National Conference with International Participation
ENGINEERING MECHANICS 2007
Svratka, Czech Republic, May 14 – 17, 2007

SIMULATION OF HUMAN GAIT OVER THE ELASTIC BRIDGE

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Summary: *The paper deals with the development of biomechanical model in order to simulate a man walking over the elastic bridge and the interaction of both objects. The current civil engineering technology enables to build bridges perfectly sustainable from mechanical point of view but with decreased eigenfrequencies that may lead bad human perception like anxiety or even seasickness. Therefore it is desirable to develop simulation models of the man-bridge interaction to be considered in the bridge design. This paper deals with the development of interaction model. The model must include the biomechanical model of human being, its control of human gait, the elastic bridge and the interaction between the biomechanical model and the bridge. The developed model is a planar one and it enables to simulate both the stable and unstable human gait.*

1. Introduction

The current improved technology of civil engineering can build bridges for human beings that satisfy all mechanical objectives of strength, reliability and lifetime, but with decreased amount of material. The consequence is the cost saving but also the decreased eigenfrequencies that result into the danger of bad human perception like anxiety or even seasickness. An example of such problems was the story of the Millenium Bridge in London.

In order to improve the design of such elastic bridges the phenomena of the man-bridge interaction is to be considered. Based on that this paper describes the development of the suitable biomechanical model for the human gait and the corresponding suitable bridge model for the interaction. The suitable model is a complex mechatronical model. The model must include the biomechanical model of human being, its control of human gait, the elastic bridge and the interaction between the biomechanical model and the bridge.

2. Biomechanical model of human gait

The human being is modeled by the planar biomechanical multibody model [2] in the Fig. 1. The concept of this model has been taken from [1]. The model consists of 8 bodies including the frame and it has 9 DOFs. The multibody model is described by 21 physical coordinates

$$\mathbf{s} = [x_2, y_2, \varphi_2, x_3, y_3, \varphi_3, x_4, y_4, \varphi_4, x_5, y_5, \varphi_5, x_6, y_6, \varphi_6, x_7, y_7, \varphi_7, x_8, y_8, \varphi_8]^T \qquad (1)$$

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that describes the position of the centers of mass of particular bodies and the orientation of the local coordinate systems firmly attached to each body at the center of mass [2, 3]. These physical coordinates are constrained by the kinematical constraints describing the connection of bodies in the revolute joints. If the revolute joint A that connects the body i and j with the coordinates $[x_{iA}, y_{iA}], [x_{jA}, y_{jA}]$ of the center A of revolute joint in the local coordinate systems then there are two constraints

$$\begin{aligned} x_i + x_{iA}\cos\varphi_i - y_{iA}\sin\varphi_i &= x_j + x_{jA}\cos\varphi_j - y_{jA}\sin\varphi_j \\ y_i + x_{iA}\sin\varphi_i + y_{iA}\cos\varphi_i &= y_j + x_{jA}\sin\varphi_j + y_{jA}\cos\varphi_j \end{aligned} \tag{2}$$

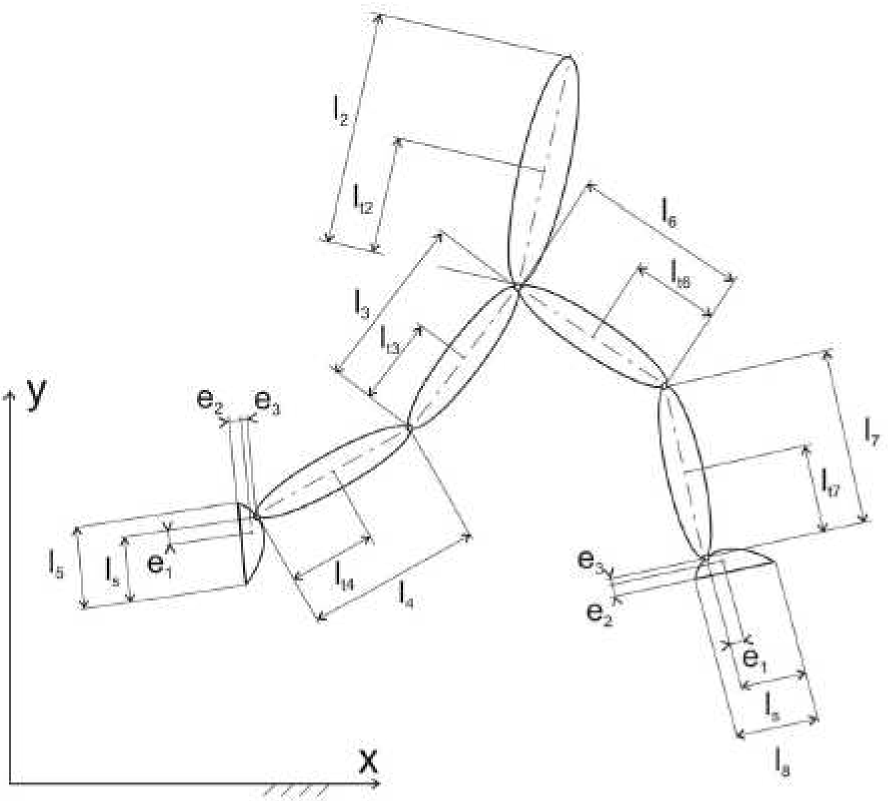

Fig. 1 The planar biomechanical model of human being

There altogether 6x2=12 such constraints [2]. The resulting equations of motion are the Lagrange equations of mixed type [3]

