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CRACK PROPAGATION CRITERIA FOR THE CRACK TERMINATING ON THE INTERFACE OF A THIN ORTHOTROPIC LAYER AND AN ORTHOTROPIC SUBSTRATE

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Summary: *The contribution discusses possible approach for the formulation of fracture criteria for the general stress concentrators – especially a surface crack terminating at the interface of two dissimilar orthotropic media. The classical differential analysis is unsuitable due to the discontinuity in the elastic properties which leads on the interface to a zero or infinite energy release rates. Theory of the Finite Fracture Mechanic is used to overcome this problem (crack increase of finite length is used instead of the infinitesimal one). The three possibilities of the crack propagation are taken into the consideration – crack deflection (single or double) and a penetration of the crack across the interface into the substrate. The so-called matched asymptotic procedure in combination with FEM is used for the calculation of appropriate changes of potential energy caused by the fracture.*

1. Introduction

The increasing use of the fibre-reinforced composites in a high performance structures has brought a renewed interest in the analysis of the cracks in anisotropic materials. Most matrices of the advanced composite material are brittle. They prone to cracking under very low applied stresses and failure frequently occur in the form of multiple matrix cracking. The orientations of these cracks may vary depending on the relative position of the reinforcement in relation to the load. The stress field in the neighbourhood of crack is governed by the overall anisotropic material response. The existence of material interfaces in composites, especially in laminates, brings other problems in the analysis of cracks – the problem of crack terminating at the interface of two anisotropic solids and the problem of interfacial crack in anisotropic solids. These problems are also encountered in the technology of protective coatings. It is also well established that the increase of the toughness of ceramics laminates or ceramic-matrix composites can be achieved by introducing weak interfaces between layers or between the fibre and the matrix. Deflection along the interface then results in a crack blunting and this effect increases the required energy for the next crack propagation. Understanding the mechanism of the crack deflection along the interface is thus essential to determine for example the suitable interlayer and the optimum interface toughness which are necessary to favour this phenomenon. The capability of an interface to deflect a crack is

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usually analyzed in terms of the competition between deflection and penetration for a stationary crack terminating at the interface at a normal or an oblique angle – e.g. Hutchinson (1994), Leguillon et al. (2000) and Martin et al. (2001).

The discontinuity in the elastic properties at the interface strongly influences the behaviour of the energy release rate of the crack in the vicinity of the interface. In the case of a strong singularity (crack lies in a stiffer material and a characteristic eigenvalue $\delta<1/2$), the energy release rates $G_p(l_p=0)$, $G_d(l_d=0)$ for a crack terminating at the interface are infinite and interface penetration or deflection is thus possible at any finite load level. In contrast, the presence of a weak singularity (crack lies in a softer material and $\delta>1/2$) implies that the energy release rates $G_p(l_p=0)$, $G_d(l_d=0)$ for a crack terminating at the interface are zero and interface penetration or deflection is not predicted for any applied load. This is a drawback of the classical differential theory which may be overcome with the help of the so-called Finite Fracture Mechanics, where the crack increment of a finite length is used instead of the infinitesimal one. The analysis is performed within the framework of two dimensional linear elasticity. Matched asymptotic analysis (Leguillon (2002)) is used to derive the change in potential energy induced by a crack growth of some finite increment. Afterwards the competition between the deflection of the main crack along the interface and the penetration into the substrate can be assessed.

2. Problem formulation

Let consider a crack lying in a thin orthotropic layer, perpendicular to the interface with an orthotropic substrate. The main goal is to work up a technique applicable for the assessment of the fracture-mechanics behaviour of such a general stress concentrator under the given loading conditions. The following Figure 1. shows three possibilities of the crack propagation which will be considered. The crack will either penetrate into the material M1 (Fig. 1b) or will be deflected along the interface (singly or doubly – Fig. 1c,d) - Martinez & Gupta (1994), Martin et al. (2001).

