac-delta function or unit-impulse function) at $x = a_i$
F_i	magnitude of the *ith* concentrated force [*N*] (positive direction of a concentrated lateral force is downward)
M_i	magnitude of the *ith* concentrated moment [*Nm*] (positive direction of a concentrated moment is clockwise)
Φ_i	magnitude of a jump discontinuity in slope of a Bernoulli-Navier beam-column in its *ith* internal hinge [*rad*]
Δ_i	magnitude of a jump discontinuity in deflection of a Bernoulli-Navier beam-column in its *ith* internal sliding connection [*m*]
$0 < a_i$	distance between the *ith* concentrated lateral force and the left end of the beam-column [*m*]
$0 < b_i$	distance between the *ith* concentrated moment and the left end of the beam-column [*m*]
$0 < c_i$	distance between the *ith* internal hinge and the left end of the beam-column [*m*]
$0 < d_i$	distance between the *ith* internal sliding connection and the left end of the beam-column [*m*]
n_1	number of concentrated lateral forces except for end ones
n_2	number of concentrated moments except for end ones
n_3	number of internal hinges
n_4	number of internal sliding connections
$\mathrm{T}(a_1 + 0)$	a directional limit of T(x) at $x = a_1$ taken from the right
$\mathrm{T}(a_1 - 0)$	a directional limit of T(x) at $x = a_1$ taken from the left
$\mathrm{M}(b_1 + 0)$	a directional limit of M(x) at $x = b_1$ taken from the right
$\mathrm{M}(b_1 - 0)$	a directional limit of M(x) at $x = b_1$ taken from the left
$\phi(c_1 + 0)$	a directional limit of $\phi(x)$ at $x = c_1$ taken from the right

$\phi\,(c_1 - 0)$	a directional limit of $\phi\,(x)$ at $x = c_1$ taken from the left
$v(d_1 + 0)$	a directional limit of v(x) at $x = d_1$ taken from the right
$v(d_1 - 0)$	a directional limit of v(x) at $x = d_1$ taken from the left

Developing Eqs. (7) to (10), we start with composition of equilibrium equations for infinitesimal elements with a concentrated lateral load or with an internal kinematic pair cut out of a beam-column.

Let us suppose that the shear force T(x) has a jump discontinuity at point $x = a_1$ of magnitude

$$\mathrm{T}(a_1 + 0) - \mathrm{T}(a_1 - 0) = -F_1 \qquad (11)$$

caused by a concentrated lateral force. Then the distributional derivative of T(x) would be in the case of only one concentrated lateral force

$$\mathrm{T}' = -q_n(x) + \left(\frac{d}{dx}(\mathrm{N}(x)\,\phi(x))\right) + [\mathrm{T}(a_1 + 0) - \mathrm{T}(a_1 - 0)].\ \mathrm{Dirac}(x - a_1) \qquad (12)$$

Substituting here Eq. (11), we arrive at

$$\mathrm{T}' = -q_n(x) + \left(\frac{d}{dx}(\mathrm{N}(x)\,\phi(x))\right) - F_1\ .\ \mathrm{Dirac}(x - a_1) \qquad (13)$$

Let the bending moment M(x) have a jump discontinuity at $x = b_1$ of magnitude

$$\mathrm{M}(b_1 + 0) - \mathrm{M}(b_1 - 0) = M_1 \qquad (14)$$

caused by a concentrated force couple. Then the distributional derivative of M(x) would be in the case of only one concentrated moment

$$\mathrm{M}' = \mathrm{T}(x) + q_m(x) + [\mathrm{M}(b_1 + 0) - \mathrm{M}(b_1 - 0)]\ .\ \mathrm{Dirac}(x - b_1) \qquad (15)$$

Introducing here Eq. (14), we arrive at

$$\mathrm{M}' = \mathrm{T}(x) + q_m(x) + M_1\ .\ \mathrm{Dirac}(x - b_1) \qquad (16)$$

Let the slope $\phi(x)$ have a jump discontinuity at $x = c_1$ of magnitude

$$\phi\,(c_1 + 0) - \phi\,(c_1 - 0) = \Phi_1 \qquad (17)$$

as a result of placing of an internal hinge. Then the distributional derivative of $\phi(x)$ would be in the case of only one internal hinge

$$\phi\,' = \frac{\mathrm{M}(x)}{E\,\mathrm{J}(x)} + [\,\phi\,(c_1 + 0) - \phi\,(c_1 - 0)].\ \mathrm{Dirac}\,(x - c_1) \qquad (18)$$