$$\frac{d}{dt}\frac{\partial E_k}{\partial \dot{s}_j} - \frac{\partial E_k}{\partial s_j} = Q_j + \sum_k \lambda_k \frac{\partial f_k}{\partial s_j} \tag{3}$$

where E_k is the kinetic energy, Q_j are the generalized forces, λ_k are the Lagrange multipliers corresponding to the constraints f_k (2). The generalized forces are derived from the acting forces. It is considered that there are the acting forces of the gravity, the control torques in the joints that correspond to the human muscles driving the human joints (Fig. 2) and the forces of the interaction with the foundation (Fig. 3). The interaction forces R_{xl}, R_{yl} with the foundation (Fig. 3) are equivalent to the resulting moments M_{rl}. The vertical forces R_{yl} are computed from the substitutive model of the interaction with the foundation by the unilateral springs and dampers (Fig. 3)

$$R_{yl} = -ky_{rel,l} - b\dot{y}_{rel,l} \tag{4}$$

where k and b are the substitutive stiffness and damping coefficient of the contact with the foundation and $y_{rel,l}$ are the relative coordinates in the contact. The horizontal forces R_{xl} are computed as the friction forces

$$R_{xl} = -\mu \left| R_{yl} \right| sign(\dot{x}_{rel,l}) \tag{5}$$

where μ is the Coulomb friction coefficient and $\dot{x}_{rel,l}$ are the relative slip velocities in the contact. In order to avoid the limit cycle around the equilibrium another friction model is introduced for very small relative slip velocities

$$R_{xl} = -g\mu \left| R_{yl} \right| \dot{x}_{rel,l} \tag{6}$$

where g is a suitable gain [2].

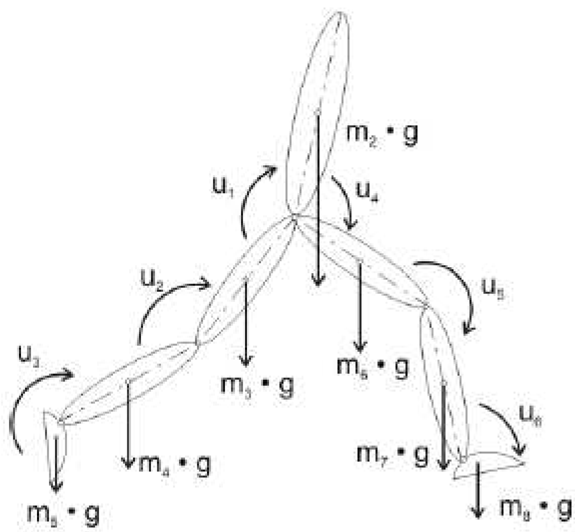

Fig. 2 The gravity and control torques acting on the planar biomechanical model

For the control synthesis the equations of motion (3) are transformed into the independent ones. The independent coordinates are selected out of the dependent ones (1). They consist of the description of the position of the human torso x_2, y_2, φ_2 and the relative coordinates in the joints ψ_i (Fig. 4). There are 9 independent coordinates

$$\mathbf{q} = [x_2, y_2, \varphi_2, \psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6]^T \tag{7}$$

