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DRY FRICTION INFLUENCE IN AN OSCILLATORY SYSTEM EXPOSED TO LOW INTENSITY RANDOM EXCITATION

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Summary: *Friction influence in analysis of a single degree of freedom oscillatory system (SDOF) is either neglected or accounted for by analytical methods, assuming steady state harmonic excitation. However, these methods are not suitable when the oscillatory system is subjected to low intensity random excitation. Two approaches to friction inclusion into oscillatory system model will be explained – the commonly used approach using the signum function and a physically correct stick-slip approach consequently solving the describing set of equations in each system state. Their relative merits will be illustrated using a SDOF system subjected to low intensity random excitation. It will be shown that the best discriminator for using either of the simulation approaches is the so-called den Hartog's factor K, relating the friction force to the excitation force. Comparison of simulation results will illustrate these findings and underline the advantages of the second approach from engineering point of view.*

1. Introduction

The analysis of translatory oscillatory systems is an essential part of machine dynamics and the starting point of further studies in engineering vibrations. The essential form of such an oscillator is a combination of rigid mass m, linear mass-less spring with spring constant k_x and an idealised viscous damper with resistance proportional to relative velocity, described by damping constant b. The mathematical treatment of such a system is well known. The external excitation can be either a time variable force $F(t)$ acting on the mass m or a kinematic excitation in form of absolute displacement $u(t)$ and its derivatives $\dot{u}(t)$ or $\ddot{u}(t)$, acting on the oscillatory system support, as is the case analysed here. The equation of motion is then:

$$m\ddot{x} + k_x(x-u) + b(\dot{x}-\dot{u}) = 0, \tag{1}$$

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which can be re-written using the time dependent relative displacement $x_r = x - u$ and its time derivatives $\dot{x}_r$, $\ddot{x}_r$ as:

$$m\ddot{x}_r + k_x x_r + b\dot{x}_r = -m\ddot{u}\,. \tag{2}$$

However, for many real mechanical oscillatory systems this description is too simple:

- Structural constraints limit the free travel i.e. stroke (relative displacement x_r) of the oscillating mass.
- Description of vibratory energy dissipation by a linear, relative velocity dependent damper is too simple.

Another important issue is the dry friction, ever-present in any mechanical systems. The friction element may be introduced intentionally, to act as a means of vibratory energy dissipation, or, more likely, is an unwanted consequence of the system design or construction. In such oscillatory systems the influence of friction cannot be neglected [1, 4, 5]. The simple linear SDOF oscillatory system may be an oversimplification of reality and its analysis may lead to erroneous conclusions. This is specially so for random or transient kinematic excitation, as is often the case in ground transportation. In this class of problems the variable of interest is the vibratory acceleration $\ddot{x}$ of mass m rather than the displacement variable. Hence it is worthwhile to analyse oscillatory systems in which both a linear damper and a friction element (described by a general friction force F_f) are present, as depicted schematically in Fig. 1, especially from the point of view of acceleration transmissibility. Then the above equations of motion are slightly modified to become:

$$m\ddot{x} + k_x(x-u) + b(\dot{x}-\dot{u}) + F_f\,\mathrm{sgn}(\dot{x}-\dot{u}) = 0, \tag{3}$$

or:
$$m\ddot{x}_r + k_x x_r + b\dot{x}_r + F_f \mathrm{sgn}(\dot{x}_r) = -m\ddot{u}. \tag{4}$$

The general damping force F_d (or mixed damping force in Den Hartog's notation [3]) is described by expression:

$$F_d = b(\dot{x}-\dot{u}) + F_f\,\mathrm{sgn}(\dot{x}-\dot{u}) = b\dot{x}_r + F_f\mathrm{sgn}(\dot{x}_r)\,. \tag{5}$$

From a mathematical point of view such an oscillatory system belongs to the class of non-conservative, non-linear systems. Non-linear oscillatory systems can be classified by various criteria, e.g. as:

- Oscillatory systems with continuous type of non-linearity (smooth systems). The mathematical solution is essentially time-invariant and can be treated by sophisticated analytical methods [7–9].
- Oscillatory systems with discontinuous type of non-linearity (dry friction, impacts, free-play, etc.), as is the case here. The discontinuous non-smooth non-linearity causes a time dependent change in the system dynamics [4, 5].

