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# CONTROL DESIGN OF ACTIVE MAGNETIC BEARING BY GENETIC ALGORITHMS

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**Summary:** *Control design of rotating shaft levitated by active magnetic bearing is described in this contribution. Genetic algorithm is used to design controller parameters. Dependence of controller parameters on rotational speed of shaft is studied.*

## 1. Introduction

An active magnetic bearing (AMB) inhibits the contact between the rotor and stator and so it eliminates the limitations of classic bearing. Therefore it is possible to use AMB in specific and extreme circumstances where classic bearing is inapplicable. Electromagnets located in stator of the bearing create a magnetic field. The force caused by magnetic field keeps the rotor levitating in desired position in the middle of air clearance. So the control of magnetic field is necessary.

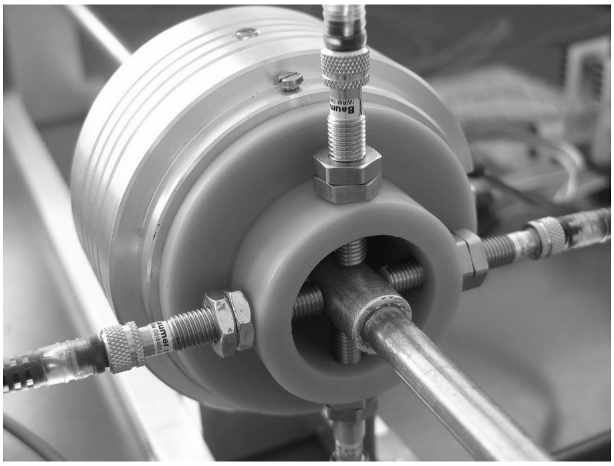

Figure 1: Active magnetic bearing

Although dependence of the magnetic force on the feeding voltage is highly nonlinear the AMB can be controlled by linear regulator with sufficient performance. Main problem is to

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find appropriate parameter values of the regulator. The standard way to design linear regulator of nonlinear system is to create the linear model of the system describing sufficiently well the nonlinear one in close proximity to operational point. This approach can be used for AMB, but common method of linearization of AMB model has disadvantages. Another approach is to use the simulation and suitable automated method to find the regulator parameters, e.g. genetic algorithms.

Genetic algorithms are inspired by biological evolution. Generally their learning is based on random selection of parents from the population (i.e. set of possible combinations of parameter values) and recombination and possibly mutation of parents to generate new members of population. The fitness (i.e. value representing performance) of new members is then computed and the members with worse fitness are removed from the population.

It is proved that population improves with time even though bad members exist whole time. So it can be assumed that genetic algorithms are capable of learning of appropriate regulator parameters.

## 2. Model of active magnetic bearing

Model used for control design is composed of two parts – model of levitated rotor and model of magnetic force. Model of the rotor is developed according to [2]. It is assumed that the rotor is fixed to the motor on the right side and on the left side is supported by AMB. The unbalance of the rotor is modeled as a mass point.