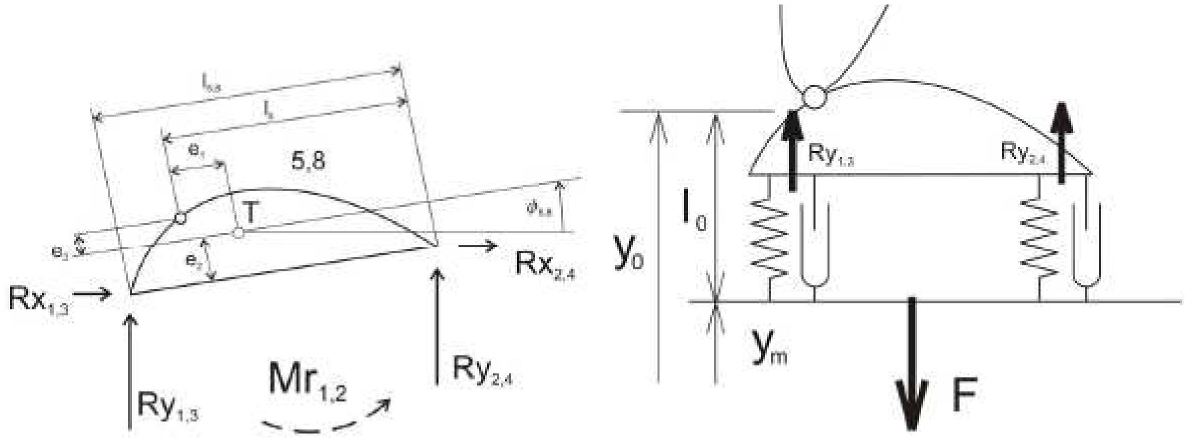

Fig. 3 The model of the interaction with the foundation

The dependent coordinates ***s*** in (1) can be expressed as the function of the independent ones ***q*** from (7), i.e. there is an inverse kinematical function

$$\mathbf{s} = \mathbf{r}(\mathbf{q}) \tag{8}$$

and by the time differentiation of (8) it is derived

$$\begin{aligned}\dot{\mathbf{s}} &= \mathbf{R}\dot{\mathbf{q}}\\ \ddot{\mathbf{s}} &= \mathbf{R}\ddot{\mathbf{q}} + \dot{\mathbf{R}}\dot{\mathbf{q}}\end{aligned} \tag{9}$$

Then the resulting independent equations of motion are [3]

$$\mathbf{R}^T\mathbf{M}\mathbf{R}\ddot{\mathbf{q}} = \mathbf{R}^T(\mathbf{Q} - \mathbf{M}\dot{\mathbf{R}}\dot{\mathbf{q}}) \tag{10}$$

where **M** is the mass matrix derived from the equation (3).

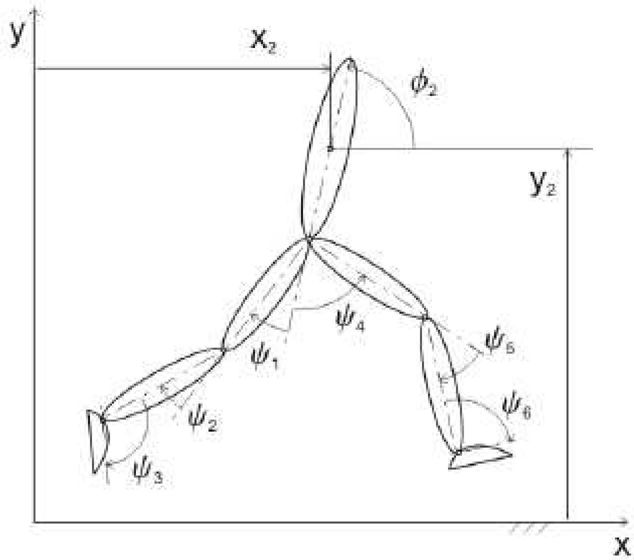

Fig. 4 The independent coordinates of the planar biomechanical model

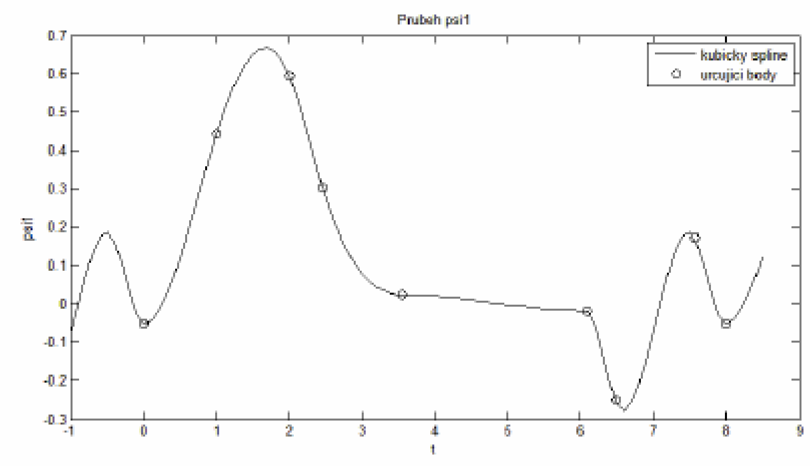

Fig. 5 The description of one periodic footstep as the spline function of the time

The fundamental problem is the way how to determine the necessary control torques u_i, i=1,…, 6 from Fig. 2. They are determined from the inverse dynamical solution of the equations of motion (10) in order to realize the periodic human footstep [1, 2]. The suitable time behaviours of the independent coordinates ψ_i are described as the spline functions of the time in the form (Fig. 5)

$$\psi_i = B_{0i} + B_{1i}t + B_{21i}t^2 + B_{3i}t^3 \qquad (11)$$

Finally the behaviour of the model is parametrized by the supporting points B_{mi} (Fig. 5).