The most common approximate analytical approaches are either to use the harmonic balance method, described first in this context fully by [3], or to solve the particular differential equations in respective time intervals [7–9]. This is laborious and may lead to transcendental equations [6]. Here the simulation approach will be followed, assuming excitation by vibratory acceleration $\ddot{u}(t)$, to arrive at solutions that are viable from engineering point of view and realisable in commercial simulation software, e.g. MATLAB®.

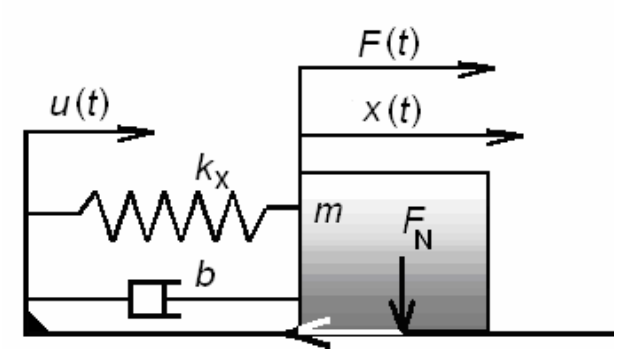

Fig. 1. Schematic layout of analysed horizontal single degree-of-freedom oscillatory system with friction influence.

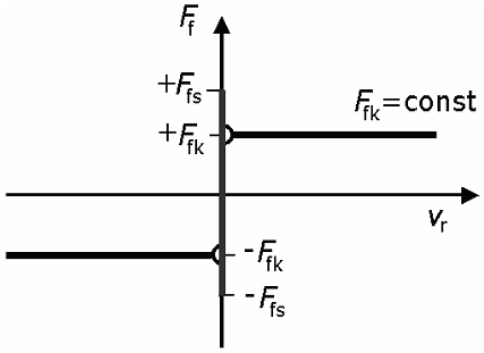

Fig. 2. Friction force F_f courses as function of relative velocity v_r.

In analysing an oscillatory system with both viscous damper and dry friction the primary question concerns the relative contribution of both dissipative terms to the total vibratory energy dissipation and thence the influence of both terms on typical system response characteristics, e.g. transfer function, time response, etc. This will be the aim of this paper.

2. Dry friction models

The first comprehensive scientific work on dry friction is attributed to Coulomb in 1785, however already the genius Leonardo da Vinci around 1500 was occupied by dry friction research [10]. Despite many years of research, the mathematical description of this phenomenon is not yet fully developed [1, 2, 10]. The phenomenon is not always reproducible, as its extent depends on surface state, lubrication, asperities, temperature, magnitude of normal force, relative velocity, etc. [1, 2, 10–12]. Various approaches to this problem are presented in the literature, e.g. [13]:

- The macro (or phenomenological) approach assumes single dissipative force acting at the interface between sliding surfaces. This approach is often denoted the static friction model.
- The micro approach takes into account detailed knowledge of characteristics of the sliding surfaces including roughness, asperities, adhesive phenomena, friction hysteresis, limit cycles, memory effects, surface lubrication, other tribological parameters, etc. This approach is often used in the so-called dynamic models [2, 11, 12]. These models are rather complicated and assume detailed knowledge of phenomena associated with the sliding surfaces.

Static friction models are based on the relation of the friction force F_f to the relative velocity v_r between the sliding surfaces in a phenomenological way. Basic approaches are described below, according to the notation of Figure 2:

A/ The Coulomb type friction characteristics F_C which may be mathematically described by the relay characteristics [1, 13, 14]:

$$F_f = F_C \operatorname{sgn}(v_r),\ F_C \equiv F_{fk} = \mu_k F_N, \tag{6}$$

where F_f is the friction force course, F_C is the Coulomb friction force and v_r is the relative velocity between the sliding surfaces. This model involves a proportional relationship between the Coulomb friction force F_C, sometimes denoted as kinetic friction force F_{fk}, and

the normal loading force F_N [11, 15] which is usually assumed to be constant. The proportionality constant μ_k is the dimension-less kinetic friction coefficient. The kinetic friction force F_{fk}, is independent of v_r; however for $v_r = 0$ it cannot be determined, i.e. the force F_f can have any value in interval $(-F_{fk}, +F_{fk})$.