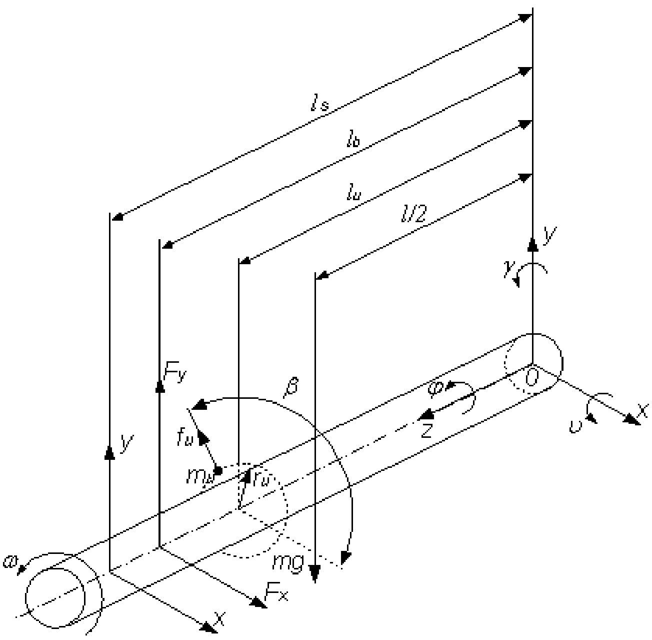


Figure 2: Model of rotor

Behavior of rotor can be described by second order differential equation

$$M\frac{d^2q}{dt^2} + \omega G\frac{dq}{dt} = Bf + f_g + \omega^2 d_u(\varphi), \tag{1}$$

where $q=\begin{bmatrix}\gamma \\ \upsilon\end{bmatrix}$, $M=\begin{bmatrix}J_r & 0 \\ 0 & J_r\end{bmatrix}$, $G=\begin{bmatrix}0 & -J_a \\ J_a & 0\end{bmatrix}$, $B=\begin{bmatrix}l_b & 0 \\ 0 & -l_b\end{bmatrix}$, $f=\begin{bmatrix}F_x \\ F_y\end{bmatrix}$, $f_g=\begin{bmatrix}0 \\ -mg\end{bmatrix}$, $d_u=\begin{bmatrix}m_u \cdot r_u \cdot l_u \cdot \cos(\varphi+\beta) \\ -m_u \cdot r_u \cdot l_u \cdot \sin(\varphi+\beta)\end{bmatrix}$, $\varphi=\int \omega dt$, $\omega$ is angular velocity of rotation, $F_x$ and $F_y$ are magnetic forces and $J_a$ and $J_r$ are inertia moments to axis of rotation and the orthogonal one respectively. Meaning of the other parameters is given by figure 2. Angular deviations can be transformed to deviations measured by sensors by equation 2.

$$\begin{bmatrix}x \\ y\end{bmatrix}=\begin{bmatrix}l_s & 0 \\ 0 & -l_s\end{bmatrix}q \tag{2}$$

Table 1: Parameter values of rotor model

| Parameter | Description | Value | Unit |
|---|---|---|---|
| $m$ | Mass of rotor | 0.4 | [kg] |
| $l$ | Length of rotor | 0.5 | [m] |
| $l_b$ | Position of AMB | 0.4 | [m] |
| $l_s$ | Position of sensors | 0.43 | [m] |
| $m_u$ | Mass of unbalance | 0.01 | [kg] |
| $r_u$ | Distance of unbalance from axis of rotation | 0.001 | [m] |
| $l_u$ | Position of unbalance | 0.4 | [m] |
| $\beta$ | Angular displacement of unbalance | 0 | [rad] |

Magnetic force is composed from forces caused by opposite electromagnets (see fig. 3). As can be seen from equations 3, magnetic force depends on feeding currents and position of the rotor and is highly nonlinear.

$$\begin{aligned} F_{m,x} &= \frac{Ai_{x,2}^2}{\left(\frac{d}{2}-x+a\right)^2}-\frac{Ai_{x,1}^2}{\left(\frac{d}{2}+x+a\right)^2} \\ F_{m,y} &= \frac{Ai_{y,2}^2}{\left(\frac{d}{2}-y+a\right)^2}-\frac{Ai_{y,1}^2}{\left(\frac{d}{2}+y+a\right)^2} \end{aligned} \tag{3}$$

Feeding voltage of electromagnets is controlled because control of voltage by PWM (pulse width modulation) is easier than direct control of current. The relation between voltage and current is given by Ohm law.

$$u=R\cdot i+L\cdot \dot{i} \tag{3}$$

Table 2: Parameters of magnetic force model

| Parameter | Description | Value | Unit |
|---|---|---|---|
| $A$ | Parameter of magnetic force | $1.42\cdot10^{-4}$ | [$\mathrm{Nm}^2/\mathrm{A}^2$] |
| $a$ | Parameter of magnetic force | $2.99\cdot10^{-5}$ | [m] |
| $d$ | Size of air clearance | $1.4\cdot10^{-3}$ | [m] |
| $R$ | Resistance of electromagnets | 266 | [Ω] |
| $L$ | Inductance of electromagnets | 0.87 | [H] |