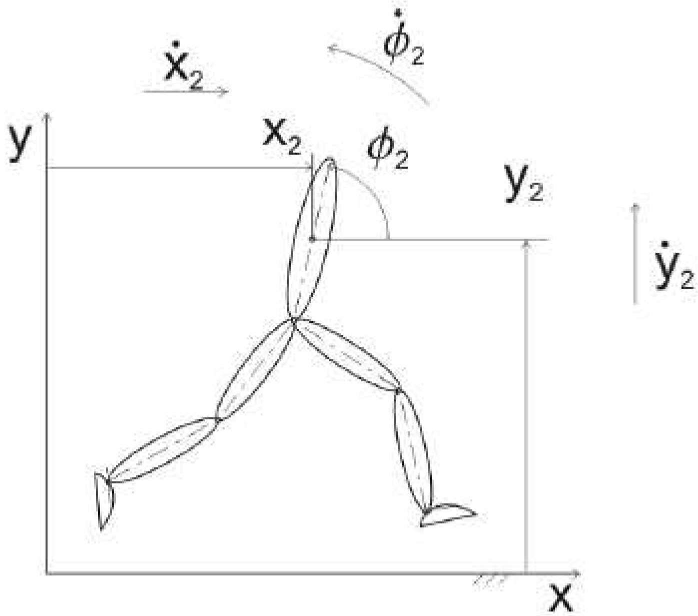

Fig. 6 The variables for the optimization performance index

The initial values of the supporting points B_{mi} have been determined after many attempts based on the observation of real human gait. Such footstep was kinematically acceptable but it has many drawbacks from the point of view of kinematics and dynamics. From the point of view of kinematics it has especially suffered from the not fully satisfied periodicity of the human footstep and from the point of view of dynamics it has especially suffered from the excessive swinging of the human torso during the footstep. The final plan for the human footstep has been determined by the optimization of the supporting points B_{mi} in order to minimize the objective performance index [2] (see Fig. 6)

$$J = (c\Delta y_2)^2 + (c\Delta \dot{y}_2)^2 + (c\Delta \dot{x}_2)^2 + (c\Delta \varphi_2)^2 + (c\Delta \dot{\varphi}_2)^2 + (\max(\varphi_2) - \min(\varphi_2)) \qquad (12)$$

with suitable gain coefficient c.

The optimization has been performed by the genetic optimization using [4]. The optimization process is depicted in the Fig. 7. During the design process of the footstep two versions of human gait have been determined. One is the gait by putting one's leg together and the other one is without putting one's leg together. There are many solutions of the problem of footstep design that corresponds to the individuality of the human gait. An example of resulting successful gait is in Fig. 11.

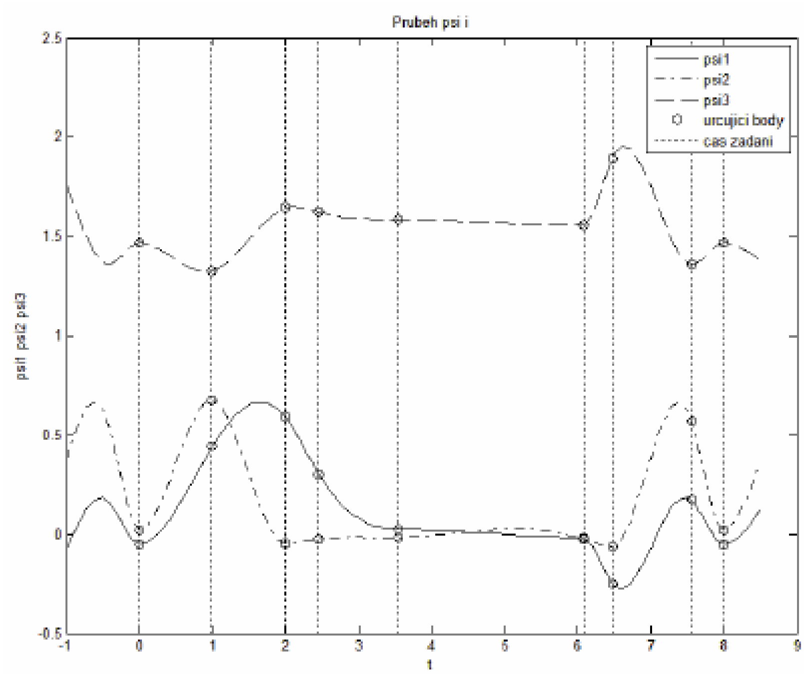

Fig. 7 The optimization of one periodic footstep by the change of supporting points

The derived human gait can operate only within static environment (foundation). The human being can adjust his behaviour according to the unexpected deviation in the environment (foundation). It is realized by the feedback balancing by the control torques in the joints. This has been considered by the introduction of PD feedback law

$$u_i = u_i(\psi_{m,des}) + k_{i1}(\psi_i - \psi_{i,des}) + k_{i2}(\dot{\psi}_i - \dot{\psi}_{i,des}) \qquad (13)$$

with suitable gains k_{i1}, k_{i2} where $\psi_{i,des}$ are the desired preplanned footstep time functions from Fig. 5 and Fig. 7.