The signum function $\mathrm{sgn}(v_r)$ is often [8, 14, 15] mathematically described as:

$$\mathrm{sgn}(v_r) = \begin{cases} +1 & \text{for } v_r > 0 \\ -1 & \text{for } v_r < 0 \end{cases}. \tag{7}$$

Different authors define different function values for the argument value $v_r = 0$ [4, 16, 24]. Note also that the signum function, as defined by expression (7) has no limit for $v_r = 0$ and is therefore not differentiable for $v_r = 0$, and hence is not a smooth continuous function.

B/ In reality a larger force is needed to start the sliding motion, i.e. for overcoming the adhesion at zero relative velocity a larger force F_{fs} is required than when the two surfaces are continuously sliding over each other [11, 15]. The friction force at $v_r = 0$ has to be described as a function of a limit force F_L, external to the dry friction interface. The limit force F_L is obtained by analysing the force balance across the interface between the sliding surfaces, and has to be compared to the static friction force value F_{fs}:

$$\text{if } |F_L| \le F_{fs} \Rightarrow v_r = 0. \tag{8}$$

If this condition is met the system is at standstill in the so-called stick state, indicated in Fig. 2 by the vertical line segment. If at a certain time instant the adhesion force F_{fs} is overcome by the external force, the oscillatory systems starts to move abruptly and the relative velocity v_r attains some non-zero value, as described in more detail in [1]. From this instant Eq. (6) is valid until v_r eventually decreases to zero and the system stops again for a certain time interval until the static friction force is overcome again. This start-slide-stop movement (stick-slip movement) leads to non-unique solution of equations describing the motion and poses mathematical difficulties [8, 14–16]. In analogy, the static friction coefficient μ_s is defined as $\mu_s = F_{fs}/F_N$, and $\mu_s > \mu_k$, because $F_{fs} > F_{fk}$.

C/ The Stribeck's effect is observed in some cases of well-lubricated surfaces [1, 2, 22] when the sliding friction force is dependent on relative velocity v_r. It exhibits a certain minimum at a relative velocity known as the Stribeck's velocity v_S and then increases with higher velocities. It is described by a velocity dependent function $f(v_r)$:

$$\begin{aligned} &F_f = f(v_r), && \text{if } v_r \neq 0, \\ &F_f = F_L, && \text{if } v_r = 0 \text{ and } F_L < |F_{fs}|, \\ &F_f = F_{fs}\,\mathrm{sgn}(F_L), && \text{if } v_r = 0 \text{ and } F_L \ge |F_{fs}|. \end{aligned} \tag{9}$$

In some simulation approaches the Stribeck's effect is modelled using a Gaussian distribution function [19] to account for the discontinuous natural dynamics of the state change at the start of the slipping motion (step transition $F_{fs} \rightarrow F_{fk}$ or $\mu_s \rightarrow \mu_k$). It is argued, that the restraining (adhesive) force is a composition of all the micro actions across the interface of contacting surfaces and their asperities [4, 12]. These actions take place consecutively and not abruptly [1, 2, 22]. The Gaussian model, introduced by Eq. (10), is a reasonable continuous approximation to this state change:

$$F_{\mathrm{f}} = \left| F_{\mathrm{fk}} + \left(F_{\mathrm{fs}} - F_{\mathrm{fk}}\right) \exp\left\{ -\left(\frac{v_{\mathrm{r}}}{v_{\mathrm{S}}} \right)^2 \right\} \right| \mathrm{sgn}(v_{\mathrm{r}}), \text{ for } v_{\mathrm{r}} \neq 0. \tag{10}$$

Further analysis will deal with an oscillatory system assuming two ways of dissipating vibratory energy by a linear viscous damping term and by a dry friction term - Eq. (5). Four different cases may arise, depending on the respective proportion of each dissipating term in Eq. (5) to the general damping force F_{d}, as illustrated in more detail in [23].