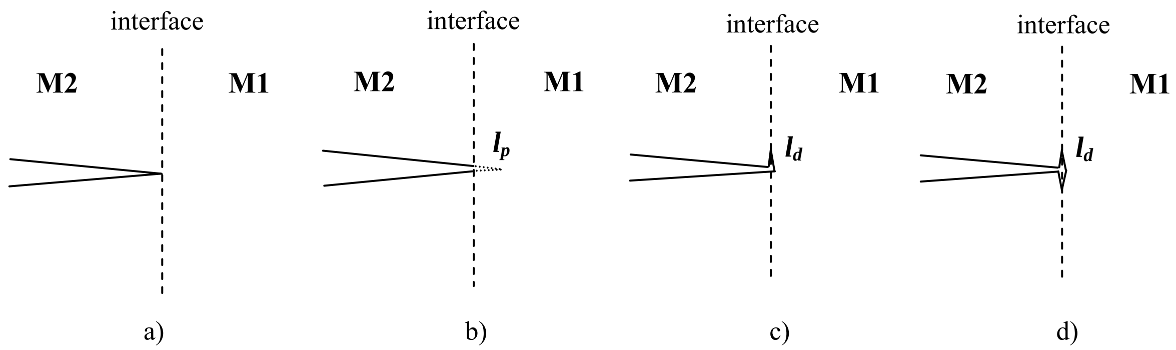

Fig. 1 a) Main crack terminating at the interface of two dissimilar materials; b) crack penetrating into the material M1; c) singly deflected crack; d) doubly deflected crack.

3. Matched asymptotic analysis

A matched asymptotic analysis (e.g. Leguillon (2002)) is used to evaluate the energy balance when the crack propagates in the vicinicity of the interface. As shown in the Figure 1, different geometries are considered (single, double deflection along the interface and the penetration into the material M1). In order to keep a validity of the asymptotic analysis, the condition of l_d, $l_p \rightarrow 0$ must hold. It means that a ratio of l/L <<1 (see Fig. 2). It is worthy of note that he asymptotic assumptions of the small crack extensions imply that the constant loading conditions have no influence on the energy balance.