3. Model of an elastic bridge

The elastic bridge has been considered based on the structure in Fig. 8 where a is the length of the footstep in order to simplify the computation of the interaction forces. The particular members of the structure have been modeled as beam elements possessing both axial and bending deformations. The resulting dynamical structural model of the elastic bridge has been derived using FEM approach

$$\mathbf{M}_s \ddot{\mathbf{w}} + \mathbf{B}_s \dot{\mathbf{w}} + \mathbf{K}_s \mathbf{w} = \mathbf{F} \tag{14}$$

where $\mathbf{w}$ is the vector of nodal coordinates of the FEM model, $\mathbf{M}_s$, $\mathbf{B}_s$, $\mathbf{K}_s$ are the mass, damping and stiffness matrices of the structure in Fig. 8 and $\mathbf{F}$ is the vector of interaction forces between the elastic bridge and the walking man. The bridge structure has been chosen with a=0.55m and the first eigenfrequency about 10Hz.

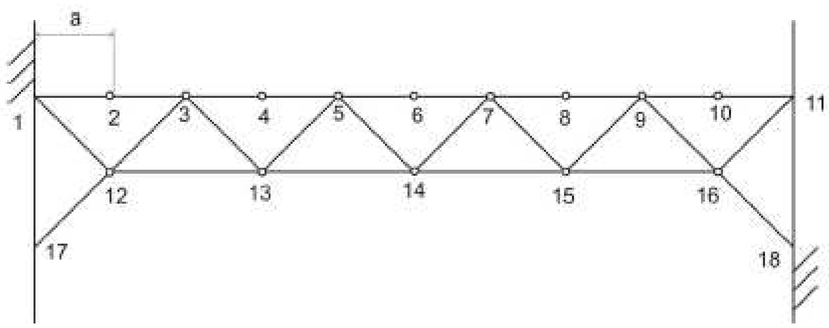

Fig. 8 The model of an elastic bridge

4. Simulation experiments of the interaction

Using the previous models the overall model of the man-bridge interaction has been assembled (Fig. 9). Many different computational experiments have been conducted.