3. Approaches to dry friction analysis and modelling

3.1. Introduction

In employing static friction models for the analysis of oscillatory systems, essentially two general approaches are feasible:

i. An approximate analytical one, based on the *harmonic balance method* approach;

ii. A simulation one, employing contemporary simulation software, making use of conditioned switching between solutions in a time-scale that is short in comparison to the dominant period of the excitation signal [4, 5]. The simulation approach enables to use either the signum function approach, described by Eqs. (6) and (7) or a more physically sophisticated approach using the limit force analysis of Eqs. (8) and (9) and in case of accounting for the Stribeck's effect also Eq. (10). The merits of both approaches have to be thoroughly assessed in the context of the specific case to be analysed.

One of the first rigorous attempts at computer simulation of the influence of friction on dynamic systems was made by Karnopp [20]. He considers the causality issues and introduces a region of small relative velocity Dv_{r} around zero, indicated by the vertical line segment in Fig. 2. Outside of this region the Coulomb approach is valid, whereas within this region F_{f} is determined by other forces acting in the system in such a way that the F_{f} remains within the region, until a breakaway force (i.e. the static friction force) is exceeded. He illustrates the advantages of this approach on various examples using the bond graph approach and in this way designing a set of appropriate conditions.

3.2. The harmonic balance method

The harmonic balance method assumes a harmonic excitation by acceleration $\ddot{u}$ (with root mean square (RMS) value $a_{0\mathrm{u}}$) or rather by absolute displacement u with amplitude u_0 and variable angular frequency ω_{x}. The method for $b = 0$ is fully explained in standard textbooks e.g. [11, 15, 17]: the equivalent damping coefficient b_{e} in vicinity of resonance of a linear oscillator is introduced, depending on the amplitude of equivalent relative displacement ζ_{e}:

$$b_{\mathrm{e}} = \frac{4F_{\mathrm{fk}}}{\pi \zeta_{\mathrm{e}} \omega_{\mathrm{x}}}. \tag{11}$$

If the assumed harmonic solution is resolved then in steady state:

$$\zeta_{\mathrm{e}}(\omega_{\mathrm{x}}) = \frac{F_0}{k_{\mathrm{x}}} \cdot \left[1 - \left(\frac{\omega_{\mathrm{x}}}{\omega_0}\right)^2\right]^{-1} \cdot \sqrt{1 - \left(\frac{4F_{\mathrm{fk}}}{\pi F_0}\right)^2}\ , \tag{12a}$$

where $F_0 = -mu_0\omega_x^2 = -\sqrt{2}ma_{0\mathrm{u}}$ is the amplitude of an equivalent excitation force and $\omega_0 = \sqrt{k_{\mathrm{x}}/m}$ is the system natural frequency.

The formula can be expressed in a more transparent way:

$$\left|\frac{\zeta_{\mathrm{e}}(\omega_{\mathrm{x}})}{\sqrt{2}a_{0\mathrm{u}}/\omega_0^2}\right| = \left|1 - \left(\frac{\omega_{\mathrm{x}}}{\omega_0}\right)^2\right|^{-1} \sqrt{1 - K^2}\ , \tag{12b}$$

with a non-dimensional factor K after [21]: $K = \frac{4}{\pi}\frac{F_{\mathrm{fk}}}{F_0}$, (12c)

The factor K will subsequently be termed Den Hartog's factor and (except for a multiplicative constant) it relates the kinetic friction force F_{fk} to the body driving force F_0. Specifically for the horizontal oscillatory system of Fig. 1 with constant normal force $F_{\mathrm{N}} = m\mathrm{g}$, the Den Hartog's factor K has the form (g is standard gravity acceleration) [17]:

$$K = \frac{4}{\pi}\frac{\mu_{\mathrm{k}}\mathrm{g}}{\sqrt{2}a_{0\mathrm{u}}}. \tag{12d}$$

Eq. (12b) describes the modulus of the frequency response function (FRF) of the relative displacement for harmonic excitation with constant displacement amplitude in vicinity of resonance. However, expressions (12a) and (12b) are approximate and valid only for $K < 1$, i.e. for $F_0 > (4/\pi)\cdot F_{\mathrm{fk}} \cong 1.273F_{\mathrm{fk}}$, i.e. for base horizontal acceleration $a_{0\mathrm{u}} > (2\sqrt{2}/\pi)\cdot(\mathrm{g}\mu_{\mathrm{k}})$. In other words the driving force has to be sufficiently large in comparison to the kinetic friction force to permit the use of Eq. (12b).