## 3. Controller

Position of levitated rotor is controlled by two discrete PID regulators. They work independently and each controls the position of the rotor in one of orthogonal axes. Required feeding voltage of electromagnets is the output of regulators. Electromagnets can cause attractive force only so control of voltage is designed so that positive voltage means switching on one of the opposite electromagnets and negative voltage means switching on of the second one (see fig. 3). Parameters of PID regulator (see table 3) are designed by genetic algorithm.

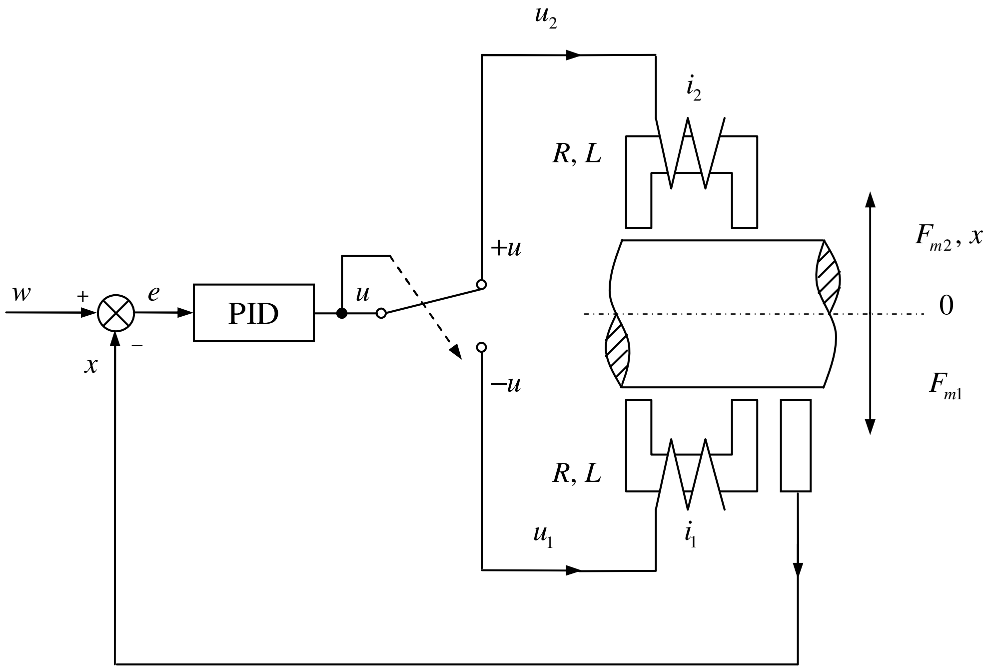


Figure 3: Control model of single axis of AMB

## 4. Implementation of genetic algorithms

Classic implementation of genetic algorithm [1] first generates given number of new members of population, computes their fitness and then removes members of population with worst fitness. Such implementation works fine if the population is big enough otherwise it almost certainly will not find best values. The problem is in this case the fitness is computed as square of control error and its computation is based on simulation. Considering thousands of simulations for different regulator's values have to be done, such fitness evaluation is slow and cannot be speed up. So the rest of algorithm has to be optimized.

Used implementation generates only one new member in every step of the algorithm. After its fitness is evaluated it replaces randomly selected member of the population. The member to be replaced is selected so as members with worst fitness have the highest probability of selection. The probability is directly proportional to the position of member in the population sorted according to fitness.

Considering even the best member of population can be replaced by the worse one, the best one has to be remembered even it is removed from population to assure it is not lost.