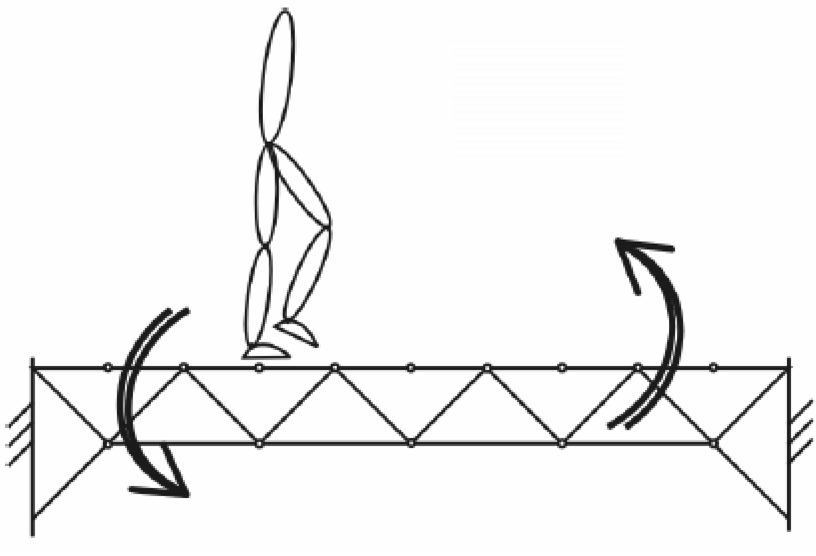

Fig. 9 The overall model of the man-bridge interaction

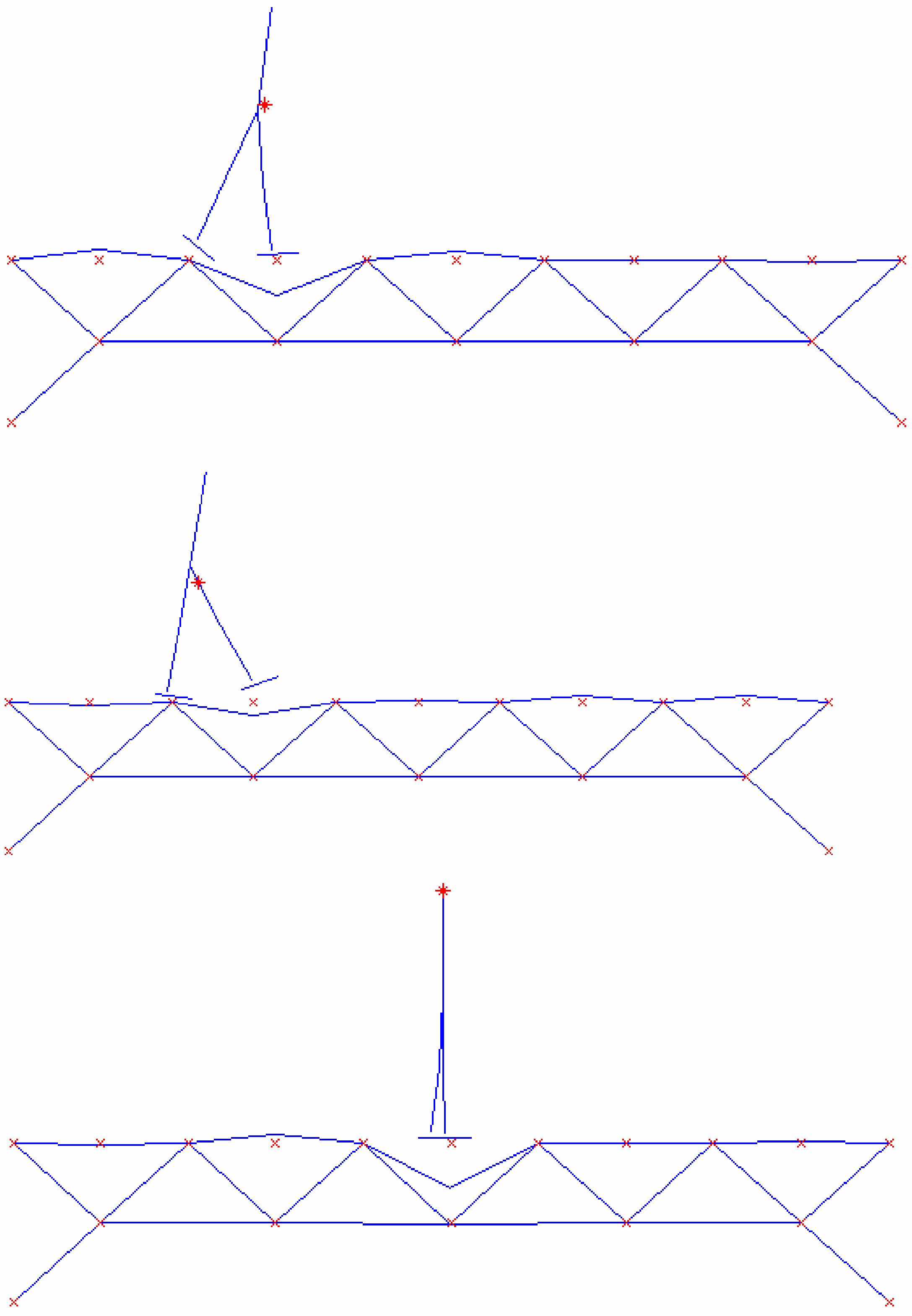

Fig. 10 The human gait with putting one's leg together on the elastic bridge

First, the human gait with putting one's leg together on the elastic bridge is on Fig. 10. Second, the human gait without putting one's leg together on the elastic bridge is on Fig. 11. Third, the difference between the behaviour of the human gait on the firm foundation and on the elastic bridge has been investigated. The result is in Fig. 12. Finally, the influence of the stiffness of the elastic bridge on the stability of the human gait has been investigated. The increased compliance of the bridge can destabilized the human gait despite the stabilizing feedback control law (13). This is demonstrated in Fig. 13 and Fig. 14.