For $F_0 < F_{\mathrm{fk}}$, or rather for $F_0 < F_{\mathrm{fs}}$ no movement is possible as the driving force would not overcome the adhesion force. If $F_0 \in (F_{\mathrm{fk}}, (4/\pi)\cdot F_{\mathrm{fk}}) \approx (F_{\mathrm{fk}},\ 1.273F_{\mathrm{fk}})$ the movement is not pure harmonic, but has one or more stops within one period [2, 11, 21] and *is not described* by the above approximate formula, as $K \geq 1$ and the term under the square root is not real. If the frequency of excitation ω_{x} approaches ω_0, the amplitude of oscillations at resonance will eventually grow beyond any limits [11, 15]. The system behaves as an undamped one with linearly increasing relative displacement amplitude x_{r} [15, 17] until structural limit is reached.

The case with both viscous and dry friction damping can be also solved using the harmonic balance method however the formulas are somehow cumbersome [21]. The Den Hartog's approach cannot account for the stick-slip phenomenon, which is accounted for by a procedure illustrated in [7, 9] for specific cases under harmonic excitation. None is applicable when random excitation is assumed, as often occurs in practice. This is especially so, if the equivalent excitation force amplitude $F_0 = \sqrt{2}ma_{0\mathrm{u}}$ would randomly fluctuate below F_{fk} and above $(4/\pi)F_{\mathrm{fk}}$ and the relative velocity v_{r} would be low, i.e. if the friction force would be commensurable with the driving force of the isolated body.

3.3. Use of the signum function

Use of the signum function for simulation of an oscillatory system with friction is an easy option that is facilitated by any simulation software. The describing equation of motion has the form (4), repeated here as Eq. (13):

$$\text{for } v_r \neq 0: \qquad m\ddot{x}_r + k_x x_r + b\dot{x}_r + F_{fk}\,\mathrm{sgn}(\dot{x}_r) = -m\ddot{u}\,. \tag{13}$$

For $v_r = 0$ the sgn function is set to zero [4, 16, 24] and so *analysis for $v_r = 0$ is completely omitted*. Sometimes the discontinuous signum function (7) is substituted by a continuous function, which approximates sgn function with required degree of accuracy [13, 14]:

$$\mathrm{sgn}(v_r) \approx \frac{2}{\pi}\arctan(cv_r) \approx \tanh(cv_r) \approx \mathrm{erf}(cv_r) \approx \frac{cv_r}{1 + c|v_r|}\,. \tag{14}$$

Constant c in each of the functions describes the numerical "match" between the sgn function and the respective continuous function used for approximation. The selection should be governed by following rules [13]:

- if it were too small, approximation would differ too much from the sought non-smooth one;
- if it were too large the effort is too great and the approximation is not sufficiently smooth.

In [14] selected numerical values are analysed. It is demonstrated, that a value of $c \geq 10^3$ suffices to fulfil both conditions and the fit with analytical solution is within 1 %. It is suggested that the last formula is better with regard to computational speed in attaining the same level of accuracy.

Any of the above approaches circumvents the problem of solving differential equation (13) with the discontinuous non-smooth signum function by introducing a continuous smooth function with arbitrary large derivative at zero crossing. The last formula of Eq. (14) was further used and a simple simulation program in MATLAB/Simulink® has been developed.

3.4. Use of the stick-slip approach

If the stick-slip phenomena are to be accounted for following approach has to be followed:

1. For $v_r \neq 0$ Eq. (13) is valid;
2. When the $v_r \neq 0$ to $v_r = 0$ transient occurs, the movement stops and the force balance condition across the friction interface has to be tested by the following set of conditions:

   i. Slipping: $|v_r| > \varepsilon$ OR $|F_L| > F_{fs}$, (15a)

   ii. Sticking: $|v_r| < \varepsilon$ AND $|F_L| < F_{fs}$, (15b)

   while: $F_L = m\ddot{x} + k_x x_r$, (16)

   and ε is a sufficiently small number, representing the vicinity of zero.