Introducing here Eq. (17), we come to

$$\phi' = \frac{M(x)}{E\,J(x)} + \Phi_1 \,.\, \text{Dirac}(x - c_1) \tag{19}$$

The axial force of the beam-column is N(c_1) at the internal hinge. Then there is a jump discontinuity in the shear force of magnitude N(c_1).sin(Φ_1) at this internal hinge of a beam-column. Assuming that Φ_1 <<1, we can use the following equation N(c_1).sin(Φ_1) = N(c_1). Φ_1 . Introducing Eq. (19) into Eq. (13), the impulse of magnitude N(c_1). Φ_1 will appear automatically. The unknown value Φ_1 may be determined by means of the deformation condition for the internal hinge (Sobotka, 2006).

Let the deflection $v(x)$ have a jump discontinuity at $x = d_1$ of magnitude

$$v(d_1 + 0) - v(d_1 - 0) = \Delta_1 \tag{20}$$

as a result of placing of an internal sliding connection. Then the distributional derivative of v(x) would be in the case of only one internal sliding connection

$$v' = \phi(x) + [v(d_1 + 0) - v(d_1 - 0)] \,.\, \text{Dirac}(x - d_1) \,. \tag{21}$$

Introducing here Eq. (20), we arrive at

$$v' = \phi(x) + \Delta_1 \,.\, \text{Dirac}(x - d_1) \tag{22}$$

The axial force of the beam-column is N(d_1) at the internal sliding connection. Then there is a jump discontinuity of magnitude Δ_1 .N(d_1) in the bending moment at this internal sliding connection. Introducing this impulse into Eq. (16), we come to the final form for the distributional derivative of M(x) in the case of one internal sliding connection and one concentrated moment along a beam-column

$$M' = T(x) + q_m(x) + M_1 \,.\, \text{Dirac}(x - b_1) + \Delta_1 \,.N(d_1).\text{Dirac}(x - d_1) \tag{23}$$

The unknown value Δ_1 may be determined by means of the deformation condition for the internal sliding connection (Sobotka, 2006).

4. The Laplace Transform of the Generalized SODE of Straight Prismatic Beam-Columns with Constant Tensile Axial Force (*N(x) = N*)

The equations (7) to (10) will be transformed as follows

$$p\,\text{laplace}(T(x), x, p) - T(0) = \\ -N\,\phi(0) - \left(\sum_{i=1}^{n1} F_i\, \mathbf{e}^{(-p\,a_i)}\right) - \text{laplace}(q_n(x), x, p) + N\,p\,\text{laplace}(\phi(x), x, p) \tag{24}$$

$$p\,\text{laplace}(\text{M}(x), x, p) - \text{M}(0) = \left(\sum_{i=1}^{n2} M_i\,\mathbf{e}^{(-p\,b_i)}\right) + N\left(\sum_{i=1}^{n4} \Delta_i\,\mathbf{e}^{(-p\,d_i)}\right) + \text{laplace}(\text{T}(x), x, p) + \text{laplace}(q_m(x), x, p) \tag{25}$$

$$p\,\text{laplace}(\phi(x), x, p) - \phi(0) = \left(\sum_{i=1}^{n3} \Phi_i\,\mathbf{e}^{(-p\,c_i)}\right) + \frac{\text{laplace}(\text{M}(x), x, p)}{E\,J} \tag{26}$$

$$p\,\text{laplace}(\text{v}(x), x, p) - \text{v}(0) = \left(\sum_{i=1}^{n4} \Delta_i\,\mathbf{e}^{(-p\,d_i)}\right) + \text{laplace}(\phi(x), x, p) \tag{27}$$

where

p	a variable for the Laplace transform
laplace	Laplace transform operator
laplace(f(x), *x*, *p*)	Laplace transform of f(x)
T(0), M(0), φ (0),v(0)	constants of integration in the form of initial parameters