## 5. Results

Learning of regulator parameters is based on the simulation. The simulation is configured so as initially the rotor is not revolving. Motor driving rotor is started after $0.5\,s$ so rotor has enough time to get in the middle of air clearance before it starts to rotate. Furthermore the influence of gravitation and small unbalance are simulated. The parameters of the rotor and bearing are given by tables 1 and 2. Results of learning are given by figure 4.

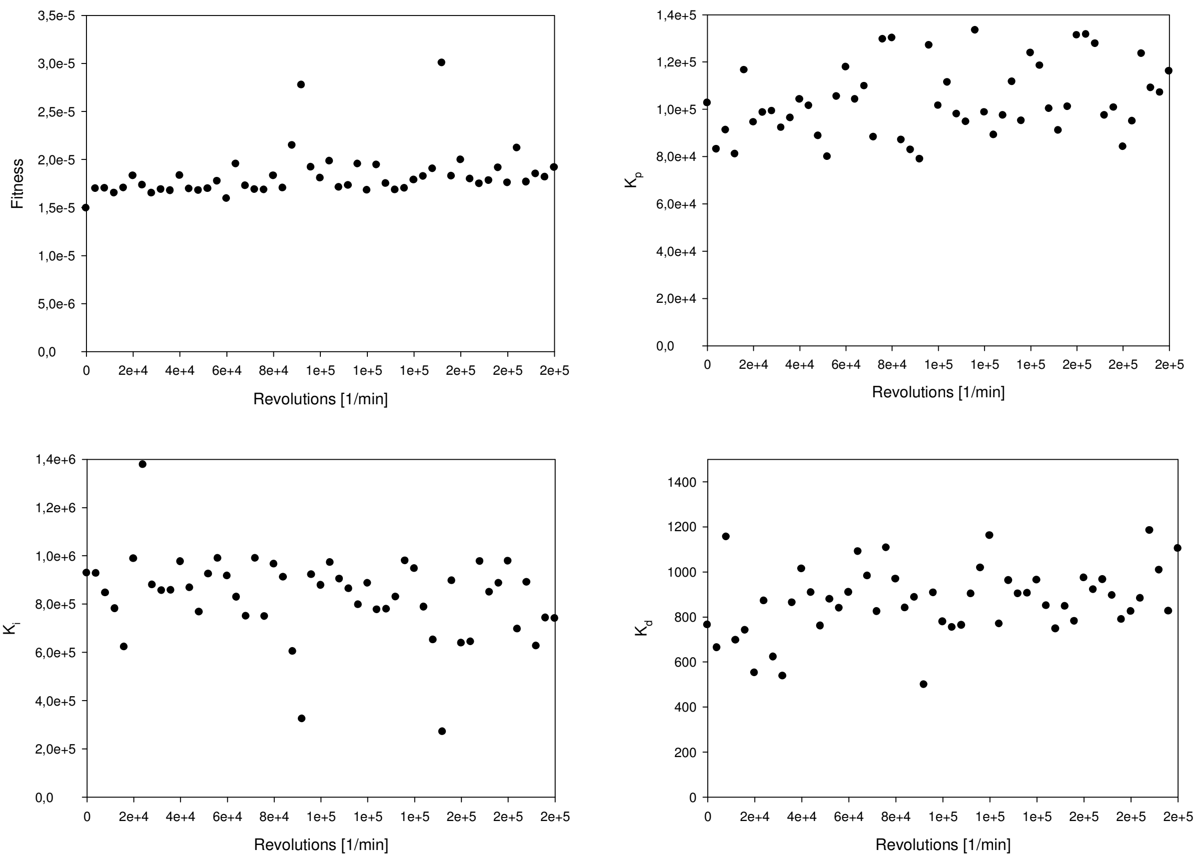


Figure 4: Dependence of learned parameters on revolutions of rotor

As can be seen dependence of regulator parameters on revolutions of rotor is minimal so it can be considered to be constant. The learned parameters are given by table 3.