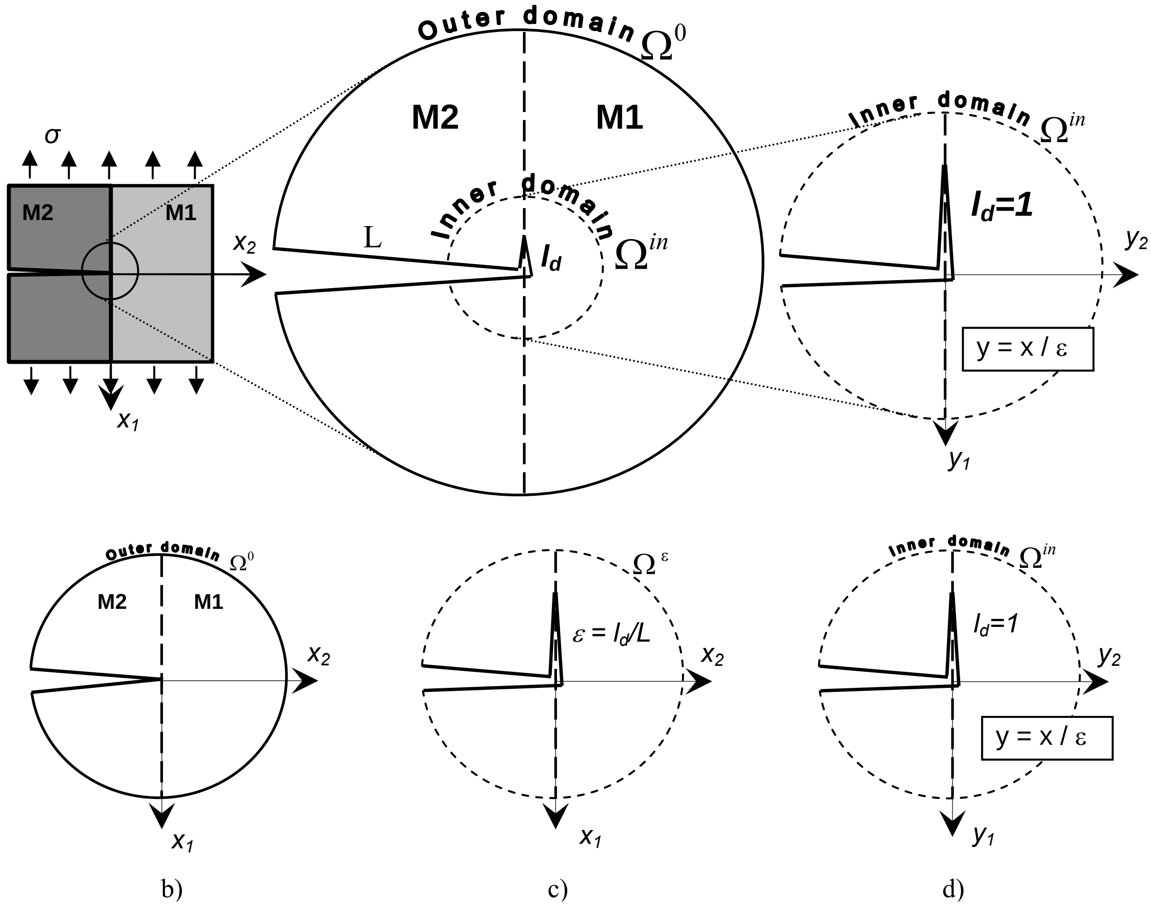

Fig. 2 a) Outer and Inner domain used in the matched asymptotic analysis (in case of the singly deflected crack). b) – d) coordinate systems of outer and inner domain

In the plane linear elasticity, we consider a domain in which the main crack is increased by some small increment l (l_p or l_d) – Fig1. The dimensionless length of this increment is denoted as ε. The solution $\underline{U}^{\varepsilon}(x_1,x_2)$ to an elasticity problem in this domain can be expressed as the unperturbed (without crack increment) solution $\underline{U}^{0}(x_1,x_2)$ defined on the outer domain Ω^0 plus a small correction - Leguillon (2002):

$$\underline{U}^{\varepsilon}(x_1,x_2)=\underline{U}^{0}(x_1,x_2)+f_1(\varepsilon)\underline{U}^{1}(x_1,x_2)+\ldots \quad, \tag{1}$$

where function $f_1(\varepsilon) \to 0$ as $\varepsilon \to 0$. Such an expansion (1) is a so-called outer expansion and is valid in the whole domain Ω^0 (or Ω^ε) except near the point where the geometry is perturbed by the crack increment. The solution $\underline{U}^0(x_1,x_2)$ is singular at the tip of the main crack and can be expanded as:

$$\underline{U}^0(x_1,x_2) = \underline{U}^0(0,0) + H\, r^\delta \underline{u}(\theta) + \dots \ . \tag{2}$$

In order to obtain a description of the near tip fields, the domain Ω^ε is stretched $\times$ $1/\varepsilon$ and as $\varepsilon \to 0$ it leads to the unbounded „inner" domain Ω^{in} described by the stretched variables $y_1 = x_1/\varepsilon$ and $y_2 = x_2/\varepsilon$. The size of the crack increment is now equal to 1. The solution can be expanded in this domain as:

$$\underline{U}^\varepsilon(x_1,x_2) = \underline{U}^\varepsilon(\varepsilon y_1, \varepsilon y_2) = F_0(\varepsilon)\underline{V}^0(y_1,y_2) + F_1(\varepsilon)\underline{V}^1(y_1,y_2) + \dots \ , \tag{3}$$

where $F_1(\varepsilon) / F_0(\varepsilon) \to 0$ as $\varepsilon \to 0$. It is the „inner" expansion. Conditions at infinity are missing to define well-posed problems for the unknown functions $\underline{V}^0(y_1,y_2)$ and $\underline{V}^1(y_1,y_2)$. They derive from the matching conditions based on the existence of an intermediate area where both expansions (1), (3) hold. In other words, the behaviour of the outer terms (in(1)) when approaching the singular point must match with that of the inner terms at infinity.

The behaviour of $\underline{V}^1(y_1,y_2)$ at infinity is prescribed:

$$\underline{V}^1(y_1,y_2) \sim \rho^\delta \underline{u}(\theta) \quad \text{as} \quad \rho \to \infty, \ \text{where} \quad \rho = \sqrt{y_1^2 + y_2^2} = r/\varepsilon \ . \tag{4}$$

The function $\underline{V}^1(y_1,y_2)$ is independent of the applied load, only depends on the local geometry. Using a superposition principle we get a definition of the second term in (3):

$$\underline{V}^1(y_1,y_2) = \rho^\delta \underline{u}(\theta) + \underline{\hat{V}}^1(y_1,y_2) \ . \tag{5}$$