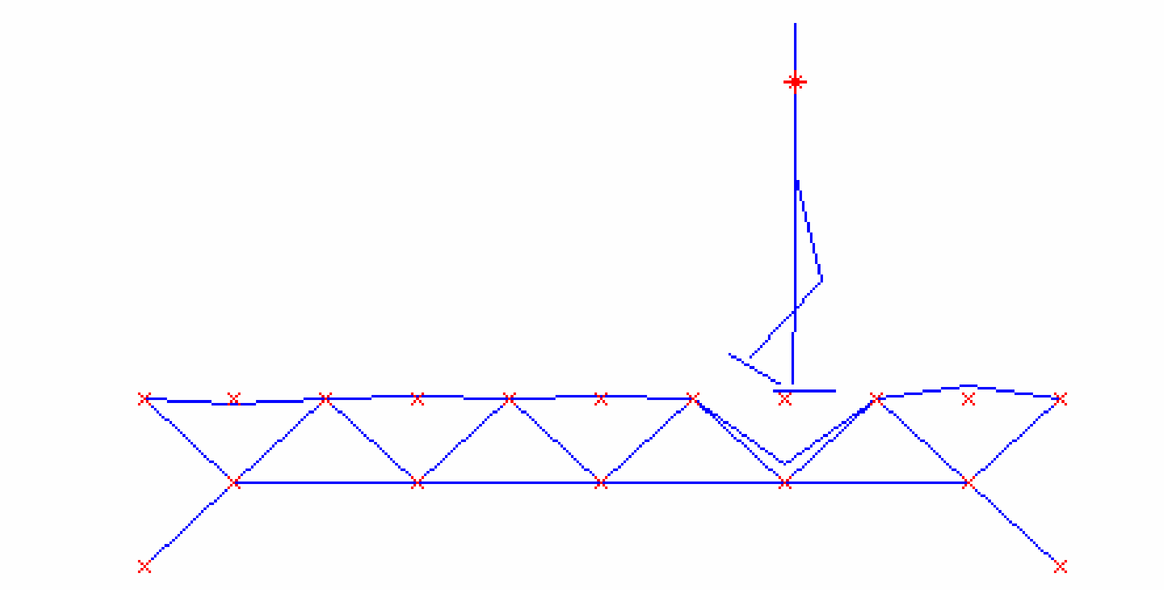

Fig. 11 The human gait without putting one's leg together on the elastic bridge

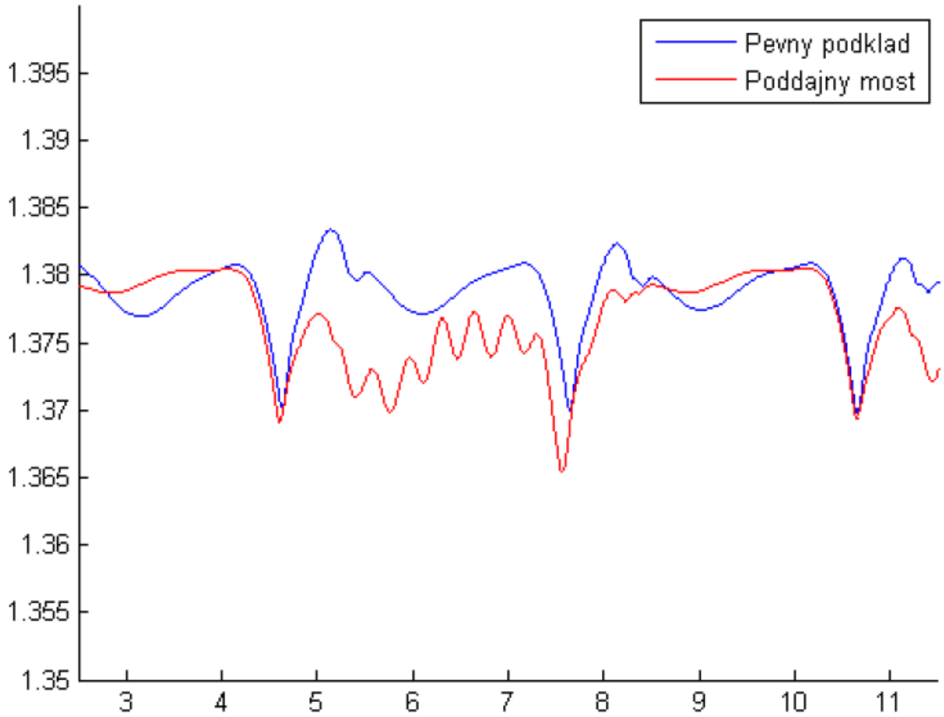

Fig. 12 The influence of the bridge elasticity on the human gait (y position of human torso)