5. The Laplace Transforms of Unknown Functions T(x), M(x), $\phi(x)$, v(x)

We use a substitution $N = E\,J\,\omega^2$ and determine the unknown Laplace transforms of T(x), M(x), $\phi(x)$, v(x) as solution to the system of linear algebraic Eqs. (24) to (27) as follows

$$\text{laplace}(\text{T}(x), x, p) = -\frac{p\,\text{laplace}(q_n(x), x, p)}{p^2 - \omega^2} - \frac{p\left(\sum_{i=1}^{n1} \frac{F_i}{\mathbf{e}^{(p\,a_i)}}\right)}{p^2 - \omega^2} + \frac{p\,\text{T}(0)}{p^2 - \omega^2} + \frac{p\left(\sum_{i=1}^{n3} \frac{\Phi_i}{\mathbf{e}^{(p\,c_i)}}\right) E\,J\,\omega^2}{p^2 - \omega^2} + \frac{\text{M}(0)\,\omega^2}{p^2 - \omega^2} + \frac{\left(\sum_{i=1}^{n2} \frac{M_i}{\mathbf{e}^{(p\,b_i)}}\right)\omega^2}{p^2 - \omega^2} + \frac{E\,J\,\omega^4\left(\sum_{i=1}^{n4} \frac{\Delta_i}{\mathbf{e}^{(p\,d_i)}}\right)}{p^2 - \omega^2} + \frac{\text{laplace}(q_m(x), x, p)\,\omega^2}{p^2 - \omega^2} \tag{28}$$

$$\mathrm{laplace}(\mathrm{M}(x), x, p) = \frac{p\,\mathrm{laplace}(q_m(x), x, p)}{p^2 - \omega^2} - \frac{\mathrm{laplace}(q_n(x), x, p)}{p^2 - \omega^2} + \frac{p\,\mathrm{M}(0)}{p^2 - \omega^2} + \frac{p\left(\sum_{i=1}^{n2} \frac{M_i}{\mathbf{e}^{(p\,b_i)}}\right)}{p^2 - \omega^2} - \frac{\sum_{i=1}^{n1} \frac{F_i}{\mathbf{e}^{(p\,a_i)}}}{p^2 - \omega^2} + \frac{\mathrm{T}(0)}{p^2 - \omega^2} + \frac{E\,J\,\omega^2\,p\left(\sum_{i=1}^{n4} \frac{\Delta_i}{\mathbf{e}^{(p\,d_i)}}\right)}{p^2 - \omega^2} + \frac{\left(\sum_{i=1}^{n3} \frac{\Phi_i}{\mathbf{e}^{(p\,c_i)}}\right) E\,J\,\omega^2}{p^2 - \omega^2} \quad (29)$$

$$\mathrm{laplace}(\phi(x), x, p) = \frac{\mathrm{T}(0)}{p\,E\,J\,(p^2 - \omega^2)} + \frac{\mathrm{M}(0)}{E\,J\,(p^2 - \omega^2)} + \left(\frac{p}{p^2 - \omega^2} - \frac{\omega^2}{p\,(p^2 - \omega^2)}\right)\phi(0) + \frac{p\left(\sum_{i=1}^{n3} \frac{\Phi_i}{\mathbf{e}^{(p\,c_i)}}\right)}{p^2 - \omega^2} + \frac{\sum_{i=1}^{n2} \frac{M_i}{\mathbf{e}^{(p\,b_i)}}}{E\,J\,(p^2 - \omega^2)} + \frac{\mathrm{laplace}(q_m(x), x, p)}{E\,J\,(p^2 - \omega^2)} - \frac{\mathrm{laplace}(q_n(x), x, p)}{p\,E\,J\,(p^2 - \omega^2)} + \frac{\omega^2\left(\sum_{i=1}^{n4} \frac{\Delta_i}{\mathbf{e}^{(p\,d_i)}}\right)}{p^2 - \omega^2} - \frac{\sum_{i=1}^{n1} \frac{F_i}{\mathbf{e}^{(p\,a_i)}}}{p\,E\,J\,(p^2 - \omega^2)} \quad (30)$$