Change of the potential energy δW_p between the solutions of unperturbed (without crack increment) $\underline{U}^0(x_1,x_2)$ and perturbed (with crack increment) $\underline{U}^\varepsilon(x_1,x_2)$ situations for unchanged boundary conditions is by use of the Betti´s theorem the following:

$$\delta W_p = \frac{1}{2}\int_\Gamma \left(\sigma\left(\underline{U}^\varepsilon\right) \underline{n}\, \underline{U}^0 - \sigma\left(\underline{U}^0\right) \underline{n}\, \underline{U}^\varepsilon \right) ds, \tag{6}$$

where Γ is any contour surrounding the corner and $\underline{n}$ its normal pointing toward the origin. The integral can be taken either in Ω^0 (or Ω^ε) or in Ω^{in}. Selecting the inner domain and substituting the asymptotics (1) – (4) into (6) (under the assumption of the constant loading conditions during the crack extension), the change of the potential energy δW per unit width between the initial position and the new crack position leads finally to:

$$\delta W_p = K \cdot H^2 \cdot \varepsilon^{2\delta} + \dots.., \tag{7}$$

where H is the Generalized Stress Intensity Factor of the given crack terminating on the interface, ε is a dimensionless parameter ($\varepsilon = l/L$, where L is a characteristic size of the outer domain), δ - characteristic eigenvalue of the given singularity (Kotoul et al. (2006), Ševeček et al. (2006)), and K is a contour integral defined as follows:

$$K = \frac{1}{2}\int_\Gamma \left(\sigma\left(\underline{V}^1\right) \underline{n}\, \rho^\delta \underline{u} - \sigma\left(\rho^\delta \underline{u}\right) \underline{n}\, \underline{V}^1 \right) ds \ . \tag{8}$$

In this integral $\underline{V}^1$ and $\sigma(\underline{V}^1)$ denote a displacements and stresses on the inner domain calculated using FE analysis, $\rho^\varepsilon\, \underline{u}$ and $\sigma\,(\rho^\varepsilon\, \underline{u})$ denote a displacements and stresses given by the singular solution – Ševeček et al. (2006). Clearly K does not depend on the actual size of the crack increment since it has been stretched to 1. Moreover it is independent of the applied loads which are included in *H*. The function $\underline{V}^1$ can be computed by finite elements and *K* can be numerically calculated using the contour independent integral (8).

4. Crack propagation criteria

After the appropriate changes of potential energy (for the cases of singly or doubly deflected and penetrating crack) are calculated, the competition between these states can be assessed. The crack will follow that path, which maximises the additional energy ΔW released by the fracture process - Martin et al. (2001). It means for example, if crack deflection occurs preferentially to the penetration, the following condition must be satisfied:

$$\Delta W_d = \delta W_d - G_i^c l_d \quad > \quad \Delta W_p = \delta W_p - G_1^c l_p \ , \tag{9}$$

and also vice versa. In (9) the G_1^c denotes the toughness of material M1 and G_i^c interface toughness. For the methods how to obtain this characteristics see e.g. O´Dowd & Shih (1992). It is also worth remarking that the differential form of the condition (9) is identical to the maximum energy release rate condition in the case of the homogenous material.

5. Numerical calculations

The determination of the change of potential energy δW_p - formulas (7), (8) due to a finite crack increment - requires a numerical solution of the stress and displacement field on the inner domain by FEM. For this purpose the FE system ANSYS 10.0 have been used. In the following figure is a demonstration of the FE mesh used for the case of singly deflected crack. Analogical mesh is used for the case of double deflection and penetration as well.