Conditions (15) can be expanded further into a more subtle set of conditions, which formed the basis of the computation algorithm [6]. For smoothing the $v_r = 0$ to $|v_r| \neq 0$ transient the Gaussian approximation (10) may be used.

This approach follows that one of Karnopp [20], using $\varepsilon \sim Dv_r$, however the bond theory is not applied. Further evaluation of condition $v_r = 0$ has to be analysed in some detail.

3.5. Determination of condition $v_r = 0$

The operation of determining when v_r reaches zero, or in numerical systems rather the condition $|v_r| < \varepsilon$, is generally called "variable zero-crossing operation" and is facilitated in standard simulation software by specific procedures (see e.g. [24]). The main difficulty for numerical systems with equal time increments is the need for precise determination of the time instant, when the zero-crossing occurs, or when $|v_r| < \varepsilon$, while the value of ε has to be assessed independently. This is very important when processing real-world data, commonly sampled at equal time increments. The standard stiff ordinary differential equations solvers with variable time increment are not applicable, unless the sampled data set could be re-interpolated in the same way. Another approach is to develop an ordinary differential equations solver with fixed time increment, which specifically caters for determining the $|v_r| < \varepsilon$ condition within the given fixed time increment Δt, as was the case here. Detailed analysis of the zero neighbourhood identification and selection of proper simulation time increment Δt is given elsewhere; interested reader is referred to [18, 22].

4. Comparison of the two dry friction force simulation approaches

In the previous chapter two approaches for accounting for dry friction influence in oscillatory system modelling were introduced. In this chapter the differences are highlighted.

In many applications time courses of different variables are less important, while aggregate and statistical characteristics are preferred, e.g. maximum and minimum values, RMS, crest factors, power spectral density (PSD), amplitude distribution etc. Here the acceleration RMS values will be used, as explained above.

When the signum function method is used a high friction value in combination with low v_r results in numerical instability causing parasitic oscillations in the time interval where v_r course is crossing the zero value. The error appears in the response acceleration and not in the relative displacement. It can be explained in following way: the sign output of the signum function for the i-th step is determined by the value in the $(i - 1)$-th step. In the vicinity of the zero crossing point the signum function forces the value for the next step to have the opposite sign and vice-versa. If the simulation interval Δt is too large, or the v_r change is too slow, false oscillations with period $2\Delta t$ occur, even if the real system would stop due to friction. In analysing real world sampled data, the time interval is set at the time of data acquisition, and later comparison by simulation means has to follow suit, or the sampled data would have to be re-sampled. Thus the choice of simulation interval Δt is to some extent limited. If the simulation interval is too large, parasitic oscillations occur when using the signum approach. These oscillations do not occur when using the physically correct stick-slip approach.

This is illustrated in Fig. 3 based on simulation of real-world data and their comparison to output acceleration course measured under field conditions with random input acceleration excitation $a_{0u} = 0.35$ ms^{-2}. Note false oscillations for $F_{fk} = 45$ N; whose consequence is a markedly different acceleration RMS value obtained by evaluating the output signal from the model using the stick-slip approach ($a_{0x} = 0.33$ ms^{-2}), compared with the signum approach, which gives $a_{0u} = 0.50$ ms^{-2}. The difference is not marked for the lower $F_{fk} = 15$ N (0.30 ms^{-2} versus 0.31 ms^{-2}), where virtually no sticking occurs in the time interval shown. This example illustrates the principal drawback of the signum method applied to oscillatory systems in

which the driving force may fluctuate around the dry friction force value. Another example, employing also the Stribeck's approximation after Eq. (10) is presented in [23].