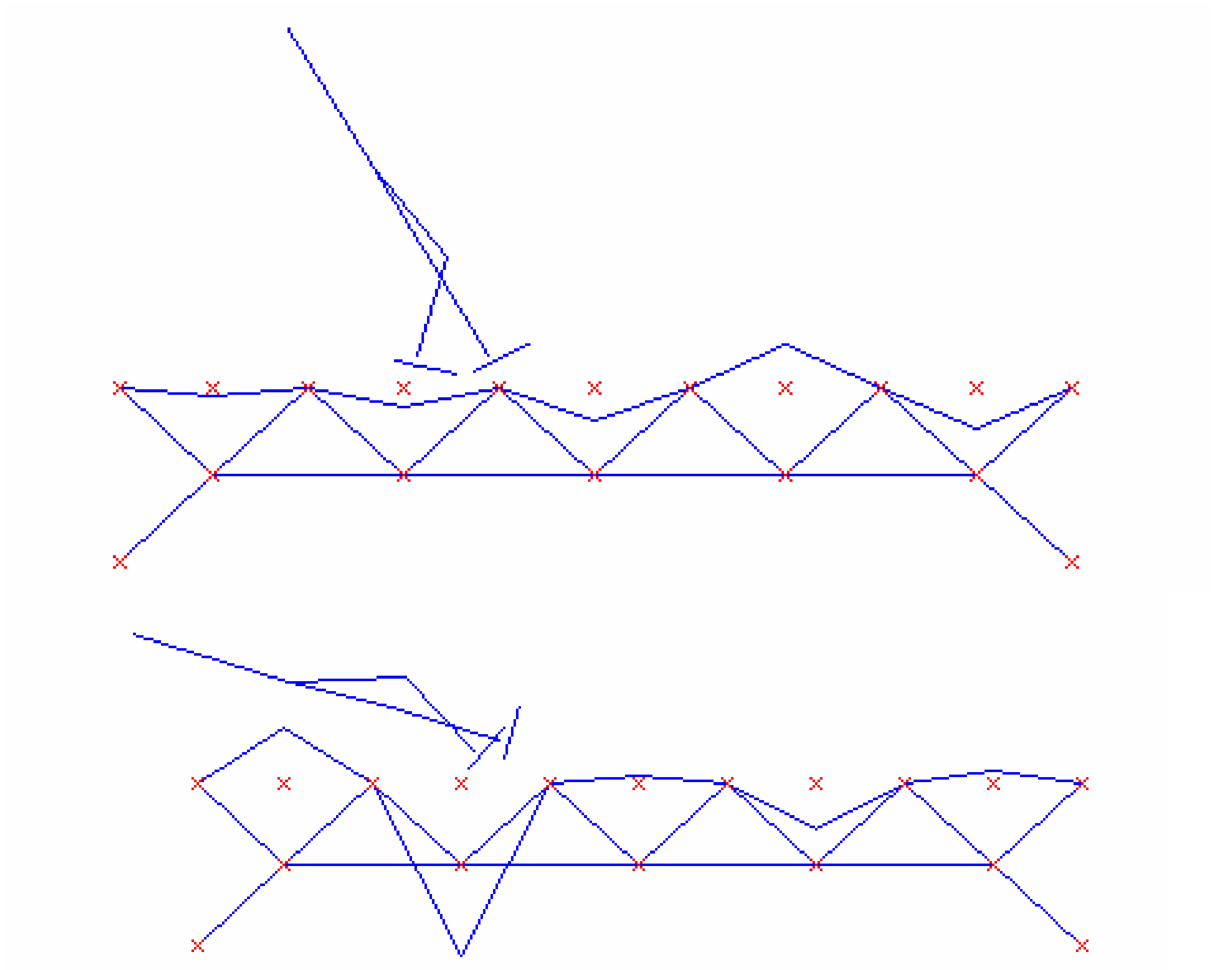

Fig. 13 The destabilization of the human gait by the increase of the bridge compliance

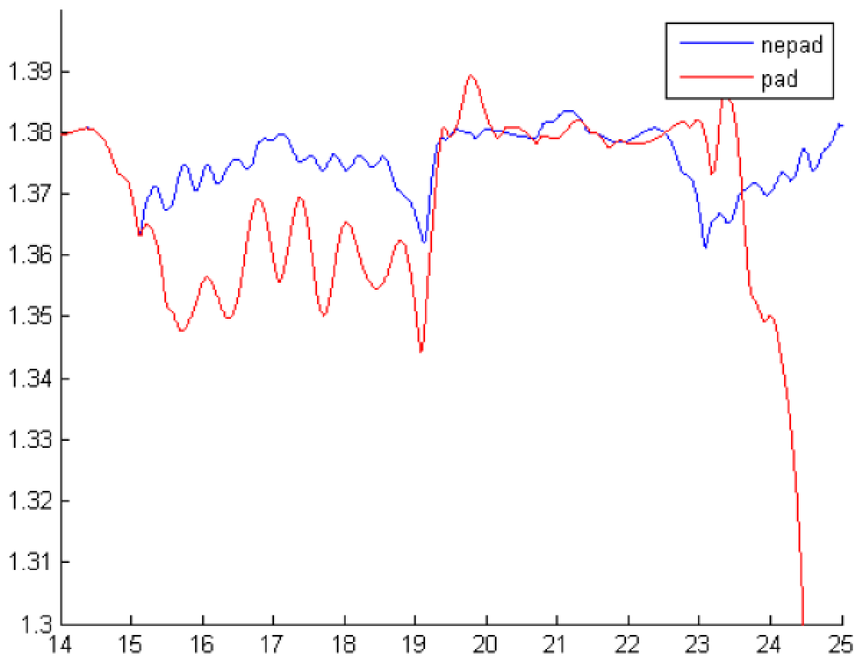

Fig. 14 The comparison of stable and unstable human gait due to the bridge compliance

5. Conclusions

The model of the man-bridge interaction including the bridge compliance has been developed. It enables to investigate many different phenomena starting with the different human gaits, different stabilizing feedback laws and ending with the influence of bridge design on the human perception.

Acknowledgment: The authors appreciates the support by the project ”Development of methods and tools of computational simulation and its applications in engineering” No. MSM 6860770003.

6. References

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