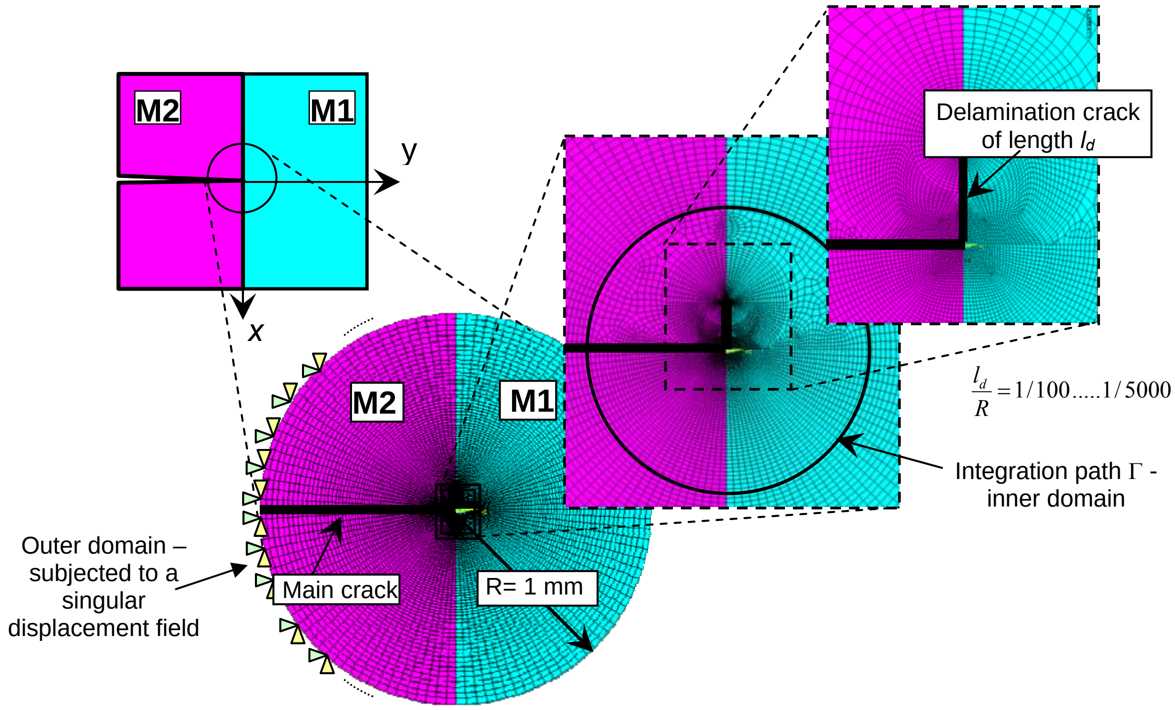

Fig. 3 Example of the FE mesh used for the case of a singly deflected crack.

The model is made of two material layers M1 and M2, where the elastic properties of both materials are identical: E_L = 137 GPa, $E_T = E_Z$ = 10,8 GPa G_{ZT} = 3,36 GPa ν_{TZ} = 0,49 $G_{ZL} = G_{TL}$ = 5,65 GPa $\nu_{ZL} = \nu_{TL}$ = 0,238 – only the main material directions are mutually turned by 90°. Practically it means that material M1 has a Young modulus E_L in the direction of the *y* axis and material M2 has E_L is in the direction of *x* axis.

The outer domain (see Fig. 3) is subjected to the displacement field on the diameter r=R:

$$\underline{U}(x_1, x_2) = H\, r^{\delta} u(\theta) + \ldots \ , \tag{10}$$

where the characteristic eigenvalue δ and the function $u(\theta)$ are taken from the singularity analysis based on the complex potential theory and the Generalized Stress Intensity Factor H is calculated for the appropriate loading conditions using a combination of the two-state ψ-integral and FEM – see Ševeček et al. (2006), Desmorat. & Leckie (1998).

The crack increments of lengths l_d, l_p are used for the case of crack deflection and penetration respectively. Each of these cases is calculated separately. Under the loading conditions (10) on the outer domain, the stresses and displacements on the inner domain are calculated using FE system ANSYS. These results are subsequently used for the calculation of the change of the potential energy evoked by the small crack increase in the chosen direction. After the change of the potential energy (for all possibilities of the crack propagation) is calculated using relations (7) and (8), the additional energy for the corresponding states can be determined using formulas (9). Crack will then follow that direction which maximises this additional energy.

Note: Factual numerical results for all possibilities of crack propagation will be presented at the conference.

6. Conclusion

In the case of the cracks terminating on the interface of two materials the differential energy analysis is unsuitable due to the discontinuity in the elastic properties which leads to a zero or infinite energy release rates (depending on the type of the singularity). Therefore the theory of the Finite Fracture Mechanics is employed for the definition of the fracture criteria – crack increment of finite length is considered instead of the infinitesimal one. The change of the potential energy due to a crack growth by some finite increment is calculated using a combination of FEM and a matched asymptotic analysis. Three possibilities of the propagation directions – penetration, single and double deflection are taken into the consideration. The crack will follow that path which maximises the additional energy released by the fracture process. This criterion can be used for cracks terminating at the interface of two dissimilar materials, lying on it or for an arbitrary multimaterial wedge as well.

7. Acknowledgements

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