$$\mathrm{laplace}(\mathrm{v}(x), x, p) = \frac{\mathrm{T}(0)}{p^2\,E\,J\,(p^2 - \omega^2)} + \frac{\mathrm{M}(0)}{p\,E\,J\,(p^2 - \omega^2)} + \left(-\frac{\omega^2}{p^2\,(p^2 - \omega^2)} + \frac{1}{p^2 - \omega^2}\right)\phi(0) + \left(\frac{p}{p^2 - \omega^2} - \frac{\omega^2}{p\,(p^2 - \omega^2)}\right)\mathrm{v}(0) + \frac{\sum_{i=1}^{n3} \frac{\Phi_i}{\mathbf{e}^{(p\,c_i)}}}{p^2 - \omega^2} - \frac{\mathrm{laplace}(q_n(x), x, p)}{p^2\,E\,J\,(p^2 - \omega^2)} + \frac{p\left(\sum_{i=1}^{n4} \frac{\Delta_i}{\mathbf{e}^{(p\,d_i)}}\right)}{p^2 - \omega^2} + \frac{\sum_{i=1}^{n2} \frac{M_i}{\mathbf{e}^{(p\,b_i)}}}{p\,E\,J\,(p^2 - \omega^2)} + \frac{\mathrm{laplace}(q_m(x), x, p)}{p\,E\,J\,(p^2 - \omega^2)} - \frac{\sum_{i=1}^{n1} \frac{F_i}{\mathbf{e}^{(p\,a_i)}}}{p^2\,E\,J\,(p^2 - \omega^2)} \quad (31)$$

6. The General Solution to the Generalized SODE, Eqs. (7) to (10), for Prismatic Beam-Columns with Constant Axial Tensile Force $N = E\,J\,\omega^2$

The unknown functions T(x), M(x), $\phi(x)$, v(x) may be determined by means of the inverse Laplace transform of Eqs. (28) to (31) as follows

$$
\begin{aligned}
\mathrm{T}(x) &= \mathrm{T}(0)\cosh(\omega\,x) + \omega\sinh(\omega\,x)\,\mathrm{M}(0)\\
&- \left(\sum_{i=1}^{n1} F_i\,\mathrm{Heaviside}(x-a_i)\cosh(\omega\,(x-a_i))\right)\\
&+ \omega\left(\sum_{i=1}^{n2} \mathrm{Heaviside}(x-b_i)\sinh(\omega\,(x-b_i))\,M_i\right)\\
&+ E\,J\,\omega^2\left(\sum_{i=1}^{n3} \cosh(\omega\,(x-c_i))\,\mathrm{Heaviside}(x-c_i)\,\Phi_i\right)\\
&+ E\,J\,\omega^3\left(\sum_{i=1}^{n4} \sinh(\omega\,(x-d_i))\,\Delta_i\,\mathrm{Heaviside}(x-d_i)\right)\\
&+ \frac{\omega\left(\int_0^x q_m(\xi)\,\mathbf{e}^{(\omega\,(x-\xi))}\,d\xi - \int_0^x q_m(\xi)\,\mathbf{e}^{(-\omega\,(x-\xi))}\,d\xi\right)}{2} - \left(\frac{1}{2}\int_0^x q_n(\xi)\,\mathbf{e}^{(\omega\,(x-\xi))}\,d\xi\right)\\
&- \left(\frac{1}{2}\int_0^x q_n(\xi)\,\mathbf{e}^{(-\omega\,(x-\xi))}\,d\xi\right)
\end{aligned}
\tag{32}
$$