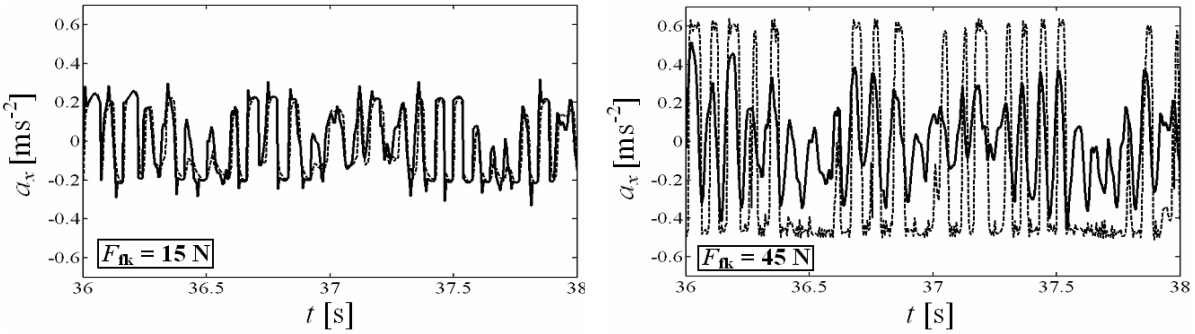

Fig. 3. Time histories of acceleration response for an oscillatory system with different friction force under random excitation: stick-slip model (——), signum model (– –).

It is of importance, whether for given conditions it is necessary to turn to the somehow complicated and more computationally demanding physically correct approach, or the signum approach suffices. As already indicated and described in more detail in [6] the decisive factor is the Den Hartog's factor K, relating the dry friction force F_{fk} to the mass driving force F_0. In analogy to the damping ratio ξ (relating the acting damping constant b to the critical one: $b_c = 2\sqrt{(mk_x)}$ the den Hartog's factor can be also termed *the relative friction coefficient*.

Further to Fig. 3 it is interesting to explore in more general way the influence of dry friction force magnitude on system behaviour. Due to system non-linearity classical description by frequency response function, assuming harmonic input and output variables (accelerations) is not applicable. Instead the acceleration transmissibility function in the x-direction T_{ax} is used, defined as: $T_{ax} = a_{0x}/a_{0u}$. Here a_{0x} and a_{0u} respectively are the RMS values of the respective acceleration time courses a_x and a_u. In this way the non-linear effects of the oscillatory system with dry-friction influence are accounted for. The oscillatory system has following parameters: $m = 88.4$ kg, $k_x = 11.5$ kNm^{-1}, i.e. $b_c = 3017$ Nsm^{-1}. Two systems are considered a system without viscous damping and a system with damping ratio $\xi = 0.25$, i.e. damping constant 504 Nsm^{-1}.The transmissibility T_{ax} courses related to factor K for two excitation levels, $a_{0u} = 0.35$ ms^{-2} and $a_{0u} = 0.67$ ms^{-2} are depicted in Fig. 4. In a companion Fig. 5 the probability of sticking ρ for the stick-slip approach is depicted, defined as the time when the system is in the stick state in relation to total duration of simulation (expressed in per-cent) for given K. The employed random excitation signals were actually measured in field tests with a heavy truck with a main vibratory power maximum around 2.0 Hz, flanged with two narrow band maxima at 6.3 Hz and 12.6 Hz respectively.

Note the large discrepancy between transmissibility T_{ax} calculated using the signum approach (solid pink line in Fig. 4) and by the physically correct model (black and green solid lines in Fig. 4) for higher K factor values. This is due to parasitic numerical oscillations in the signum model, illustrated in Fig. 3. From Figs. 4 and 5 it can be seen that these differences emerge when ρ is larger than some 50 %. The corresponding K factor threshold value is $K \cong ¾ = 0.75$. As factor K increases the sticking becomes more often and transmissibility T_{ax} asymptotically converges to unity, i.e. the oscillatory system becomes stuck and moves as a rigid body. Note there are oscillations for $K \geq 1$ which are not accounted for at all by Eq. (12).