Table 3: Learned parameters of regulator

| Parameter | Description | Value | Unit |
|---|---|---|---|
| $K_p$ | Proportional gain | $10^5$ | [Vm$^{-1}$] |
| $K_i$ | Integrative gain | $9 \cdot 10^5$ | [Vm$^{-1}$s$^{-1}$] |
| $K_d$ | Derivative gain | $9 \cdot 10^2$ | [Vm$^{-1}$s] |
| $T$ | Sampling period | $10^{-3}$ | [s] |

The controllability by the regulator with given coefficients is proven by simulation study. Figure 5 shows simulation of control for frequencies of the rotor rotation from zero to 200000 min$^-$1 with step 20000 min$^{-1}$.

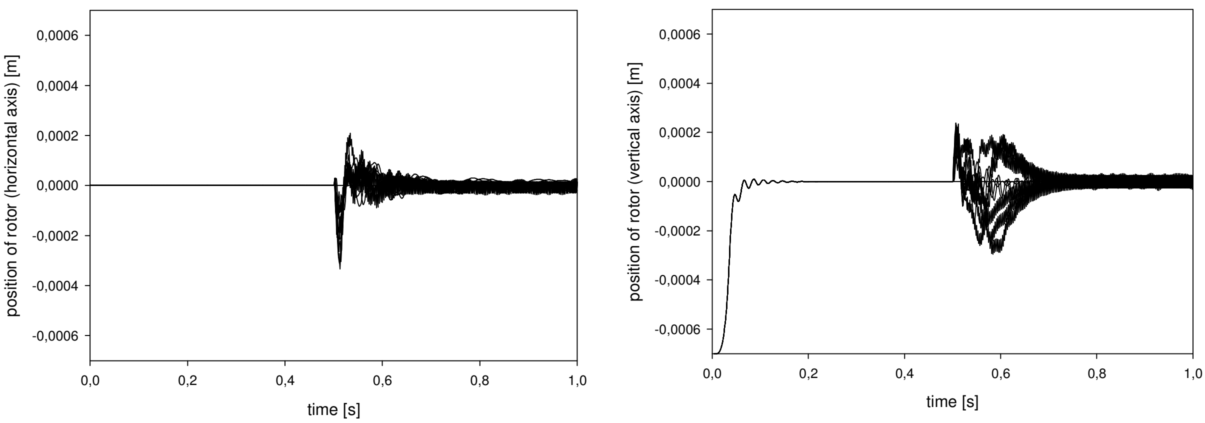


Figure 5: Simulation of control for rotational speed of rotor from 0 to 200000 min$^{-1}$

From previous text it can be assumed that rotor can be stabilized in central position of air clearance with given regulator parameters for any speed of rotation. But closer inspection of graph of fitness values on figure 4 shows two values of fitness that are remarkably higher then other ones. Also values in close proximity to them have raising tendency. Furthermore learned values of integrative gain at given frequencies also remarkably differ from other ones.

The reason is that at given frequencies (see table 4) exists resonance between oscillations caused by unbalance and regulator. It means the position of rotor cannot be controlled by regulator with learned parameters at these frequencies (and close ones).

Table 4: Resonance frequencies

| Parameter | Description | Value | Unit |
|---|---|---|---|
| $f_1$ | Resonance frequency | $9.2 \cdot 10^4$ | [min$^{-1}$] |
| $f_2$ | Resonance frequency | $1.52 \cdot 10^5$ | [min$^{-1}$] |

## 6. Conclusion

Published simulation results show that although behavior of AMB is highly nonlinear, it can be controlled by common PID regulator. The quality of control is sufficient but if unbalance of rotor is nonzero than frequencies of rotation of rotor at which resonance occurs exist. It means that AMB cannot be controlled by PID regulator with given parameters at these frequencies.


## 7. Acknowledgement

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## 8. Literature


[1] Goldberg, D., E., "Genetic Algorithms in Search, Optimization, and Machine Learning (Hardcover)", Addison Wesley, 1988

[2] Krämer, E., "Dynamics of Rotors and Foundations," Berlin Heidelberg: Springer-Verlag, 1993