$$
\begin{aligned}
\mathrm{M}(x) &= \frac{\sinh(\omega\,x)\,\mathrm{T}(0)}{\omega} + \mathrm{M}(0)\cosh(\omega\,x) - \frac{\sum_{i=1}^{n1} \mathrm{Heaviside}(x-a_i)\sinh(\omega\,(x-a_i))\,F_i}{\omega}\\
&+ \left(\sum_{i=1}^{n2} M_i\,\mathrm{Heaviside}(x-b_i)\cosh(\omega\,(x-b_i))\right)\\
&+ \omega\,J\,E\left(\sum_{i=1}^{n3} \mathrm{Heaviside}(x-c_i)\sinh(\omega\,(x-c_i))\,\Phi_i\right)\\
&+ \omega^2\,J\,E\left(\sum_{i=1}^{n4} \mathrm{Heaviside}(x-d_i)\cosh(\omega\,(x-d_i))\,\Delta_i\right)\\
&+ \frac{\int_0^x q_n(\xi)\,\mathbf{e}^{(-\omega\,(x-\xi))}\,d\xi - \int_0^x q_n(\xi)\,\mathbf{e}^{(\omega\,(x-\xi))}\,d\xi}{2\,\omega} + \frac{1}{2}\int_0^x q_m(\xi)\,\mathbf{e}^{(\omega\,(x-\xi))}\,d\xi\\
&+ \frac{1}{2}\int_0^x q_m(\xi)\,\mathbf{e}^{(-\omega\,(x-\xi))}\,d\xi
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
\phi(x) = {} & \frac{(-1+\cosh(\omega\, x))\,\mathrm{T}(0)}{\omega^2\, E\, J} + \frac{\sinh(\omega\, x)\,\mathrm{M}(0)}{\omega\, E\, J} + \phi(0) \\
& - \frac{2\left(\sum_{i=1}^{n1} \mathrm{Heaviside}(x-a_i)\,\sinh\left(\frac{1\,\omega\,(x-a_i)}{2}\right)^2 F_i\right)}{E\, J\, \omega^2} \\
& + \frac{\sum_{i=1}^{n2} \sinh(\omega\,(x-b_i))\,\mathrm{Heaviside}(x-b_i)\, M_i}{\omega\, E\, J} \\
& + \left(\sum_{i=1}^{n3} \Phi_i\, \mathrm{Heaviside}(x-c_i)\cosh(\omega\,(x-c_i))\right) \\
& + \omega\left(\sum_{i=1}^{n4} \Delta_i \sinh(\omega\,(x-d_i))\,\mathrm{Heaviside}(x-d_i)\right) + \left(-\int_0^x q_n(\xi)\,\mathbf{e}^{(-\omega\,(x-\xi))}\,d\xi\right. \\
& - \int_0^x q_n(\xi)\,\mathbf{e}^{(\omega\,(x-\xi))}\,d\xi + 2\int_0^x q_n(\xi)\,d\xi \\
& \left. + \omega\left(-\int_0^x q_m(\xi)\,\mathbf{e}^{(-\omega\,(x-\xi))}\,d\xi + \int_0^x q_m(\xi)\,\mathbf{e}^{(\omega\,(x-\xi))}\,d\xi\right)\right) \Big/ (2\,\omega^2\, E\, J)
\end{aligned}
\qquad (34)
$$

$$
\begin{aligned}
\mathrm{v}(x) = {} & \frac{(-\omega\, x + \sinh(\omega\, x))\,\mathrm{T}(0)}{\omega^3\, E\, J} + \frac{(-1+\cosh(\omega\, x))\,\mathrm{M}(0)}{\omega^2\, E\, J} + \phi(0)\, x + \mathrm{v}(0) \\
& + \frac{\sum_{i=1}^{n1} (-\sinh(\omega\,(x-a_i)) + \omega\,(x-a_i))\,\mathrm{Heaviside}(x-a_i)\, F_i}{\omega^3\, E\, J} \\
& + \frac{2\left(\sum_{i=1}^{n2} \sinh\left(\frac{1\,\omega\,(x-b_i)}{2}\right)^2 \mathrm{Heaviside}(x-b_i)\, M_i\right)}{J\, E\, \omega^2} \\
& + \frac{\sum_{i=1}^{n3} \Phi_i\, \mathrm{Heaviside}(x-c_i)\sinh(\omega\,(x-c_i))}{\omega} \\
& + \left(\sum_{i=1}^{n4} \Delta_i\, \mathrm{Heaviside}(x-d_i)\cosh(\omega\,(x-d_i))\right) + \left(-\int_0^x q_n(\xi)\,\mathbf{e}^{(\omega\,(x-\xi))}\,d\xi\right. \\
& + \int_0^x q_n(\xi)\,\mathbf{e}^{(-\omega\,(x-\xi))}\,d\xi + \omega\left(2\int_0^x q_n(\xi)\,(x-\xi)\,d\xi - 2\int_0^x q_m(\xi)\,d\xi\right. \\
& \left.\left. + \int_0^x q_m(\xi)\,\mathbf{e}^{(-\omega\,(x-\xi))}\,d\xi + \int_0^x q_m(\xi)\,\mathbf{e}^{(\omega\,(x-\xi))}\,d\xi\right)\right) \Big/ (2\,\omega^3\, E\, J)
\end{aligned}
\qquad (35)
$$

7. Conclusions

The contribution of this paper is that the generalized system of ordinary differential equations (7) to (10) derived for straight beam-columns holds true also for discontinuous unknown functions. The general solution to the generalized SODE for prismatic beam-columns with constant axial force is the generalization of the classical initial parameters method for discontinuous beam-columns, i.e. containing internal hinges or internal sliding connections.

The general solution (32) to (35) was found for prismatic beam-columns with constant axial tensile force by means of the Laplace transform method using symbolic programming approach.

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