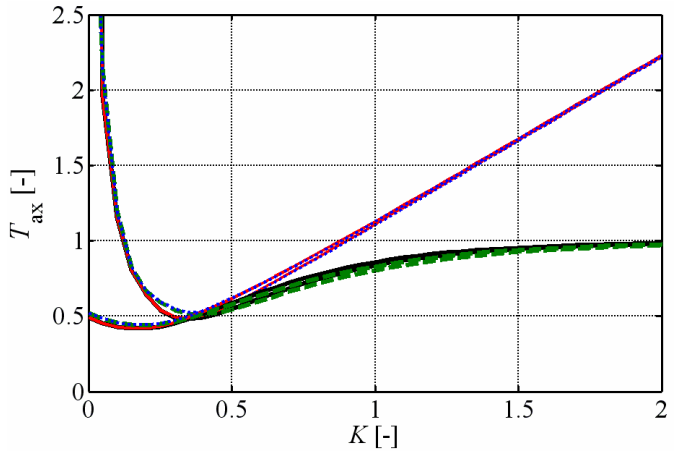

Fig. 4. Transmissibility T_{ax} dependence on factor K (—— a_{0u} = 0.35 ms^{-2}; – – a_{0u} = 0.69 ms^{-2}).

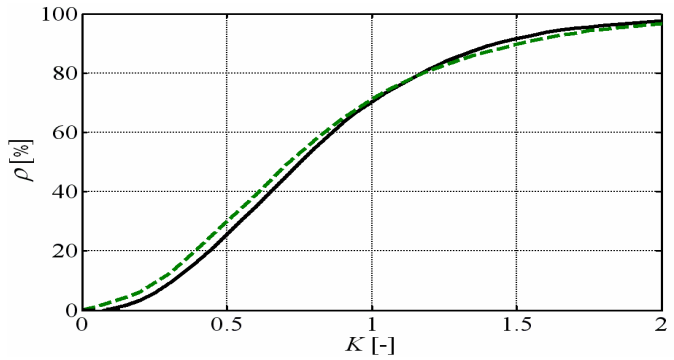

Fig. 5. Probability ρ in dependence on factor K (—— a_{0u} = 0.35 ms^{-2}; – – a_{0u} = 0.69 ms^{-2}).

On the lover end of the K factor value (say, for $K \leq ¼ = 0.25$), there is seen an obvious difference between the damped system (lower pair of lines) and the undamped system (upper pair of lines). Due to low friction the oscillatory system without viscous damping is not able to extract sufficient amount of the vibratory energy transmitted throughout and the transmissibility T_{ax} increases beyond acceptable limits (lines approaching upper frame border) Both approaches to dry friction modelling furnish the same T_{ax} dependence on K for $K < 0.50$.

5. Conclusion

The contribution deals with the analysis and simulation of a general single of degree of freedom oscillatory system with vibratory energy dissipation by both an idealised linear viscous damper and a dry friction interface. For modelling the dry friction interface the phenomenological macro-slip approach is employed, described first by the approximate harmonic balance approach, then by the signum function approach and by the physically correct stick-slip approach. The last two approaches both are illustrated by a simulation example using as the input stationary random excitation acceleration from a real situation. The differences in the two approaches are highlighted, indicating that the physically correct stick-slip approach describes the reality better than the computationally simpler signum

approach. The signum approach is prone to false numerical oscillations that completely distort the acceleration response signal. These effects are dependent on the relation between the dry friction force value and isolated body driving force, best described by Den Hartog's factor K (the relative friction coefficient).

It can be concluded that the simpler model, employing the signum approach, is suitable for oscillatory systems with low inherent dry friction and high driving force, whereas for correct modelling of systems with higher friction and low driving force the limit force analysis approach is essential. A possible discrimination between using of either of these models is the Den Hartog's factor value of $K \approx 0.50 \div 0.75$. For lover K values the signum model suffices, whereas for higher values, when the probability of sticking rises above 50 % of the simulation time, the physically correct model use is inevitable.

The limit force analysis approach describes reality correctly from a physical point of view, including as it does also the static friction. It is universally applicable to any oscillatory system with dry friction in a generic way, irrespective of the magnitude of the Den Hartog's factor. The SDOF oscillatory system including a physically correct dry friction model that has been developed and described here is of a generic nature and can be widely used for systems modelling in transport industries. Its application circumvents deeper knowledge of advanced methods of non-linear systems analysis and enables more effective exploitation of available simulation software for better understanding of the performance of oscillatory systems.

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